AUTOPILOT DESIGN FOR AGILE MISSILE USING LTV CONTROL AND NONLINEAR DYNAMIC INVERSION

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Abstract: This paper is concerned with autopilot design for an agile missile with aerodynamic fins, thrust vectoring control, and side-jet thrusters. Two-time scale dynamic inversion is used as a nonlinear flight control law. To deal with the inherently weak robustness property of dynamic inversion, Ackermann-like formula which is a time-varying version of Ackermann formula for LTI systems is applied to control the aerodynamic fins to stabilize LTV tracking error dynamics. In addition, control allocation algorithms for the effective distribution of the required total control efforts to the individual actuators are suggested, which are capable of extracting the maximum performance by combining each control effector. Finally, the main results are validated through nonlinear simulations with aerodynamic data.

Keywords: Time-Varying Eigenvalue Assignment, Control Allocation, Nonlinear Dynamic Inversion, Autopilot

1. INTRODUCTION

The modern control system of an agile missile has the many challenges due to the stringent required performance such as fast time response, high angle of attack, and high maneuverability. Usually, to achieve the required performance, the agile missiles combine the new control effectors (thrust vectoring, side thrusters) with the conventional control surface (aerodynamic fin) because thrust vectoring control and side-jet thrusters can provide additional moments and forces to achieve the reference command (Wise and Broy, 1998). However, managing each of a group of control devices with the independent control logic sometimes can result in reduced missile controllability and efficiency (Paradiso, 1991).

On the other hand, an agile missile has nonlinear, time-varying and highly coupled dynamics. Furthermore, this has many uncertainties due to the difficulty to obtain exact aerodynamic data for vehicles operating under such conditions and may in fact be poorly approximated to the actual dynamics. These and other concerns have prompted researchers to look beyond the classical methods. Despite the well-known limitations, two of most methods, gain-scheduling and nonlinear dynamic inversion (feedback linearization), appeared to be the focus of the current prominent research efforts.

This paper is concerned with autopilot design for an agile missile with aerodynamic fin, thrust vectoring control, side-jet thrusters. Two control schemes are used for autopilot design. One is two-time scale dynamic inversion used as a nonlinear control law which can determine the nominal states trajectories. The other is LTV control technique which is applied to stabilize LTV tracking error dynamics that can be obtained by trajectory linearization. For LTV control, Ackermann-like formula based on the time-varying eigenvalues theory will be proposed. This is a time-varying version of Ackermann formula for linear time-invariant system. Closed-loop stability of LTV tracking error dynamics can be achieved by assigning the time-varying eigenvalues to the desired trajectories with the negative real parts. The control allocation algorithms which distribute control demand among the individual control effectors, generate the nominal control inputs of each control effector to achieved the required moment which can be obtained from two-time scale dynamic inversion. They are capable of extracting the maximum performance from each control effector by combining the action of them. The main results will be validated through the nonlinear simulations with aerodynamic data.

2. AGILE MISSILE DYNAMICS

The considered agile model with additional control effectors is a nonlinear pitch dynamics model. The equation of motion is given by

$$\begin{aligned}\dot{\alpha} = &\frac{\frac{1}{2}\rho V S}{m}[C_{Z_0}(\alpha, M) + C_{Z_\delta}(\alpha, M, \delta_{fin})] + q \\ &+\frac{T}{mV}\delta_{tvc} + \frac{1}{mV}T_{sjt},\end{aligned} \tag{1}$$

$$\begin{aligned}\dot{q} = &\frac{\frac{1}{2}\rho V^2 S C}{I_{yy}}[C_{m_0}(\alpha, M) + C_{m_\delta}(\alpha, M, \delta_{fin}) \\ &+\frac{C}{2V}C_{mq}(M)] + \frac{T l_{tvc}}{I_{yy}}\delta_{tvc} - \frac{l_{sjt}}{I_{yy}}T_{sjt},\end{aligned} \tag{2}$$

$$\begin{aligned}\dot{V} = &\frac{1}{m}[\tfrac{1}{2}\rho V^2 S\{C_{X_0}(\alpha, M) + C_{X_\delta}(\alpha, M, \delta_{fin})\} \\ &+T\cos(\delta_{tvc})]\cos(\alpha) - \frac{1}{m}[\tfrac{1}{2}\rho V^2 S\{C_{Z_0}(\alpha, M) \\ &+C_{Z_\delta}(\alpha, M, \delta_{fin}\} + T\delta_{tvc} + T_{sjt}]\sin(\alpha)\end{aligned} \tag{3}$$

where α, q, V, δ_{fin}, δ_{tvc}, T_{sjt}, M are angle of attack, pitch rate, missile velocity, aerodynamic fin deflection angle, thrust vectoring control deflection angle, side-jet thrust and Mach number, and m, ρ, S, C, T, l_{tvc}, l_{sjt} are mass, air density, reference area, reference length, thrust, moment arm of thrust vectoring control, and moment arm of side-jet thrust, respectively. C_{X_0}, C_{Z_0}, C_{m_0} are aerodynamic coefficients at $\delta_{fin} = 0$, and C_{X_δ}, C_{Z_δ}, C_{m_δ} are variations of aerodynamic coefficients due to δ_{fin} deflection. Aerodynamic fin and TVC actuators have the limits within ±30° and ±5.5°, and second-order dynamics with $\zeta = 0.7$, $\omega_n = 150$ and $\zeta = 0.7$, $\omega_n = 50$, respectively. A side-jet thruster has constant thrust during $30ms$ burning time like a pulse signal and maximum 10 side-jet thrusters can be simultaneously ignited at once.

Aerodynamic coefficients in Eqs. (1)-(2) are represented as the approximated function of angle of attack at fixed Mach number for control design as follows:

$$\begin{aligned}\tilde{C}_{Z_0}(\alpha) &= a_1\alpha^4 + b_1\alpha^3 + c_1\alpha^2 + d_1\alpha \\ \tilde{C}_{Z_\delta}(\alpha, \delta_{fin}) &= (a_2\alpha^3 + b_2\alpha^2 + c_2\alpha + d_2)\delta_{fin} \quad (4) \\ \tilde{C}_{m_0}(\alpha) &= a_3\alpha^4 + b_3\alpha^3 + c_3\alpha^2 + d_3\alpha \\ \tilde{C}_{m_\delta}(\alpha, \delta_{fin}) &= (a_4\alpha^3 + b_4\alpha^2 + c_4\alpha + d_4)\delta_{tvc} \quad (5)\end{aligned}$$

where the coefficients a_i, b_i, c_i, d_i in Eqs.(4)-(5) are constants obtained from curve-fitting of aerodynamic data.

3. NONLINEAR DYNAMIC INVERSION

Nonlinear dynamic inversion is used to obtain the required pitch moment for angle of attack command tracking. The structure of this paper is two-time scale dynamic inversion. Fast dynamic inversion, q inversion, calculates the required moment needed for the actual pitch rate, q, to follow the commanded pitch rate q_{cmd} given by slow dynamic inversion, α inversion (Reiner et al., 1996).

First, the slow dynamic inversion which transforms the angle of attack command into the derived pitch rate command has the following form:

$$\begin{aligned}q_{cmd} = &\dot{\alpha}_d - \frac{\frac{1}{2}\rho V S}{m}[\tilde{C}_{Z_0}(\alpha) + \tilde{C}_{Z_\delta}(\alpha)\bar{\delta}_{fin}] \\ &-\frac{T}{mV}\bar{\delta}_{tvc} - \frac{1}{mV}\bar{T}_{sjt}\end{aligned} \tag{6}$$

where $\bar{\delta}_{fin}$, $\bar{\delta}_{tvc}$, and $\bar{T}_{sjt}$ are the nominal fin deflection, thrust vectoring control deflection, and side-jet thrust, respectively. $\dot{\alpha}_d$ is the desired angle of attack dynamics and defined by

$$\dot{\alpha}_d = \omega_\alpha(\alpha_{cmd} - \alpha_{meas}) \tag{7}$$

where α_{cmd} is angle of attack command and α_{meas} is measured(or estimated) angle of attack. ω_α is a design parameter.

Second, the fast dynamic inversion is applied to the dynamics of pitch rate q and calculates the required moment to achieve the reference command. With Eq. (2), the fast dynamic inversion has the following form:

$$\begin{aligned}&\dot{q}_d - \frac{\frac{1}{2}\rho V^2 S C}{I_{yy}}[\tilde{C}_{m_0}(\alpha) + \frac{C}{2V}\tilde{C}_{mq}] = \\ &\frac{\frac{1}{2}\rho V^2 S C}{I_{yy}}\tilde{C}_{m_\delta}(\alpha)\bar{\delta}_{fin} + \frac{T l_{tvc}}{I_{yy}}\bar{\delta}_{tvc} - \frac{l_{sjt}}{I_{yy}}\bar{T}_{sjt}\end{aligned} \tag{8}$$

Let this equation be briefly represented as follows:

$$M_d = M_f\bar{\delta}_{fin} + M_t\bar{\delta}_{tvc} + M_s\bar{T}_{sjt} \tag{9}$$

where M_f, M_t and M_s mean the control distribution functions of aerodynamic fin, thrust vectoring control, and side-jet thruster, respectively. In Eq. (9), the left-hand term means the required moment which makes pitch rate have the desired dynamics and can be given by

$$M_d = \dot{q}_d - \frac{\frac{1}{2}\rho V^2 SC}{I_{yy}}[\tilde{C}_{m_0}(\alpha) + \frac{C}{2V}\tilde{C}_{mq}] \tag{10}$$

where $\dot{q}_d$ is the desired pitch rate dynamics and defined by

$$\dot{q}_d = \omega_q(q_{cmd} - q_{meas}) \tag{11}$$

where q_{cmd} is the derived pitch rate command obtained from the slow dynamic inversion and q_{meas} is measured pitch rate. ω_q is a design parameter. The right-hand term is the achievable moment which is generated by using the aerodynamic fin, thrust vectoring control and side-jet thruster.

4. CONTROL ALLOCATION

The family of the control effectors of the agile missile can be divided into two groups according to the usage phase. One(Group A) is a group of the aerodynamic fin and the thrust vectoring control, and the other(Group B) is a group of the aerodynamic fin and the side-jet thruster. The former is used during thrust propulsion, while the latter is used after burning out. Two control allocation techniques - a pseudo control method for Group A and a daisy-chain method for Group B - are used for allocating the pitch control moment.

4.1 Pseudo Control Method

Pseudo control allocation technique for aerodynamic fin and thrust-vectoring control is representative of the ganged configurations (Paradiso, 1991). The ganged effectors always cooperate, that is, their control effort is coordinated and control effectiveness of each control effector is adjusted by the time-varying weighting functions w_i. For the case of Group A, to accomplish the desired command, the following equality must be satisfied with

$$\begin{aligned} M_d &= M_f\bar{\delta}_{fin} + M_t\bar{\delta}_{tvc} \\ &= \begin{bmatrix} M_f & M_t \end{bmatrix} \begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} \end{aligned} \tag{12}$$

From Eq. (12), the amount of the deflection of each control effector can be determined by matrix inversion as follows:

$$\begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = \begin{bmatrix} M_f & M_t \end{bmatrix}^{-1} M_d \tag{13}$$

where the inverse of control distribution function matrix is not unique because of rank redundancy. Hence the control allocation function of each control effector can be obtained from using the pseudo-inverse property minimizing the following object function:

$$\begin{aligned} &\min\ J = \begin{bmatrix} \bar{\delta}_{fin} & \bar{\delta}_{tvc} \end{bmatrix} \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} \begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} \\ &\text{subject to } \begin{bmatrix} M_f & M_t \end{bmatrix} \begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = v \end{aligned} \tag{14}$$

where v is pseudo control. The pseudo control v is distributed in such a way that the weighted energy of the actual control input is minimized. The above optimization problem has an explicit solution which can be using several technique. But, by using the Lagrange multipliers, the optimal inputs are given by

$$\begin{bmatrix} \bar{\delta}_{fin} \\ \bar{\delta}_{tvc} \end{bmatrix} = \begin{bmatrix} \dfrac{M_f}{(M_f)^2 + (\frac{w_1}{w_2})(M_t)^2} \\ \dfrac{(\frac{w_1}{w_2})M_t}{(M_f)^2 + (\frac{w_1}{w_2})(M_t)^2} \end{bmatrix} M_d \tag{15}$$

where w_1, w_2 are the weighting values of each control effector, respectively.

4.2 Daisy-Chain Method

Daisy-chain allocation technique for aerodynamic fin and side-jet thruster allocates control effectors in prioritized manner (Berg et al., 1996). This means that the control effector family with high priority is first used, and then others are used later. In this study, side-jet thruster is high priority actuator. Therefore, aerodynamic fin is not used until at least one side-jet thruster is ignited except for a case that the required moment is less than a side-jet thruster can generate. Daisy-chain control allocation for Group B is given by the following equation:

$$\begin{bmatrix} \bar{T}_{sjt} \\ \bar{\delta}_{fin} \end{bmatrix} = \begin{bmatrix} M_s^{-1}M_d \\ M_f^{-1}\{M_d - M_s\bar{T}_{sjt}\} \end{bmatrix} \tag{16}$$

Since the side-jet thruster which has constant thrust during burning time is a pulse-like signal, the nominal control command of side-jet thrust in Eq. (16) must be discretized.

5. TIME-VARYING CONTROL TECHNIQUE

In this section, time-varying eigenvalue (PD-eigenvalue) is introduced into Ackermann-like formula which is the time-varying version of Ackermann formula for LTI systems. LTV control is applied to stabilize a tracking error dynamics which is derived by linearizing a nonlinear dynamics through the nominal trajectories.

5.1 LTV Spectral Theory

Some technical preliminaries to propose the Ackermann-like formula for linear time-varying systems are presented in this section. In order to obtain the eigenvalue for linear time-varying systems, an unified spectral theory for Nth-order scalar linear time-varying systems is introduced as follows:

$$y^{(N)} + a_N(t)y^{(N-1)} + \cdots + a_1(t)y = 0 \quad (17)$$

(17) can be conveniently represented as $\mathcal{D}_a\{y\} = 0$ using the scalar polynomial differential operator (SPDO)

$$\mathcal{D}_a = \delta^N + a_N(t)\delta^{N-1} + \cdots + a_1(t) \quad (18)$$

where $\delta = d/dt$ is the derivative operator.

Definition 1. (Zhu and Johnson, 1991)

Let $\mathcal{D}_\alpha$ be an Nth-order SPDO and let $\{y_i\}_{i=1}^N$ be any fundamental set of solutions to $\mathcal{D}_\alpha\{y\} = 0$. Let

$$W(t) = \begin{bmatrix} y_1 & \cdots & y_N \\ \dot{y}_1 & \cdots & \dot{y}_N \\ \vdots & \ddots & \vdots \\ y_1^{(N-1)} & \cdots & y_N^{(N-1)} \end{bmatrix} \quad (19)$$

be the Wronskian matrix associated with $\{y_i\}_{i=1}^N$. Denote by $D(t)$ the diagonal matrix

$$D(t) = \text{diag}\,[y_1, y_2, \cdots, y_N]. \quad (20)$$

Then

$$\begin{aligned} P_N(\rho_1(t), \cdots, \rho_N(t)) &= W(t)D^{-1}(t) \\ &= \begin{bmatrix} 1 & \cdots & 1 \\ \mathcal{D}_{\rho_1}\{1\} & \cdots & \mathcal{D}_{\rho_N}\{1\} \\ \mathcal{D}_{\rho_1}^2\{1\} & \cdots & \mathcal{D}_{\rho_N}^2\{1\} \\ \vdots & \ddots & \vdots \\ \mathcal{D}_{\rho_1}^{N-1}\{1\} & \cdots & \mathcal{D}_{\rho_N}^{N-1}\{1\} \end{bmatrix} \end{aligned} \quad (21)$$

where $\mathcal{D}_{\rho_i} = (\delta + \rho_i)$, $\mathcal{D}_{\rho_i}^k = \mathcal{D}_{\rho_i}\mathcal{D}_{\rho_i}^{k-1}$. The canonical coordinate transformation matrix $P_N(t)$ is called the modal canonical matrix for $\mathcal{D}_\alpha$ associated with the PD-spectrum $\{\rho_i(t)\}_{i=1}^N$.

The column vectors $p_i(t)$ of $P_N(t)$ satisfying

$$A_c(t)p_i(t) - \rho_i(t)p_i(t) = \dot{p}_i(t) \quad (22)$$

and row vectors $q_i^T(t)$ of $Q_N(t) = P_N^{-1}(t)$ satisfying

$$q_i^T(t)A_c(t) - \rho_i(t)q_i^T(t) = -\dot{q}_i^T(t) \quad (23)$$

are called right PD-eigenvectors and left PD-eigenvectors, respectively, of $\mathcal{D}_\alpha$ associated with $\rho_i(t)$ where $A_c(t)$ is the companion matrix (phase-variable form matrix) associated with $\mathcal{D}_a$.

□

The relationship between the coefficients of *Eq.*(18) and PD-spectrum is given by the following *Lemma 1.* This will be used to determine the phase-variable form matrix of the closed-loop system with the desired PD-eigenvalues.

Lemma 1.(Zhu and Johnson, 1991)

If the linear time-varying system is synthesized from a PD-spectrum $\{\rho_i(t)\}_{i=1}^N$, then the coefficients $\{a_i(t)\}_{i=1}^N$ are given by the synthesis formula

$$a_k(t) = \frac{\tilde{p}_{k,N+1}(t)}{\det P_N\left(\rho_1(t), \cdots, \rho_N(t)\right)} \quad (24)$$

where $P_N\left(\rho_1(t), \cdots, \rho_N(t)\right)$ is the canonical PD-modal matrix associated with $\{\rho_i(t)\}_{i=1}^N$ given by *Eq.*(21), and $\tilde{p}_{k,N+1}(t)$ denotes the algebraic cofactor of $p_{k,N+1}(t)$ in the $(N+1)\times(N+1)$ matrix

$$P_{N+1}(t) = [p_{ij}(t)] = \left[\begin{array}{ccc|c} & & & 1 \\ & P_N(t) & & \mathcal{D}_\rho\{1\} \\ & & & \vdots \\ \hline \mathcal{D}_{\rho_1}^N\{1\} & \cdots & \mathcal{D}_{\rho_N}^N\{1\} & \mathcal{D}_\rho^N\{1\} \end{array}\right]. \quad (25)$$

□

5.2 Ackermann-like Formula

In this section, the Ackermann-like formula for SISO linear time-varying system is proposed. Consider a controllable SISO linear time-varying system of the form :

$$\dot{x} = A(t)x + b(t)u \quad (26)$$

with the state vector $x \in \mathbb{R}^{N\times1}$ and the scalar input $u(t)$. The system can be stabilized by means of a state feedback. Since a given system is controllable, there exists a nonsingular controllability matrix $\mathcal{C}(t) = \begin{bmatrix} b_1(t) & b_2(t) & \cdots & b_N(t) \end{bmatrix}$ where $b_{i+1}(t) = A(t)b_i(t) - \dot{b}_i(t)$ with $b_1(t) = b(t)$ and an inverse matrix of $\mathcal{C}^{-1}(t)$ satisfied with

$$\mathcal{C}^{-1}(t)\mathcal{C}(t) = \begin{bmatrix} \tilde{C}_{N-1}(t) \\ \tilde{C}_{N-2}(t) \\ \vdots \\ \tilde{C}_0(t) \end{bmatrix}$$

$$\times \begin{bmatrix} b_1(t) & b_2(t) & \cdots & b_N(t) \end{bmatrix} = I \tag{27}$$

where $\tilde{C}_i(t)$ is the $(i+1)$th row vector of $\mathcal{C}^{-1}(t)$. Let

$$z = \tilde{C}'_0(t)x \tag{28}$$

with $\tilde{C}'_0(t) = \tilde{C}_0(t)$, and

$$\dot{\tilde{C}}'_p(t) = \dot{\tilde{C}}'_{p-1}(t) + \tilde{C}'_{p-1}(t)A(t) \tag{29}$$

for each $p = 1, 2, \cdots, N$.

It then follows that for each p,

$$\begin{array}{rcl} z & = & \tilde{C}'_0(t)x \\ \dot{z} & = & \tilde{C}'_1(t)x \\ \vdots & & \vdots \\ z^{(N-1)} & = & \tilde{C}'_{N-1}(t)x \end{array} \tag{30}$$

and that

$$z^{(N)} = \tilde{C}'_N(t)x + \tilde{C}'_{N-1}(t)b(t)u. \tag{31}$$

The rows $\tilde{C}'_p(t)$ are of dimension $(1 \times N)$. Thus the Nth row $\tilde{C}'_N(t)$ is a linear combination of the $N-1$ previous rows. Consequently the linear combination coefficients $a_i(t)$ exist. Further $\tilde{C}'_{N-1}(t)b(t)$ is unity. Therefore, from *Eq.*(31), we have

$$\begin{aligned} z^{(N)} = & \left[a_0(t)\tilde{C}'_0(t) + \cdots + a_{N-1}(t)\tilde{C}'_{N-1}(t)\right] x \\ & + \; u. \end{aligned} \tag{32}$$

To assign the PD-spectrum of the given system to the desired locations, the feedback control can be determined as follows;

$$\begin{aligned} u &= k(t)x \\ &= -\left(\begin{array}{l} \begin{bmatrix} a_0(t) & a_1(t) & \cdots & a_N(t) \end{bmatrix} \\ \quad + \begin{bmatrix} d_1(t) & d_2(t) & \cdots & d_N(t) \end{bmatrix} \end{array} \right) \\ &\quad \times \begin{bmatrix} \tilde{C}'_0(t) \\ \tilde{C}'_1(t) \\ \vdots \\ \tilde{C}'_{N-1}(t) \end{bmatrix} x \end{aligned} \tag{33}$$

where the coefficients $d_i(t)$ are synthesized from the desired PD-spectrum. They can be easily obtained from *Lemma 1.* If the desired PD-spectrum is satisfied with stability criterion (Zhu, 1996), the close-loop system can be stabilized.

5.3 LTV Autopilot Design

For LTV autopilot design, let

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \begin{bmatrix} \alpha \\ q \end{bmatrix} \tag{34}$$

be the state vector of the missile. Then, from Eqs. (1)-(2), the state equation is given by

$$\begin{aligned} \dot{\xi} &= f(\xi, \delta_{fin}) \\ &= \begin{bmatrix} f_1(\xi_1, \xi_2, \delta_{fin}) \\ f_2(\xi_1, \xi_2, \delta_{fin}) \end{bmatrix}. \end{aligned} \tag{35}$$

Now, for the given angle of attack command α_{cmd} and the derived pitch rate command q_{cmd} from slow dynamic inversion, let $\bar{\xi}$ be the nominal state trajectory and $\bar{\delta}_{fin}$ be the nominal fin deflection such that

$$\dot{\bar{\xi}} = f[\bar{\xi}, \bar{\delta}_{fin}]. \tag{36}$$

Define the tracking errors by

$$x = \xi - \bar{\xi}, \tag{37}$$

and the tracking error control by

$$v = \delta - \bar{\delta}_{fin}. \tag{38}$$

Then the linearized tracking error dynamics is given by

$$\dot{x} = A(t)x + B(t)v \tag{39}$$

where

$$\begin{aligned} A(t) &= \left.\frac{\partial f}{\partial \xi}\right|_{\bar{\xi}, \bar{\delta}_{fin}} = \begin{bmatrix} a_{11}(t) & 1 \\ a_{21}(t) & a_{22}(t) \end{bmatrix}, \\ B(t) &= \left.\frac{\partial f}{\partial \delta}\right|_{\bar{\xi}, \bar{\delta}_{fin}} = \begin{bmatrix} b_1(t) \\ b_2(t) \end{bmatrix}. \end{aligned} \tag{40}$$

The autopilot design task amounts to finding a control law such that the tracking error becomes zero exponentially for any admissible angle of attack command. This can be achieved using an LTV controller. Now an LTV control law v can be designed for LTV tracking error dynamics (39) using the Ackermann-like formula outlined in previous statements.

6. SIMULATION RESULTS

Simulations with aerodynamic data are performed to validate the proposed schemes. In this study, there are two scenarios. One is subsonic flight condition($M = 0.6$) for Group A, and the other is hypersonic($M = 6.0$) for Group B.

Results for Scenario 1 and Scenario 2 are presented in Figures 1 - 2, and Figures 3 - 4, respectively. Figure 1 shows that an angle of attack command for Scenario 1 is well tracked within

5% steady-state error under various uncertainties such as poorly approximated aerodynamic data in curve-fitting, missile velocity variation, etc. The distributed control efforts to follow the command are depicted in Figure 2. As approaching the steady state, the deflection of thrust vectoring control is growing down less and less while the deflection of aerodynamic fin is growing up more and more. It is because the authorities of control effectors are dependent on flight condition. Therefore, this fact reveals that pseudo control method for Group A is the efficient control allocation algorithm reflected on flight condition. Under similar circumstances, the angle of attack tracking performance for Scenario 2 is depicted in Figure 3. This shows that after burning out, the angle of attack command can be achieved by using side-jet thrust. The allocated control efforts by daisy-chain method for Group B are depicted in Figure 4. It can be inferred from this that side-jet thrust usage prior to aerodynamic fin increases the maneuverability of the missile in homing phase.

7. CONCLUSIONS

In this paper, a new autopilot is proposed for an agile missile with conventional control surface, aerodynamic fin, and additional thrusts such as thrust vectoring control, side-jet thrusters. The features of the proposed schemes include (1) effective control allocation for each control effector (aerodynamic fin, thrust vectoring control, side-jet thrusters) to achieve the angle of attack command, (2) good tracking performance for angle of attack command without scheduling of any constant design parameters throughout a wide range of angle of attack, and (3) time-varying control gains to improve the robustness for the unstructured uncertainties. The proposed schemes have been validated by nonlinear simulations with aerodynamic data.

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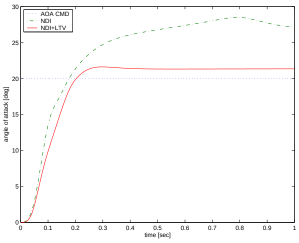

Fig. 1. Angle of attack output for scenario 1

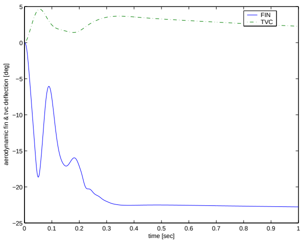

Fig. 2. Fin vs. TVC for scenario 1

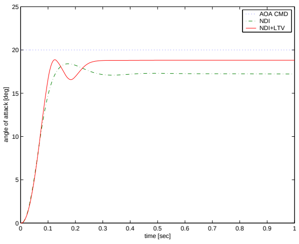

Fig. 3. Angle of attack output for scenario 2

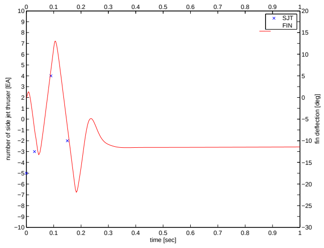

Fig. 4. Fin vs. SJT for scenario 2