

ONLINE APPROXIMATORS FOR THE ACTUATORS FDI OF A SMALL COMMERCIAL AIRCRAFT

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Abstract: In this paper the problem of detecting and isolating faults on the actuators of a small commercial aircraft using online approximators is dealt with. In particular the capabilities of radial basis neural networks to approximate faults on the engines and elevators of an AP-68 Vulcanair commercial aircraft are investigated also in the presence of model uncertainties and external disturbances. The structure of the aircraft and of the fault models appears to be well suited for this kind of application. Although some effort has to be spent in tuning a certain number of parameters, the numerical results obtained on a full nonlinear Matlab-Simulink simulator of the aircraft seem to be encouraging. *Copyright©2005 IFAC.*

Keywords: Fault detection and isolation, Fault approximation, Aircraft control, Nonlinear adaptive estimators, Radial base function networks.

1. INTRODUCTION

The application of FDI (Fault Detection and Isolation) concepts to aircrafts is mainly concentrated on sensors and actuators which are subsystems strictly connected to the flight control system. Most of the current research activities are focused on the so-called analytical redundancy techniques making use of a priori information on the system behavior to detect and isolate faults. If a "reliable" model of the system is available, a possible way to achieve FDI goals is to use a bank of dynamic observers to generate residuals for each monitored system output signal. A decision making system is then designed and tuned to detect and isolate faults. Different observer based techniques have been studied in the

literature: Kalman filtering, *Thau* observers, *parity space* approaches, *dedicated observers*, H_2/H_∞ based techniques and others (Chen and Patton, 1999), (Frank and Seliger, 2000). Several works are focused on actuator FDI: unknown input observer for disturbance decoupling (Park *et al.*, 2000), (Yaz and Azemi, 1998), PI observer for detection of unknown actuator faults (Linder *et al.*, 1998), adaptive observers based techniques (Wang and Daley, 1996), H_∞ techniques (Marcos *et al.*, 2004).

FDI problems are particularly challenging in aircraft which are generally modelled as multi-input multi-output systems with nonlinearities, uncertainties and external disturbances. Satisfactory answers have been given in the case that the aircraft is modelled as a linear time invariant plant (Liao *et al.*, 2000), (Persis *et al.*, 2001). However, in the presence of large maneuvers within the operating envelope with robustness and disturbance

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rejection requirements, the adaptation of linear techniques to the nonlinear context may require practical solutions needing large experience on the particular problem which is dealt with.

Considerable research activity aimed at the design and analysis of observer based actuator fault diagnosis schemes specific for nonlinear systems has been produced recently. An interesting field of research concerning the use of nonlinear observers in conjunction with adaptive and online approximation methods is described in (Vemuri and Polycarpou, 1998), (Zhang *et al.*, 2002) and the bibliography therein.

In the present paper an experience on the application of the above mentioned techniques for FDI on the actuators of a small commercial aircraft is described. In particular abrupt and incipient faults on engines and elevators are considered. Both identification and isolation problem are treated. The numerical results obtained on a model of a AP-68 Vulcanair commercial aircraft show satisfactory performance also in the presence of aerodynamic and mass uncertainties and of atmospheric disturbances.

2. THE AIRCRAFT MODEL AND THE ACTUATOR FAULTS

The mathematical model of the aircraft dynamics can be written as follows (Stevens and Lewis, 1992)(see Table 1 for the list of symbols):

$$WV \cos \beta \dot{\alpha} = -T \sin(\alpha + \mu_T) - L \\ + WVq + Wg_1 \quad (1a)$$

$$W\dot{V} = T \cos(\alpha + \mu_T) \cos \beta - D + Wg_2 \quad (1b)$$

$$VW\dot{\beta} = -T \cos(\alpha + \mu_T) \sin \beta + Y + \\ - WVr + Wg_3 \quad (1c)$$

$$J_x \dot{p} - J_{xz} \dot{r} = R + qr(J_y - J_z) + pqJ_{xz} \quad (1d)$$

$$J_y \dot{q} = M + rp(J_z - J_x) + (r^2 - p^2)J_{xz} \quad (1e)$$

$$-J_{xz} \dot{p} + J_z \dot{r} = N + pq(J_x - J_y) - qrJ_{xz} \quad (1f)$$

$$\dot{\phi} = p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \quad (1g)$$

$$\dot{\theta} = q \cos \phi - r \sin \phi \quad (1h)$$

$$\dot{\psi} = r \cos \phi \sec \theta + q \sin \phi \sec \theta \quad (1i)$$

where:

$$g_1 = g(\sin \alpha \sin \theta + \cos \alpha \cos \phi \cos \theta) \\ g_2 = g(-\cos \alpha \cos \beta \sin \theta + \sin \beta \sin \phi \cos \theta + \\ + \sin \alpha \cos \beta \cos \phi \cos \theta) \\ g_3 = g(\cos \alpha \sin \beta \sin \theta + \cos \beta \sin \phi \cos \theta + \\ - \sin \alpha \sin \beta \cos \phi \cos \theta),$$

and:

$$D = \frac{1}{2} \rho V^2 S C_D \quad L = \frac{1}{2} \rho V^2 S C_L \\ Y = \frac{1}{2} \rho V^2 S C_Y \quad R = \frac{1}{2} \rho V^2 S b C_R \\ M = \frac{1}{2} \rho V^2 S \bar{C}_M \quad N = \frac{1}{2} \rho V^2 S b C_N$$

The aerodynamic coefficients can be expressed as the sum of a number of terms depending on different variables. For the longitudinal dynamics, the lift, drag, and pitch moment coefficients, neglecting the contribute of the Reynolds number and of the other control surfaces as flaps, slats, speed brakes, can be written as:

$$C_L = C_L(q, \alpha, \dot{\alpha}, Mach, \delta_e) \\ C_D = C_D(q, \alpha, \dot{\alpha}, Mach, \delta_e) \\ C_M = C_M(q, \alpha, \dot{\alpha}, Mach, \delta_e).$$

As for the main uncertainties affecting the aircraft we have aerodynamic and mass properties uncertainties. The aerodynamic uncertainties are introduced as multiplicative perturbations affecting some of the addenda composing the total coefficients. For example the lift coefficient can be written as:

$$C_L = C_L(q, \alpha, \dot{\alpha}, Mach, \delta_e) = C_{L\text{basic}}(\alpha, Mach) + \\ + C_{L\alpha}(\alpha, Mach)(1 + \Delta C_{L\alpha})\alpha + \\ + C_{L\delta_e}(\alpha, Mach, \delta_e)(1 + \Delta C_{L\delta_e})\delta_e + \dots \quad (2)$$

where the ellipses denotes further terms which do not explicitly depend on the uncertainties. In the lift (C_L) case we have two perturbed coefficients, that is the uncertain contribution due to the elevator and to the angle of attack; these uncertainties introduce a percentage variation of the nominal value of the terms they affect. Similar uncertainties have to be considered for the other aerodynamic coefficients.

The aircraft mass and the centre of gravity (CG) position are also uncertain in different ways. First, during a given flight, the mass varies due to fuel consumption; also, the mass distribution may vary with the consequence of changing the position of the aircraft CG. Moreover the aircraft mass and its distribution may vary as different flights are considered. Therefore the aircraft weight and the CG coordinates are uncertain parameters which also strongly affects some of aerodynamic derivatives ($C_{M\alpha}$).

In our application we cope with two kind of faults, an abrupt loss of the engine thrust and a slow increase of the angular position of the elevator.

Both fault functions are assumed to belong to a known finite set of possible nonlinear fault functions:

$$\mathcal{F} \triangleq \{\Phi^1(x, u), \Phi^2(x, u)\} \quad (3)$$

where each $\Phi^s(x, u)$, $s = 1, 2$, describing the fault profiles, is described by:

$$\Phi^s(x, u) \triangleq \left[(\theta_1^s)^T g_1^s(x, u), \dots, (\theta_n^s)^T g_n^s(x, u) \right]^T,$$

$\theta_i^s \in \Re^{q_i^s}$, $i = 1, \dots, n$, being an unknown parameter vector that belong to a known compact set $\Xi_i^s \subset \Re^{q_i^s}$ and $g_i^s(x, u) \in \Re^n \times \Re^m \rightarrow \Re^{q_i^s}$ being a known smooth vector field. With the above notation it is possible to characterize a general class of faults where g_i^s is the functional structure of the s -th fault affecting the i -state equation and θ_i^s the magnitude of the fault. If we assume $u = (\delta_e \ \delta_T \ \delta_a \ \delta_r \ \delta_f)^T$, $y = x = (V \ \alpha \ q \ \theta \ p \ r \ \phi \ \beta)^T$, equation (1) can be readily compacted in the following form to take into account the faulty situations:

$$\dot{x} = f(x, u) + \xi(x, u, t) + \beta(t - t_0)\Phi(x, u) \quad (4)$$

where $x \in \Re^n$ is the state vector, $x \in \Re^m$ is the input vector, and $\beta(t - t_0)$ is a matrix function representing the time behavior of the faults, t_0 being the unknown fault occurrence time.

The vector fields f, Φ, ξ are the aircraft dynamics, the change in the system dynamics due to a fault occurrence and the modeling uncertainty functions, respectively.

To assure some stability properties (see (Zhang *et al.*, 2002)) the modeling uncertainties represented by the vector field ξ are assumed to be bounded by some known functional:

$$|\xi_i(x, u, t)| \leq \bar{\xi}_i(x, u, t), \quad i = 1, \dots, n$$

$$\forall (x, u) \in \mathcal{D}, \quad \forall t \geq 0$$

\mathcal{D} being an assigned stability region.

For our application we choose the fault time profile $\beta(\cdot)$ to be a diagonal matrix of the form:

$$\beta(t - t_0) \triangleq \text{diag}\left(\beta_1(t - t_0), \dots, \beta_n(t - t_0)\right) \quad (5)$$

where:

$$\beta_i(t - t_0) = \begin{cases} 0 & \text{if } t < t_0 \\ 1 - \exp(-m_i(t - t_0)) & \text{if } t \geq t_0 \end{cases}$$

$$i = 1, \dots, n.$$

The positive scalars m_i in the above functions are the so-called fault evolution rates. A value of m_i very close to zero is used to characterize a sudden loss of thrust (abrupt faults) whereas incipient faults on the elevators are simulated with higher values of m_i .

3. THE FDI TECHNIQUE BASED ON THE ON-LINE APPROXIMATORS

The basic idea for process fault detection and isolation developed in (Zhang *et al.*, 2002) and the bibliography therein is illustrated in figure 1. In our application case, a bank of three nonlinear adaptive estimators is needed: the first one for the detection (Fault Detection and Approximation Estimator, FDAE), the remaining two (Fault Isolation Estimators, FIEs) for the isolation of

faults on engines and elevators respectively. As for the FDAE, the following structure adopted:

$$\dot{x} = f(x, u) + \hat{\Phi}(x, u, \hat{\theta}) + L(x - \hat{x}) \quad (6)$$

where $\hat{x} \in \Re^n$ is the estimated state vector, $\hat{\Phi}(\cdot, \cdot, \cdot)$ represents an on-line approximation model of the fault effect, $\hat{\theta} \in \Re^q$ being a vector of adjustable parameters. L is a constant square stability matrix of dimension n . The initial value of the estimated parameter vector $\hat{\theta}(0) = \hat{\theta}^0$ is chosen such that $\hat{\Phi}(x, u, \hat{\theta}^0) = 0$ for all $(x, u) \in \mathcal{D}$. An on-line approximator to adjust the parameter

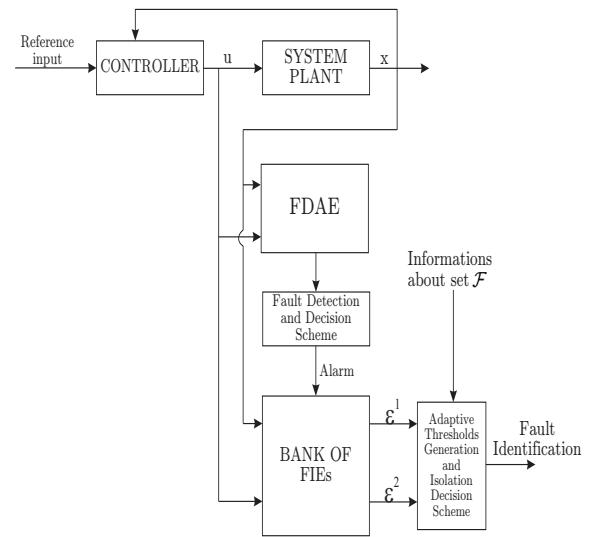


Fig. 1. FDI scheme proposed.

estimate $\hat{\theta}$ at each time t so that $\hat{\Phi}(x, u, \hat{\theta})$ approximates the unknown function $\beta(t - t_0)\Phi(x, u)$ as well as possible has to be chosen.

If a linear parameterization (Hertz *et al.*, 1991) of the function $\hat{\Phi}(x, u, \hat{\theta})$ is adopted:

$$\hat{\Phi}(x, u, \hat{\theta}) = \hat{\Psi}(x, u)^T \hat{\theta} \quad (7)$$

with $\hat{\Psi}(x, u) \in \Re^{n \times m}$ sufficiently smooth, some stability properties for the learning scheme described by (6) can be assured (Polycarpou and Helmicki, 1995).

As for the choice of the on-line approximation function $\hat{\Phi}(x, u, \hat{\theta})$, in our application case, we adopted a Radial-Basis-Function (RBF) based neural network whose output is in the form:

$$\hat{\Phi}(x, u, \hat{\theta}) = \left\{ \sum_{i=1}^n \hat{\theta}_i \psi_i(x, u) : \hat{\theta}_i \in \Re \right\} \quad (8)$$

ψ_i being Gaussian functions:

$$\psi_i(x, u) = e^{-\frac{\left(\begin{bmatrix} x \\ u \end{bmatrix} - c_i\right)^2}{\sigma_i^2}}$$

where the Gaussian's centers c_i and widths σ_i are tunable parameters.

As for the learning algorithm for updating the weights $\hat{\theta}$, if $\epsilon = x - \hat{x}$ is the state estimation error it is possible to adopt the following in the continuous time:

$$\dot{\hat{\theta}} = \Pi_{\Xi} \{ \Gamma Z^T \epsilon \} \quad (9)$$

where the projection operator Π_{Ξ} (Ioannou and Sun, 1995), (Polycarpou and Helmicki, 1995) restricts the parameter estimation vector $\hat{\theta}$ to a predefined convex set $\Xi \subset \Re^p$, $\Gamma = \Gamma^T > 0$ is the learning rate matrix, and $Z \triangleq \partial \hat{\Phi} / \partial \hat{\theta}$ is the gradient matrix of the online approximator with respect to its adjustable weights. From Lyapunov theory (Ioannou and Sun, 1995) some stability properties of this learning algorithm can be proven.

A fault is declared when state estimation errors exceed a certain fixed threshold $\bar{\epsilon}$ (see Section 4 for the choice of its value):

$$\begin{cases} |\epsilon_i(t)| \leq \bar{\epsilon}_i \forall i \in \{1, \dots, n\} & \text{healthy operation} \\ |\epsilon_i(t)| > \bar{\epsilon}_i \text{ for some } i & \text{faulty operation} \end{cases}$$

For isolation purposes two instances of the following nonlinear adaptive estimators are used: one for the engine faults and the other for the elevator faults.

$$\dot{\hat{x}}^s = f(x, u) + \hat{\Phi}^s(x, u, \hat{\theta}^s) + L^s(x - \hat{x}^s) \quad (10)$$

with:

$$\hat{\Phi}^s(x, u, \hat{\theta}^s) = \left[(\hat{\theta}_1^s)^T g_1^s(x, u), \dots, (\hat{\theta}_n^s)^T g_n^s(x, u) \right]^T$$

where $\hat{\theta}_i^s \in \Re^{q_i^s}$ ($i = 1, \dots, n$, $s = 1, 2$) is the estimate of the s -th fault parameter vector in the i -th state variable, and L is a square stability matrix of dimension n .

The design procedure for the two FIEs is quite similar to that adopted for the FDAE.

The first isolation estimator is oriented to the identification of abrupt faults on engine thrust, the second one to the identification of incipient faults on the position of the elevators.

Also in this case a learning algorithm for updating $\hat{\theta}_i^s$ can be derived using the Lyapunov synthesis approach (Ioannou and Sun, 1995). If we let $\epsilon_i^s = x_i - \hat{x}_i^s$ be the i -th component of the state estimation error vector of the s -th estimator, the learning algorithm becomes:

$$\dot{\hat{\theta}} = \Pi_{\Xi_i^s} \{ \Gamma_i^s g_i^s(x, u) \epsilon_i^s \} \quad (11)$$

where $\Gamma_i^s = \Gamma_i^{sT} > 0$ is a symmetric, positive definite learning rate matrix.

Once a fault is detected by FDAE the bank of FIEs is activated providing information to the decision-logic block.

If the s -th fault occurs and is detected at time t_d the i -th component of the estimation error associated with the s -th FIE satisfies $|\epsilon_i^s(t)| \leq \mu_i^s(t) \forall i \in \{1, \dots, n\}$ and for $t \geq t_d$ while for each $r \in \{1, 2\}/\{s\}$ there exist some finite time

$t^r > t_d$ and some $i \in \{1, \dots, n\}$ such that $|\epsilon_i^r(t^r)| > \mu_i^r(t^r)$; $\mu_i(t)$, $i = 1, \dots, n$ is a set of adaptive thresholds that clearly play a crucial role in the fault isolation scheme. A detailed procedure to compute non conservative thresholds associated with the residual of each fault isolation estimator is shown in (Zhang *et al.*, 2002). As the estimate $\hat{\theta}_i^s$ belongs to a known compact set Ξ_i^s , e.g. a hypersphere with center O_i^s and radius R_i^s , we have $|\theta_i^s - \hat{\theta}_i^s| \leq R_i^s + |\hat{\theta}_i^s - O_i^s|$ and the following threshold function for fault isolation is chosen:

$$\begin{aligned} \mu_i^s(t) = & \int_{t_d}^t e^{-\lambda_i(t-\tau)} \\ & \cdot \left[\left(k_i^s(\tau) + e^{-\bar{m}_i(t-t_d)} |\hat{\theta}_i^s(\tau)| \right) \right. \\ & \cdot \left. |g_i^s(x(\tau), u(\tau))| + \bar{\xi}(x(\tau), u(\tau), \tau) \right] d\tau \\ & + |\epsilon_i^s(t_d)| e^{-\lambda_i(t-t_d)} \end{aligned} \quad (12)$$

where $k_i^s(t) = R_i^s + |\hat{\theta}_i^s - O_i^s|$, \bar{m}_i being a known lower bound tuning parameter for the unknown fault evolution rate m_i .

4. SIMULATION RESULTS ON THE AP-68 VULCANAIR COMMERCIAL AIRCRAFT

In order to apply the FDI technique described in the previous section we firstly had to rewrite the aircraft model equations to fit the form (4). One of the main advantages in using the proposed technique is that it can take into account the nonlinear function mapping faults directly into the system dynamics.

For the sake of presentation simplicity we focus on equation (1a) that can be rewritten as:

$$\begin{aligned} \dot{\alpha} = & \frac{1}{\cos \beta} \left[q + \frac{g}{V} \left(\cos \alpha \cos \theta \cos \phi + \sin \alpha \sin \theta \right) \right. \\ & - \frac{\bar{q}S}{WV} \left(C_{Lbasic} + C_{L\alpha}\alpha + \dots \right) - \frac{\bar{q}S}{WV} C_{L\delta_e} \delta_e + \\ & \left. - \frac{T}{WV} \sin(\alpha + \mu_T) \right] + \xi_1(x, u, t) \end{aligned} \quad (13)$$

where $\bar{q} = \rho V^2 / 2$ and ξ_1 represents the uncertain term. This equation can be rewritten as:

$$\begin{aligned} \dot{\alpha} = & \frac{1}{\cos \beta} \left[q + \frac{g}{V} \left(\cos \alpha \cos \theta \cos \phi + \sin \alpha \sin \theta \right) \right. \\ & - \frac{\bar{q}S}{WV} \left(C_{Lbasic} + C_{L\alpha}\alpha + \dots \right) - \frac{\bar{q}S}{WV} C_{L\delta_e} (\delta_e + \\ & \left. + F_{\delta_e}) - \frac{T(1+F_T)}{WV} \sin(\alpha + \mu_T) \right] + \xi_1(x, u, t) \end{aligned} \quad (14)$$

The last two terms are explicitly affected by actuator faults on engine and elevator, F_T and F_{δ_e} respectively. Rewriting (14) we have:

$$\dot{\alpha} = \frac{1}{\cos \beta} \left[q + \frac{g}{V} \left(\cos \alpha \cos \theta \cos \phi + \sin \alpha \sin \theta \right) - \frac{\bar{q}S}{WV} \left(C_{L_{basic}} + C_{L\alpha}\alpha + \dots \right) - \frac{\bar{q}S}{WV} C_{L\delta_e} \delta_e + \right. \\ \left. - \frac{T}{WV} \sin(\alpha + \mu_T) \right] + \xi_1(x, u, t) + \\ - F_{\delta_e} \frac{\bar{q}SC_{L\delta_e}}{WV \cos \beta} - F_T \frac{\sin(\alpha + \mu_T)}{WV \cos \beta} T \quad (15)$$

We have the desired form

$$\dot{\alpha} = f_1(x, u) + \xi_1(x, u, t) + (\theta_1^1)^T g_1^1 + (\theta_1^2)^T g_1^2$$

with

$$\theta_1^1 \triangleq F_{\delta_e} \quad g_1^1 \triangleq \left[\frac{\bar{q}SC_{L\delta_e}}{WV \cos \beta} \right] \\ \theta_1^2 \triangleq F_T \quad g_1^2 \triangleq \left[\frac{\sin(\alpha + \mu_T)}{WV \cos \beta} \right] T.$$

To make the technique work on the proposed application a number of parameters had to be tuned both for FDAE and FIEs.

The FDAE required the calibration of the $\bar{\epsilon}_i$, ($i = 1, \dots, n$) thresholds. To this end a wide campaign of simulations in non-faulty operating conditions was performed in the presence of model uncertainties and external disturbances (atmospheric turbulence) and also in the presence of different manoeuvres (both longitudinal and latero-directional) exciting as much as possible the system nonlinearities. For the online approximator we also chose a uniform width $\sigma = 0.7$, 22 fixed centers c_i evenly distributed in the interval [-2 2] and $\Gamma = \gamma I$ with $\gamma = 10$.

For the FIEs the following parameters were fixed:

$$\Xi_i^1 = [0.05 \ 1], \quad \Xi_i^2 = [4 \ 20],$$

whereas

$$\Gamma^1 = \gamma^1 I, \quad \Gamma^2 = \gamma^2 I$$

with $\gamma^1 = 1$, $\gamma^2 = 0.01$.

Several simulations have been performed to test the above FDI scheme in faulty operating conditions also in the presence of model uncertainties and external disturbances. The simulations were performed in the Matlab/Simulink environment using a detailed full nonlinear model of the AP-68 Vulcanair commercial aircraft.

In Figure 2-5 the simulation results obtained for an abrupt fault on the engine and an incipient fault on the elevator are shown. Figures 2 and 3 show the effect of a 60% ($F_T = -0.6$, $m_2 = 0.9$) abrupt loss of engine thrust at time $t_0 = 20$ s on the FDAE and FIEs estimation errors. It can be seen that the estimation error, generated by the FIE which was designed for the identification of elevators faults exceeds the threshold, the other still providing an estimation error under the threshold. Figure 5 shows the effect of an incipient fault of 10 degrees of the elevator deflection angle ($F_{\delta_e} = 10$, $m_1 = 0.3$) at time $t_0 = 15$ s on the FIEs estimation errors.

5. CONCLUSIONS

The problem of detecting and isolating actuator faults, both incipient and abrupt kind, on the engines and elevators of an AP-68 Vulcanair commercial aircraft in the presence of external disturbances and uncertainties has been considered. A procedure to design a bank of observers with on line approximators to identify the kind of fault acting on the plant is described. A campaign of simulations on the full nonlinear model of the aircraft have shown the effectiveness of the proposed technique. The following topics have to be further investigated: the procedure to tune the FDAE and FIEs parameters, the optimal choice of matrix L , the possibility to have an on-line approximation of the model uncertainties. All these topics are directed to a practical application of the technique on real world aircrafts.

6. ACKNOWLEDGMENTS

This work was carried out under the G.A.F.A.C.S. project and was partially supported by the CARICAL foundation.

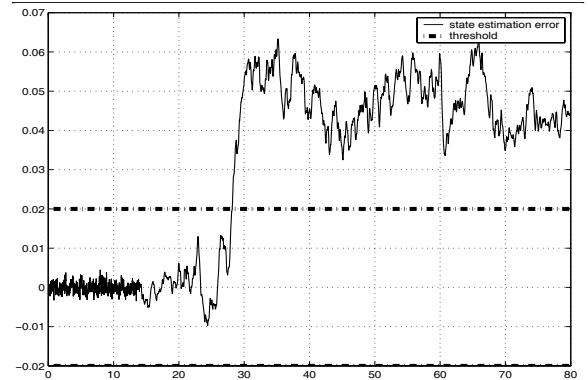


Fig. 2. Time behavior of the FDAE state estimation error $\epsilon_3 = x_3 - \hat{x}_3$ in the case of abrupt fault on engine. The estimation error exceeds threshold at $t_d \approx 28s$.

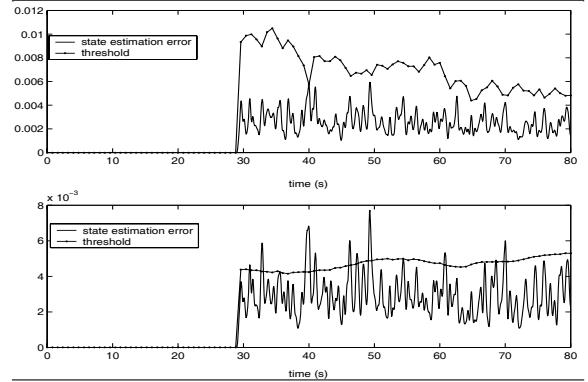


Fig. 3. Time behaviors of the state estimation errors $\epsilon_3^{1,2} = x_3 - \hat{x}_3^{1,2}$ (continues line) and corresponding adaptive thresholds $\mu_3^{1,2}$ (dash-dotted line) associated with the two isolation estimators in the case of abrupt fault on engine.

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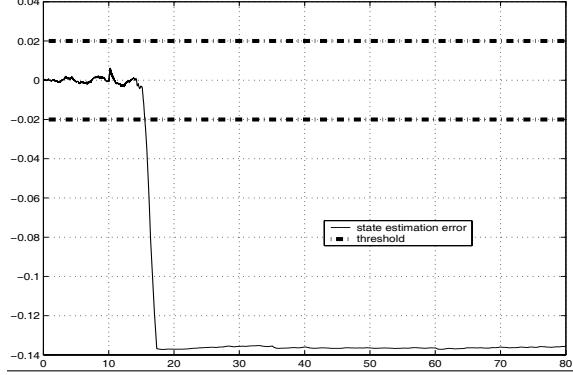


Fig. 4. Time behavior of the FDAE state estimation error $\epsilon_1 = x_1 - \hat{x}_1$ in the case of incipient fault on elevator position. The estimation error exceeds threshold at $t_d \approx 15s$.

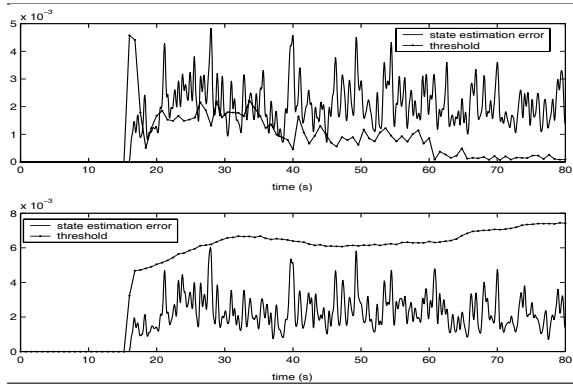


Fig. 5. Time-behaviors of the state estimation errors $\epsilon_1^{1,2} = x_1 - \hat{x}_1^{1,2}$ (continues line) and corresponding adaptive thresholds $\mu_1^{1,2}$ (dash-dotted line) associated with the two isolation estimators in the case of incipient fault on elevator position.

Variable	
$[p \ q \ r]^T$	Roll, Pitch Yaw rates
$[\phi \ \theta \ \psi]^T$	Roll, Pitch Yaw angles
V	True air speed
α	Angle of attack
β	Sideslip angle
h	Altitude
T	Thrust
μ_T	Angle between the thrust direction and the X -body axis
W	Aircraft weight
g	Gravity acceleration constant
J_x, J_y, J_z	Momentum of inertia about (X, Y, Z)-body axes
J_{xz}	Cross product of inertia
C_D, C_L, C_Y	Drag, Lift, Side force coefficients
C_R, C_M, C_N	Roll, Pitch, Yaw moment coefficients
$\rho = (\rho(h))$	Air density
\bar{c}	Mean aerodynamic chord
b	Wing span
S	Wing reference area
$\delta_e, \delta_a, \delta_r$	Elevator, Ailerons, Rudder deflection
δ_T	Throttle command
δ_f	Flap deflection

Table 1. Nomenclature

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