BACKLASH COMPENSATION USING RECEDING HORIZON CONTROL

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Abstract: We present a receding horizon control algorithm for compensation of backlash at the input of a stable linear system under control rate constraints. The problem is first posed as a receding horizon optimal control problem by modelling backlash as a piecewise affine system with a state space partition consisting of three regions. This optimal control problem involves solving, at each step, 3^N quadratic programmes, where N is the optimisation horizon. As an alternative to solving the quadratic programmes, we propose a strategy based on a recently devised suboptimal receding horizon control algorithm which utilises a singular value decomposition of the Hessian of the quadratic programme. This alternative strategy leads, at the cost of some performance degradation, to much smaller computational load since a feasible rather than optimal solution has to be obtained at each step.

Keywords: Receding horizon control, backlash, piecewise affine systems, rate constraints, model predictive control, SVD algorithm.

1. INTRODUCTION

Backlash is a common nonlinearity that limits control performance in many industrial applications, notably mechanical and hydraulic systems. According to the survey paper Nordin and Gutman (2002), few control innovations aimed at this problem have been presented since the early strategies based on describing function analysis (Gelb and Vander Velde, 1968). A novel scheme was introduced in Tao and Kokotović (1993) based on adaptive inversion of the backlash nonlinearity. Other nonlinear techniques such as dynamic inversion using neural networks and backstepping have since been proposed (Selmic and Lewis, 2001).

More recently, the idea of using the receding horizon optimal control [RHOptC] (or model predictive control) framework for backlash compensation under actuator magnitude constraints has been suggested in Zabiri and Samyudia (2004). The RHOptC controller proposed by Zabiri and Samyudia (2004) incorporates an inverse model of the backlash function and logic variables are introduced which permit the use of mixed-integer quadratic programming for the computations. The resulting system falls into the general class of mixed logical dynamical [MLD] systems introduced by Bemporad and Morari (1999).

MLD systems have been shown to be equivalent to piecewise affine [PWA] systems in Bemporad et al. (2000). RHOptC of PWA systems is a subject of current research and several algorithms have been proposed in recent literature (Mayne and Raković, 2003; Borrelli et al., 2003; Grieder et al., 2004). A key issue in controlling these systems is the inherent computational complexity of controller synthesis and analysis (Grieder et al., 2004).

In this paper we consider backlash compensation under the receding horizon control [RHC] framework. By modelling the backlash nonlinearity as a PWA system with three regions, we first pose a RHOptC problem with horizon N, which involves, in the worst case scenario, the solution of 3^N quadratic programmes [QP] (Mayne and Raković, 2003). To circumvent the complexity issue, we approximate the QP solutions using the principal directions of the Hessians as provided by singular value decompositions [SVD]. Then, the controller is implemented in RHC fashion following the algorithm of Rojas et al. (2003).

The remainder of the paper proceeds as follows. In Section 2 we formulate the RHOptC problem for backlash compensation under rate constraints. In Section 3 we describe the SVD-based strategy and associated suboptimal RHC for the same problem. We also prove stability of the closed loop system. In Section 4 we provide simulation results and finally conclusions are given in Section 5.

2. THE RECEDING HORIZON OPTIMAL CONTROL PROBLEM

We consider the following model of a linear discrete-time system with a backlash nonlinearity at the input:

$$\xi_{k+1} = A\xi_k + Bv_k, \quad \xi_k \in \mathbb{R}^n, v_k \in \mathbb{R}, \tag{1}$$

$$v_k = \mathcal{B}(v_{k-1}, u_k), \quad u_k \in \mathbb{R}. \tag{2}$$

The backlash nonlinearity is given by

$$\mathcal{B}(v_{k-1}, u_k) = \begin{cases} m(u_k - \ell) & \text{if } m(u_k - \ell) \le v_{k-1}, \\ v_{k-1} & \text{if } v_{k-1} + m\ell \le mu_k \le v_{k-1} + mr, \\ m(u_k - r) & \text{if } m(u_k - r) \ge v_{k-1}, \end{cases} \tag{3}$$

where $m > 0$, $r > 0$ and $\ell < 0$. Figure 1 shows its characteristic. We assume that the eigenvalues of A in (1) are inside the unit circle.

The backlash function (3) can be represented as a PWA system with state $z_k = v_{k-1}$ and dynamics given by

$$z_{k+1} = A_i z_k + B_i u_k + G_i, \tag{4}$$

$$v_k = C_i z_k + D_i u_k + E_i, \tag{5}$$

$$\text{if} \quad (u_k, z_k) \in R_i \triangleq \{(u, z) : L_i u + J_i z \le W_i\}, \tag{6}$$

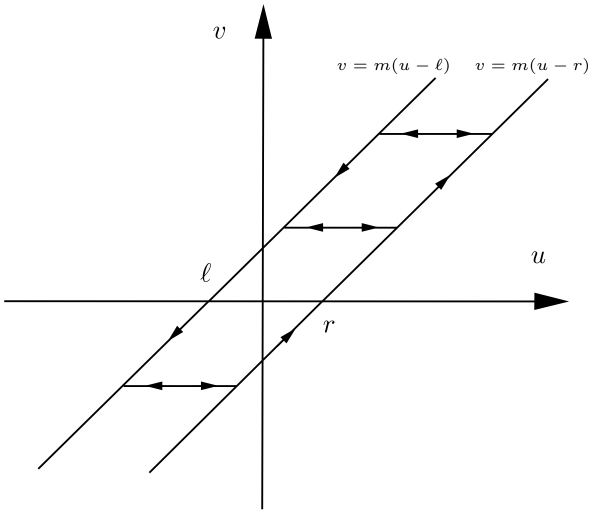

Fig. 1. Backlash characteristic.

for $i = 1, 2, 3$, where

$A_1 = 0$, $B_1 = m$, $G_1 = -m\ell$,
$L_1 = m$, $J_1 = -1$, $W_1 = m\ell$,
$A_2 = 1$, $B_2 = 0$, $G_2 = 0$,
$L_2 = m[1 \;\; -1]^{\mathrm{T}}$, $J_2 = [-1 \;\; 1]^{\mathrm{T}}$, $W_2 = m[r \;\; -\ell]^{\mathrm{T}}$,
$A_3 = 0$, $B_3 = m$, $G_3 = -mr$,
$L_3 = -m$, $J_3 = 1$, $W_1 = -mr$,

and $C_i = A_i$, $D_i = B_i$, $E_i = G_i$ for $i = 1, 2, 3$.

We now define $x_k \triangleq \begin{bmatrix} z_k \; \xi_k \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^{n+1}$, and combine (1) and (4)–(6) into a single nonlinear (PWA) equation $x_{k+1} = f(x_k, u_k)$. As the base for the RHOptC design, we pose, at time k and for the current state $x_k = x$ and the previous input $u_{k-1} = u$, the following fixed-horizon optimisation problem:

$$V_N^{\text{OPT}}(x, u) \triangleq \min V_N(x_j, u_j), \tag{7}$$

subject to:

$$x_{j+1} = f(x_j, u_j) \quad \text{for } j = 0, \ldots, N-1, \tag{8}$$

$$x_0 = x, \tag{9}$$

$$|u_j - u_{j-1}| \le \Delta \quad \text{for } j = 0, \ldots, N-1, \tag{10}$$

$$u_{-1} = u, \tag{11}$$

$$[1 \; 0 \; \ldots \; 0]x_N = 0, \tag{12}$$

where

$$V_N(x_j, u_j) \triangleq x_N^{\mathrm{T}} P x_N + \sum_{j=0}^{N-1} (x_j^{\mathrm{T}} Q x_j + u_j^{\mathrm{T}} R u_j), \tag{13}$$

$$P = \begin{bmatrix} 0 & 0 \\ 0 & \bar{P} \end{bmatrix}, \quad \bar{P} = A^{\mathrm{T}} \bar{P} A + Q. \tag{14}$$

Problem (7)–(14) is the minimisation of the quadratic objective function (13)–(14) for the PWA system (8) under rate constraints [1] (10) and a terminal state constraint (12). Its solution can be found by solving 3^N QPs, which correspond

[1] Magnitude constraints can also be included.

to all the possibilities $(u_j, z_j) \in R_i$ for $i = 1, 2, 3$ and $j = 0, 1, \ldots, N-1$. Some simplifications are possible in certain cases. For example, if the rate limit $\Delta > 0$ is greater than the backlash "dead-zone" $r - \ell$, then we can impose the condition that $(u_j, z_j) \notin R_2$ for $j = 0, 1, \ldots, N-1$, resulting in only 2^N QPs to solve. Also, if we impose the condition $(u_{N-1}, z_{N-1}) \notin R_2$, which we assume, then it is easy to show that the terminal state constraint (12) takes the form

$$u_{N-1} = \begin{cases} \ell & \text{if} \quad (u_{N-1}, z_{N-1}) \in R_1, \\ r & \text{if} \quad (u_{N-1}, z_{N-1}) \in R_3. \end{cases} \tag{15}$$

Equation (15) can be substituted in (7)–(14) to obtain QPs having $N-1$ decision variables. Let $N_{\mathrm{QP}} \leq 3^N$ be the number of QPs to solve. Each of the QPs has the form

$$\min \mathbf{u}^{\mathrm{T}} H \mathbf{u} + 2\mathbf{u}^{\mathrm{T}}(Fx + a) + b, \tag{16}$$

subject to:

$$L\mathbf{u} + J \begin{bmatrix} u \\ x \end{bmatrix} \leq W, \tag{17}$$

where $\mathbf{u} = [u_0, \ldots, u_{N-2}]^{\mathrm{T}} \in \mathbb{R}^{N-2}$, and H, F, a, b, L, J and W change with each of the N_{QP} possibilities. The vector a and the scalar b are independent of $\mathbf{u}$ (but a depends on u_{N-1} and b on x and u_{N-1}). Note that b does not affect the minimiser of (16)–(17) but it affects the optimal value and hence has to be considered in the evaluation.

Once the N_{QP} QPs of the form (16)–(17) have been solved, the optimal solution to problem (7)–(14) is computed as the minimum of the QPs. Let the minimiser be $\mathbf{u}^{\mathrm{OPT}} = [u_0^{\mathrm{OPT}}, \ldots, u_{N-2}^{\mathrm{OPT}}]$. Then the RHOptC strategy applies the first element of this vector, that is, $u_k = u_0^{\mathrm{OPT}}$. Time is then stepped forward and the whole procedure is repeated at the next time instant. The configuration for RHOptC is depicted in Figure 2.

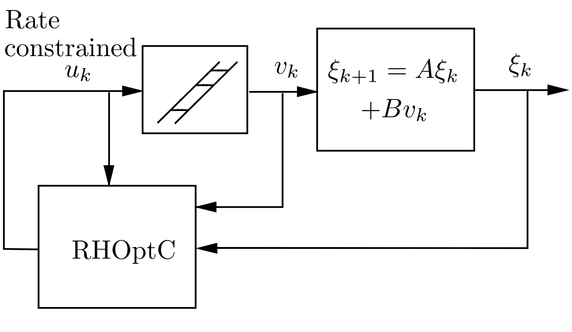

Fig. 2. Configuration for RHC.

3. THE SVD ALGORITHM FOR BACKLASH COMPENSATION

Instead of solving the QPs (16)–(17), we will approximate their solution using an algorithm recently introduced by Rojas et al. (2003). This algorithm is based on an SVD of the Hessian of the QP. The singular vectors of the Hessian provide a set of orthogonal basis vectors spanning the control space. At each time step, a variable number of components of the unconstrained optimal control solution along the singular vectors is selected so that no constraints are violated.

More precisely, consider the QP (16)–(17). Let

$$H = VSV^{\mathrm{T}} \tag{18}$$

be the SVD of the Hessian $H = H^{\mathrm{T}} > 0$. In (18), V contains the singular vectors of H and S is the diagonal matrix

$$S = \operatorname{diag}\left\{\sigma_1, \, \sigma_2, \, \ldots, \, \sigma_{N-1}\right\},$$

with the singular values of H arranged in decreasing order.

Using the coordinate transformation $\mathbf{u} = V\tilde{\mathbf{u}}$, the vector of control moves $\mathbf{u}$ in (16) can be expressed as a linear combination of the singular vectors of H:

$$\mathbf{u} = V\tilde{\mathbf{u}} = \sum_{j=1}^{N-1} \mathbf{v}_j \tilde{\mathbf{u}}(j),$$

where $\mathbf{v}_j$, $j = 1, \ldots, N-1$, are the columns of V and $\tilde{\mathbf{u}}(j)$ are the components of the vector $\tilde{\mathbf{u}}$. We can then express the objective function in (16) as

$$\tilde{\mathbf{u}}^{\mathrm{T}} S \tilde{\mathbf{u}} + 2\tilde{\mathbf{u}}^{\mathrm{T}} V^{\mathrm{T}}(Fx + a) + b,$$

whose unconstrained minimum is

$$\tilde{\mathbf{u}}^{\mathrm{UC}} = -S^{-1} V^{\mathrm{T}}(Fx + a). \tag{19}$$

The key idea of the algorithm is to construct a suboptimal solution $\mathbf{u}_s^{\mathrm{SVD}}$ to the QP problem (16)–(17) by approximating (19) using the basis vectors starting from the largest singular values and proceeding downwards until a constraint boundary is reached. Heuristically, this algorithm examines the components of the input having largest benefit-to-cost ratio first, as discussed in Goodwin et al. (2005, p. 258).

The same procedure is repeated for each of the N_{QP} QPs of the form (16)–(17) and the vector $\mathbf{u}_s^{\mathrm{SVD}}$ that achieves the minimum objective function value is selected as a suboptimal solution to problem (7)–(14). The procedure is summarised below.

SVD control vector construction:

(1) For each QP of the form (16)–(17):
   (a) Let $\bar{\mathbf{u}}$ be a feasible interior point for constraints (17).
   (b) Calculate $\tilde{\mathbf{u}}^{\mathrm{UC}}$ as in (19).
   (c) Increase r from $r = 0$ to $r = N-2$ and, for each r, increase $\alpha \in (0, 1]$ while the vector [2]

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[2] $\sum_{j=1}^{0} \mathbf{v}_j \tilde{\mathbf{u}}^{\mathrm{UC}}(j) \triangleq 0$.

$$\mathbf{u}_{r,\alpha} \triangleq \bar{\mathbf{u}} + \sum_{j=1}^{r} \mathbf{v}_j \tilde{\mathbf{u}}^{\text{UC}}(j) + \alpha \mathbf{v}_{r+1} \tilde{\mathbf{u}}^{\text{UC}}(r+1)$$

satisfies the constraints (17), namely

$$L\mathbf{u}_{r,\alpha} + J \begin{bmatrix} u \\ x \end{bmatrix} \leq W.$$

(d) For the values of r and α resulting from step (1c), set

$$\mathbf{u}_s^{\text{SVD}} = \mathbf{u}_{r,\alpha},$$
$$V_s^{\text{SVD}} = (\mathbf{u}_s^{\text{SVD}})^{\mathrm{T}}[H\mathbf{u}_s^{\text{SVD}} + 2(Fx + a)] + b.$$

(2) Compute

$$\mathbf{u}^{\text{SVD}} = \arg\min_{s=1,\dots,N_{\text{QP}}} \{V_s^{\text{SVD}}\},$$
$$V^{\text{SVD}} = \min_{s=1,\dots,N_{\text{QP}}} \{V_s^{\text{SVD}}\}, \qquad (20)$$

and let [3]

$$\mathcal{U}^{\text{SVD}} = \{u_k^{\text{SVD}}, u_{k+1}^{\text{SVD}}, \dots, u_{k+N-1}^{\text{SVD}}\} \qquad (21)$$

be the associated control sequence. ◦

The above SVD construction is used in an RHC algorithm that ensures closed-loop stability provided the initial state belongs to an *admissible set*, defined next.

Definition 1. (Admissible Set $\mathbb{Z}_{\mathbf{N}}$). Let $\mathbb{Z}_N$ be the set of all initial states $\zeta = [u \quad x^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n+2}$ for which there exists an admissible control sequence $\mathcal{U}^{\text{SVD}}$ computed as in (21). ◦

To prove stability, we will use the objective function (13)–(14) as a Lyapunov function $\mathcal{V}^*(\zeta)$ as is standard in RHC (Mayne et al., 2000). An algorithm with provable stabilising properties can then be developed as follows. Starting with a state $\zeta \in \mathbb{Z}_N$, we compute the SVD sequence as in (21). At the next time step, we check if the successor state ζ^+ belongs to $\mathbb{Z}_N$. If $\zeta^+ \notin \mathbb{Z}_N$, we apply the second move of the SVD sequence $\mathcal{U}^{\text{SVD}}$ obtained in the previous step. If $\zeta^+ \in \mathbb{Z}_N$, we compute a new SVD sequence $\mathcal{U}^{\text{SVD}+}$ as in (21) and check whether applying the first move of $\mathcal{U}^{\text{SVD}+}$ would decrease the value of the Lyapunov function $\mathcal{V}^*(\zeta)$. If so, we apply the first control move of $\mathcal{U}^{\text{SVD}+}$. If not, we apply the second move of the sequence $\mathcal{U}^{\text{SVD}}$ obtained in the previous step. At each time step, the value of $\mathcal{V}^*(\zeta)$ is updated and the procedure is repeated. The resulting algorithm ensures that the Lyapunov function $\mathcal{V}^*(\zeta)$ decreases at each step.

We formalise the above procedure in the following:

SVD Algorithm:

(1) At time $k = 0$, and given an initial state $\zeta_0 = [u_{-1} \quad x_0^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{Z}_N$:

(a) Compute $\mathcal{U}_0^* = \mathcal{U}^{\text{SVD}} = \{u_0^*, , \dots, u_{N-1}^*\}$ as in (21) and let $\mathcal{V}(\zeta_0) = V^{\text{SVD}}$ as per (20).

(b) Apply as the initial control action the first element of $\mathcal{U}_0^*$, that is,

$$u_0 = \mathcal{K}^{\text{SVD}}(\zeta_0) \triangleq u_0^*.$$

(c) Set $k = 1$, $\zeta_1 = [u_0 \quad f(x_0, u_0)^{\mathrm{T}}]^{\mathrm{T}}$, and go to step (2).

(2) At time k:

(a) If $\zeta_k \notin \mathbb{Z}_N$, go to step (3).

(b) If $\zeta_k \in \mathbb{Z}_N$, compute $\mathcal{U}^{\text{SVD}}$ and V^{SVD} as in (21) and (20).

(c) If $V^{\text{SVD}} > \mathcal{V}^*(\zeta_{k-1}) - x_{k-1}^{\mathrm{T}} Q x_{k-1} - u_{k-1}^{\mathrm{T}} R u_{k-1}$, go to step (3).

(d) If $V^{\text{SVD}} \leq \mathcal{V}^*(\zeta_{k-1}) - x_{k-1}^{\mathrm{T}} Q x_{k-1} - u_{k-1}^{\mathrm{T}} R u_{k-1}$, set

$$\mathcal{U}_k^* = \mathcal{U}^{\text{SVD}} = \{u_0^*, u_1^*, \dots, u_{N-1}^*\}, \qquad (22)$$
$$\mathcal{V}^*(\zeta_k) = V^{\text{SVD}}.$$

(e) Apply as the current control action the first element of $\mathcal{U}_k^*$, that is,

$$u_k = \mathcal{K}^{\text{SVD}}(\zeta_k) \triangleq u_0^*. \qquad (23)$$

(f) Set $k = k + 1$, $\zeta_k = [u_k \quad f(x_k, u_k)^{\mathrm{T}}]^{\mathrm{T}}$, and return to step (2).

(3) (a) Apply as the current control action the second element of $\mathcal{U}_{k-1}^*$ in (22), that is,

$$u_k = \mathcal{K}^{\text{SVD}}(\zeta_k) \triangleq u_1^*. \qquad (24)$$

(b) Update the sequence of control moves $\mathcal{U}_k^*$ by retaining the last $N - 1$ elements of the previous $\mathcal{U}_{k-1}^*$ in (22) and adding zero as the last element, that is,

$$\mathcal{U}_k^* = \{u_1^*, u_2^*, \dots, u_{N-1}^*, 0\}. \qquad (25)$$

Let $\mathcal{V}^*(\zeta_k)$ be the objective value corresponding to (25) as per (13)–(14).

(c) Set $k = k + 1$, $\zeta_k = [u_k \quad f(x_k, u_k)^{\mathrm{T}}]^{\mathrm{T}}$, and go to step (2). ◦

The above algorithm ensures that the closed-loop trajectories converge to the origin, as proved in the following theorem.

Theorem 2. (Closed-loop Stability). For all initial states in $\mathbb{Z}_N$ the origin is an attractive equilibrium point for the closed-loop system

$$\zeta^+ = \begin{bmatrix} \mathcal{K}^{\text{SVD}}(\zeta) \\ f(x, \mathcal{K}^{\text{SVD}}(\zeta)) \end{bmatrix}. \qquad (26)$$

Proof: We first note that constraint (12) ensures that the terminal state z_N is zero. Hence (see (1) and (4)–(6)),

$$x_N = \begin{bmatrix} 0 \\ \xi_N \end{bmatrix} \quad \text{and} \quad f(x_N, 0) = \begin{bmatrix} 0 \\ A\xi_N \end{bmatrix}.$$

Let $\mathcal{U}^* = \{u_0^*, u_1^*, \dots, u_{N-1}^*\}$ be the SVD sequence used at some step, and let

$$\mathcal{X}^* = \{x_1^*, x_2^*, \dots, \begin{bmatrix} 0 \\ \xi_N \end{bmatrix}\} \qquad (27)$$

[3] The last element u_{k+N-1}^{SVD} is obtained from the equality constraint (15).

be the corresponding x-state sequence. If, at the next step, the SVD sequence (25) is used then the resulting x-state sequence is

$$\mathcal{X}^* = \{x_2^*, \ldots, \begin{bmatrix} 0 \\ \xi_N \end{bmatrix}, \begin{bmatrix} 0 \\ A\xi_N \end{bmatrix}\}. \tag{28}$$

Now, let $\zeta = [u \quad x^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{Z}_N$ and let ζ^+ given by (26) be the successor state. If the control sequence $\mathcal{U}^{*+}$ is computed as in (22), we have

$$\mathcal{V}^*(\zeta^+) \leq \mathcal{V}^*(\zeta) - x^{\mathrm{T}}Qx - u^{\mathrm{T}}Ru$$

by construction. On the other hand, if the control sequence $\mathcal{U}^{*+}$ is computed as in (25), we obtain, using (13)–(14) and the fact that the control sequences and corresponding state sequences (27), (28) share common terms:

$$\begin{aligned} \mathcal{V}^*(\zeta^+) &= \mathcal{V}^*(\zeta) - x^{\mathrm{T}}Qx - u_0^* R^{\mathrm{T}} u_0^* \\ &\quad + \xi_N^{\mathrm{T}}(A^{\mathrm{T}}\bar{P}A + Q - \bar{P})\xi_N \\ &= \mathcal{V}^*(\zeta) - x^{\mathrm{T}}Qx - u^{\mathrm{T}}Ru. \end{aligned}$$

Thus, $\mathcal{V}^*(\zeta)$ decreases along the trajectories of the closed-loop system (26). Attractivity of the origin then follows from standard Lyapunov arguments; see, for example, Goodwin et al. (2005, Theorem 4.3.1). □

4. SIMULATION RESULTS

Consider the linear system (1) with matrices

$$A = \begin{bmatrix} 1.70 & -0.72 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and output $y_k = [1 \quad 0.2]\xi_k$. The parameters of the backlash function in (3) are $m = 1$, $r = 0.3$, $\ell = -0.3$. The rate constraint in (10) is $\Delta = 1$. In the objective function (13)–(14) we set $N = 5$, $Q = I$ and $R = 0.01$.

We first designed a RHOptC computed for the linear system only under rate constraints, that is, *without backlash compensation*. Figure 3 shows the resulting output and input responses when there is no backlash in the loop. The same controller was simulated after introducing backlash in the loop as in Figure 2. The resulting output response and the signals at the input and output of the backlash nonlinearity are plotted in Figure 4. We can see that the presence of backlash introduces oscillations in the responses.

Secondly, we simulated the closed loop system of Figure 2 under RHOptC *with backlash compensation*, as described in Section 2. The resulting output response and the signals at the input and output of the backlash nonlinearity are plotted in Figure 5. We can see that the optimal controller compensates the backlash oscillation effect while maintaining the performance close to that without backlash (Figure 3).

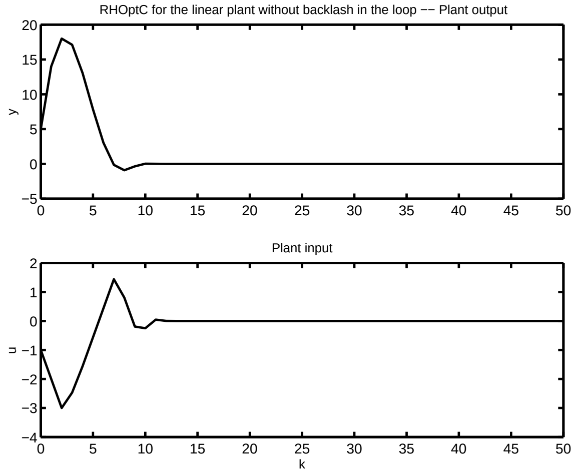

Fig. 3. Linear system output (top) and input (bottom) for RHOptC for the linear system without backlash in the loop.

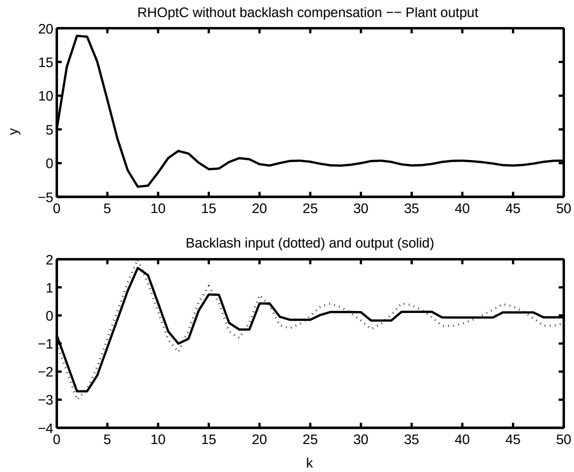

Fig. 4. Linear system output (top) and backlash input and output (bottom) for RHOptC for the linear system without backlash compensation.

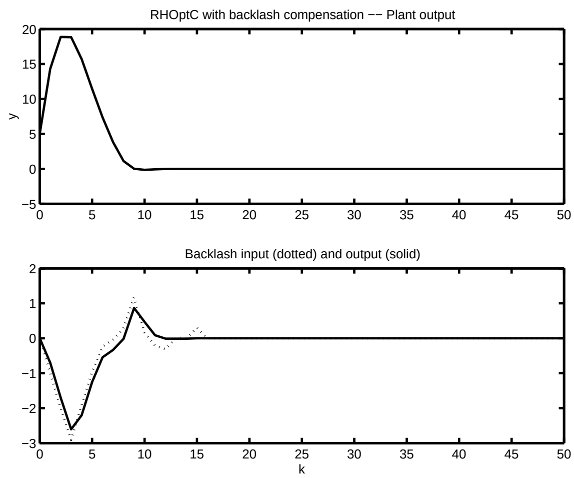

Fig. 5. Linear system output (top) and backlash input and output (bottom) for RHOptC with backlash compensation.

We next simulated the closed loop system of Figure 2 under the SVD algorithm *with backlash compensation*, as described in Section 3. An interior feasible solution was obtained by making the constraints tighter and finding a feasible boundary point for this more restrictive problem. The resulting output response and the signals at the input and output of the backlash nonlinearity are plotted in Figure 6. We can see that the SVD strategy eliminates the oscillation and gives a slightly slower response than RHOptC.

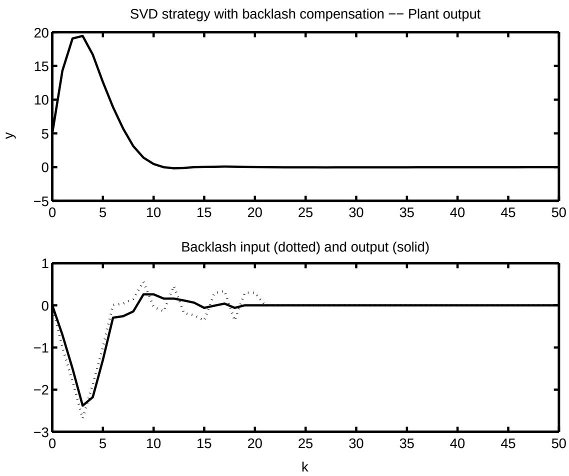

Fig. 6. Linear system output (top) and backlash input and output (bottom) for the SVD algorithm with backlash compensation.

Finally, Figure 7 shows a comparison of the output responses for the three controllers just discussed.

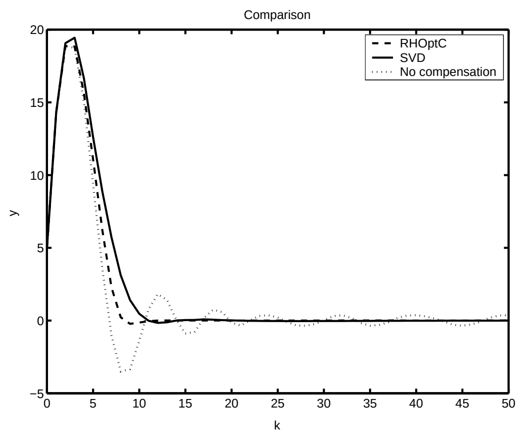

Fig. 7. Linear system output under the three different controllers discussed.

5. CONCLUSIONS

We have presented an algorithm for backlash compensation at the input of a stable linear system under rate constraints. The algorithm is based on the SVD of the Hessian of the QPs arising in the receding horizon optimal control problem for the same system. Simulation examples have shown that performance degradation is small with respect to the optimal solution. In addition, the computational load is smaller since a feasible rather than optimal solution has to be obtained at each step.

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