MODEL PREDICTIVE CONTROL OF AUTOMOTIVE POWERTRAINS WITH BACKLASH

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Abstract: In automotive powertrains, the existence of backlash causes driveability problems, which to some extent are remedied by the engine control system. The control problem has a constrained, minimum-time character, which motivates an investigation of the usability of model predictive control, MPC, in this application. Recent developments in MPC theory make an off-line calculation of the control law possible. This makes MPC more attractive for implementation in fast control loops such as the one under study here. The results indicate that MPC has a potential in this application. However, the off-line computation time is significant, and further robustness investigations are needed.

Keywords: Backlash, Automotive control, Predictive control, Piecewise linear controllers, Linear quadratic regulators

1. INTRODUCTION

Backlash is a problem in powertrain control. The sources of backlash are mainly play between gears in the final drive and in the gearbox. Backlash introduces a hard nonlinearity in the powertrain.

When the driver goes from engine braking to acceleration or vice versa, so called tip-in / tip-out maneuvers, the backlash is traversed, and potentially uncomfortable "shunt and shuffle" phenomena are experienced. During the backlash traverse, no torque is transmitted through the shaft. Then, when contact is achieved, the impact results in a large shaft torque and a sudden acceleration of the vehicle. Engine control systems must compensate for the backlash. The goal of the control system is to traverse the backlash as fast as possible, but without a large acceleration derivative (*jerk*) at the contact instant. One existing control strategy today is to first control the engine torque to a low positive value. After a pre-specified time, under which the backlash is supposed to have been traversed, the controller starts to follow the driver's torque request. With respect to the backlash traverse, this is an open-loop controller. A short review of feedback strategies is found in (Lagerberg and Egardt, 2002), where some of the strategies are also evaluated.

In (Lagerberg, 2004) is described how *open-loop* optimization can be used to find the optimal control signal trajectory, and get a theoretical lower limit on the time needed to perform a backlash traverse. The results presented there resemble the results in this paper, but here, *feedback* control laws are also computed.

Model predictive control, MPC, (Maciejowski, 2002; Mayne *et al.*, 2000), is a method for combining open-loop optimal control calculations with feedback. In each sampling interval of the controller, an optimal open-loop solution is calculated, for a specified prediction horizon, and the first control step in this solution is taken. For the next sampling instant, a new optimal solution is calculated. This implies a large computational burden on the controller, and MPC is mostly used in processes with very slow dynamics, such as in the process industry. MPC is recently used also in automotive applications, e.g. (Terwen *et al.*, 2004; Rückert *et al.*, 2004).

The MPC theory is recently made attractive for implementation also in systems with fast dynamics. For linear and piecewise affine systems, it is possible to perform an off-line calculation of an optimal feedback control law that satisfies given constraints. See e.g. (Morari *et al.*, 2003; Kerrigan and Mayne, 2002). This paper describes an application of this theory to the automotive powertrain with backlash. Other automotive applications are reported e.g. in (Bemporad *et al.*, 2001; Borrelli *et al.*, 2001).

The paper is organized as follows: Next section gives a brief description of model predictive control design for piecewise affine systems. Section 3 presents the powertrain model used in this paper. In Section 4, the control problem is further defined. Section 5 describes the synthesis of the MPC-based controller, and simulations of the resulting controller are discussed in Section 6. The paper ends with conclusions in Section 7.

2. MPC FOR PIECEWISE AFFINE SYSTEMS

A piecewise affine, PWA, system is defined as

$$x(k+1) = A_i x(k) + B_i u(k) + f_i \quad (1)$$
$$L_i x + E_i u \leq W_i \quad (2)$$
$$\text{if} \quad x(k) \in \mathcal{D}_i, \quad i = 1, \ldots, I_f \quad (3)$$

where $\mathcal{D}_i$ is a polytope defined by

$$\mathcal{D}_i = \{x \in \mathbb{R}^n | H_i x \leq K_i\}, \quad i = 1, \ldots, I_f \quad (4)$$

and I_f is the number of dynamics that represent the system. ($H_i \in \mathbb{R}^{s_i \times n}$, $K_i \in \mathbb{R}^{s_i}$ where s_i is the number of scalar inequalities required to define the polytope.)

For this class of systems it is possible to make an off-line control law synthesis. In this paper, a currently developed MATLAB toolbox, the Multi-Parametric Toolbox, MPT, (Kvasnica *et al.*, 2004) is used for this synthesis. The toolbox contains a collection of algorithms for solving constrained optimal control problems by multi-parametric methods. For PWA systems, the general optimization problem that the MPT toolbox solves is

$$\min_{u(0),\ldots,u(N-1)} \|Q_f x(N)\|_l + \sum_{k=0}^{N-1} \|Qx(k)\|_l + \|Ru(k)\|_l \quad (5)$$

subject to

$$x(k+1) = A_i x(k) + B_i u(k) + f_i \quad (6)$$
$$L_i x + E_i u \leq W_i, \quad x(k) \in \mathcal{D}_i, \quad (7)$$
$$i = 1, \ldots, I_f, \quad k = 1, \ldots, N-1 \quad (8)$$
$$x(N) \in \mathcal{X}_{set} \quad (9)$$

and where Q, Q_f, R are appropriate weighting matrices for the chosen norm $l \in \{1, 2, \infty\}$. $\mathcal{X}_{set}$ in (9) is a polytope that defines a terminal constraint. This is used as a "setpoint" for the controller. It is also possible to define the minimum time optimization problem

$$\min_{u(0),\ldots,u(N-1)} N \quad (10)$$

subject to the same constraints as above.

Multi-parametric programming is used to find the optimal control sequence $\{u(k)\}_{k=0}^{N-1}$ parameterized by the initial state $x(0)$. This parameterization is piecewise affine, and the first control signal in the sequence can be regarded as a state feedback control law

$$u(k) = F_r x(k) + G_r, \quad if \quad x(k) \in \mathcal{P}_r \quad (11)$$

where $\mathcal{P}_r$ is a polytope defined by

$$\mathcal{P}_r = \{x \in \mathbb{R}^n | H_r^c x \leq K_r^c\}, \quad r = 1, \ldots, R \quad (12)$$

Note that in general, each system dynamics $\mathcal{D}_i$ may contain more than one controller partition, $\mathcal{P}_r$.

The MPT synthesis algorithm for PWA systems, described in (Grieder *et al.*, 2004), starts in the target set, $\mathcal{X}_{set}$ and iteratively searches the state space for polytopes from where the target set can be reached in 1,2,3,... steps. This iteration proceeds until all of the allowed state space, $\bigcup_{i=1}^{I_f} \mathcal{D}_i$, is explored, or until no more polytopes can be found. After this iteration, multi-parametric programming is used to find feedback controllers of the form (11) for all the found polytopes, and stored in a table together with the polytopes they are valid on. The union of all found polytopes is called the controllable set, $\mathcal{K}_\infty^{\mathrm{PWA}}$.

The synthesis algorithm is computationally intense, and already for the relatively small sized problem under study here, the computation time is substantial (hours to days on a standard desktop PC). Therefore, the iterations described above may be interrupted when a large enough subset of the state space is explored, resulting in the controllable set $\mathcal{K}_N^{\mathrm{PWA}}$, where N is the maximum number of steps needed to reach the target set.

Although the off-line synthesis is time-consuming, the on-line implementation may be very fast. At each sampling instant, the current state is used to find the appropriate controller partition, $\{\mathcal{P}_r\}_{r=1}^{R}$. The corresponding control law (11) is found in a look-up table, and the control signal is applied to the system. However, the look-up table may become large, in this paper approximately 10000 partitions. This may lead to extensive look-up times and memory requirements in the implementation.

3. POWERTRAIN MODEL

The powertrain model under consideration in this paper is seen in Figure 1, and the following notation is used: The indices m and l refer to motor and load respectively. J_m, J_l [kgm^2] are moments of inertia and

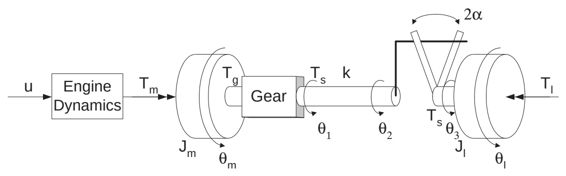

Fig. 1. Powertrain model.

b_m and b_l [Nm/(rad/s)] are viscous friction constants. k [Nm/rad] is the shaft stiffness. T_m, T_g, T_s and T_l [Nm] are torques at the engine output, at the gearbox input, at the gearbox output and the load input, and the road load respectively. u [Nm] is the requested engine torque. i [rad/rad] is the gearbox ratio. 2α [rad] is the backlash gap size. θ_m and θ_l [rad] are the angular positions of motor and load. θ_1, θ_2 and θ_3 [rad] are the angular positions of the indicated positions on the shaft.

As seen in the figure, the powertrain consists of two rotating masses, one representing the engine flywheel (*motor*) and one representing the vehicle mass (*load*) respectively:

$$J_m\ddot{\theta}_m + b_m\dot{\theta}_m = T_m - T_g \tag{13}$$

$$J_l\ddot{\theta}_l + b_l\dot{\theta}_l = T_s - T_l \tag{14}$$

The masses are connected by a gearbox (with fixed gear ratio),

$$T_g = T_s/i, \theta_3 = \theta_l, \theta_1 = \theta_m/i \tag{15}$$

and a flexible shaft with a backlash of size 2α. With the backlash modeled as a dead-zone nonlinearity, the shaft torque is described by:

$$T_s = k\begin{cases}\theta_1 - \theta_3 - \alpha & \text{if } \theta_1 - \theta_3 \geq \alpha \\ 0 & \text{if } |\theta_1 - \theta_3| < \alpha \\ \theta_1 - \theta_3 + \alpha & \text{if } \theta_1 - \theta_3 \leq -\alpha\end{cases} \tag{16}$$

where the three modes are referred to as the positive contact ($co+$), backlash (bl) and negative contact ($co-$) modes respectively.

The engine dynamics is modeled as a first order system with time constant τ_{eng}:

$$\dot{T}_m = (u - T_m)/\tau_{eng} \tag{17}$$

For the MPC-solution, the computational complexity of the controller design is increasing rapidly with the size of the plant model. Therefore the model used here is a simplification as compared to the model in e.g. (Lagerberg and Egardt, 2002) or (Lagerberg, 2004). Specifically, shaft damping and engine delay are neglected here.

4. PROBLEM FORMULATION

The powertrain control system is suggested to be switching between two different controllers. One is a vehicle acceleration controller, designed for the powertrain strictly in one of the contact modes. The other controller is an MPC-based controller, which is designed to traverse the backlash gap in an optimal way.

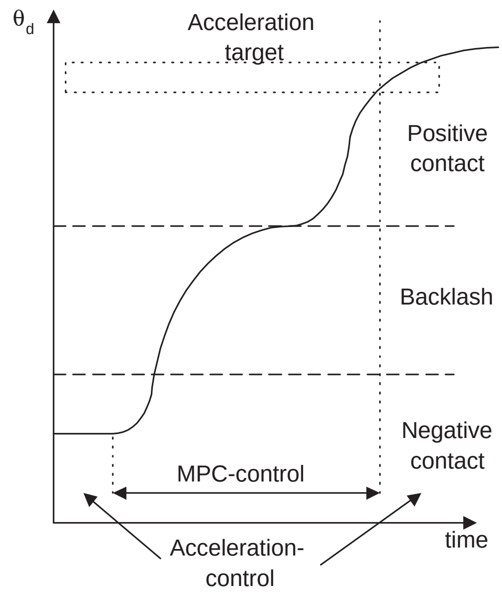

Fig. 2. A schematic tip-in sequence. The state-space is divided into three regions with different dynamics. The tip-in sequence starts and ends in acceleration control, while during the backlash traverse, the MPC-controller is active. The MPC-controller is designed to reach the target in minimum time, with constraints on the jerk at the contact instant. $\theta_d = \theta_1 - \theta_3$

The MPC-based controller will typically be involved in tip-in and tip-out maneuvers, and it is the controller under study in this paper.

In terms of the powertrain model described above, the control problem related to a tip-in maneuver can be formulated as follows, see also Figure 2: The initial condition for a tip-in is the powertrain in negative contact mode, with a retardation of the system. At the starting point, the driver steps on the accelerator pedal, and requests a positive vehicle acceleration. This initiates the MPC-controller, which has as its goal to control the powertrain into positive contact, and then achieve a pre-specified acceleration. The transition into positive contact mode should be made with the vehicle jerk (acceleration derivative) below a specified level. This means that at the contact instant, the relative speed between engine and vehicle sides should be low. The MPT formulation of the problem does not allow such explicit constraints at a switching instant between two affine dynamics. Instead, the problem is divided into two consecutive subproblems: First, the system is controlled to almost contact between the sides, and with a small relative speed. Then, the system is controlled into the positive contact mode, and to the specified acceleration. The subproblems are solved individually, and will be referred to as *phases* in the following.

The MPC controller is designed to reach a pre-specified acceleration. However, the driver's requested acceleration may vary. It is possible to reformulate the MPC control problem into a tracking problem, but this implies the inclusion of more states in the control design model, and hence increased computation time for the controller synthesis. In (Lagerberg and Egardt, 2004), other approaches to the MPC formulation of this problem are presented.

As the acceleration controller mentioned above, a linear state feedback controller (LQ) is used in this paper. It is roughly tuned to reduce the oscillations when the system is in contact mode. Integral action is used to reach the desired acceleration. Bumpless transfer is used to initialize the control signal to the same value as for the MPC controller when the LQ controller is activated.

5. MPC CONTROLLER DESIGN

In this section, the problem definition above is transformed into the form presented in Section 2. To summarize, the problem set-up comprises the following: The system dynamics is formulated as a PWA system and the "setpoint", $\mathcal{X}_{set}$ is defined. Constraints on the allowed state and control variables are defined on the form (2).

5.1 PWA-formulation of dynamics

Define $\omega_m \hat{=} \dot{\theta}_m$, $\omega_l \hat{=} \dot{\theta}_l$ and $\theta_d \hat{=} \theta_1 - \theta_3 = \theta_m/i - \theta_l$, and the state vector $x = \begin{bmatrix}\omega_m & \omega_l & \theta_d & T_m\end{bmatrix}^T$. The model (13-17) can then be written as the PWA-system

$$\dot{x} = \begin{cases} A_{co+}x + Bu + f_{co+} & \text{if } \theta_d \geq \alpha \\ A_{co-}x + Bu + f_{co-} & \text{if } \theta_d \leq -\alpha \\ A_{bl}x + Bu + f_{bl} & \text{if } -\alpha < \theta_d < \alpha \end{cases} \tag{18}$$

where

$$A_{co+} = \begin{bmatrix} -\frac{b_m}{J_m} & 0 & -\frac{k}{J_m i} & \frac{1}{J_m} \\ 0 & -\frac{b_l}{J_l} & \frac{k}{J_l} & 0 \\ \frac{1}{i} & -1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_{eng}} \end{bmatrix} \tag{19}$$

$$A_{co-} = A_{co+} \tag{20}$$

$$A_{bl} = \begin{bmatrix} -\frac{b_m}{J_m} & 0 & 0 & \frac{1}{J_m} \\ 0 & -\frac{b_l}{J_l} & 0 & 0 \\ \frac{1}{i} & -1 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_{eng}} \end{bmatrix} \tag{21}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau_{eng}} \end{bmatrix}^T \tag{22}$$

$$f_{co+} = \begin{bmatrix} \frac{k\alpha}{J_m i} & \frac{-k\alpha - T_l}{J_l} & 0 & 0 \end{bmatrix}^T \tag{23}$$

$$f_{co-} = \begin{bmatrix} -\frac{k\alpha}{J_m i} & \frac{k\alpha - T_l}{J_l} & 0 & 0 \end{bmatrix}^T \tag{24}$$

$$f_{bl} = \begin{bmatrix} 0 & -\frac{T_l}{J_l} & 0 & 0 \end{bmatrix}^T \tag{25}$$

Here, the load disturbance, T_l, is taken as a constant, which furthermore is set to zero in the current setting. This is due to the restrictions on the allowable class of systems in the current MPT implementation, that the origin should be an equilibrium point for some of the dynamics. The road friction and air resistance parts of T_l are instead included in the load friction coefficient b_l. The MPT problem formulation can also be extended with a description of disturbances in the states. This can be used to gain some robustness to e.g. road slope disturbances.

The regions where the respective affine dynamics are valid is defined by the inequalities in (18), which are written as polytopes $\mathcal{D}_i$, $i \in \{co+, co-, bl\}$ from (4).

The MPC theory is based on a discrete-time system description, so a discretization of the PWA model above is used for the MPT solution.

5.2 Target sets

The setpoints or target sets, $\mathcal{X}_{set}$, are polytopes, defined by inequalities of the form $Hx \leq K$.

5.2.1. Target set for phase 1 The first phase should end when the engine and vehicle sides of the backlash is almost in contact, and with a small relative speed. The engine torque should be close to zero at the contact instant, in order to get a globally optimal solution for the total backlash traverse. This is seen in the open-loop optimal results in (Lagerberg, 2004). These conditions are formulated as

$$\theta_{d,\min,1} \leq \quad \theta_d \quad \leq \theta_{d,\max,1} \tag{26}$$

$$\omega_{\text{diff},\min,1} \leq \omega_m/i - \omega_l \leq \omega_{\text{diff},\max,1} \tag{27}$$

$$T_{m,\min,1} \leq \quad T_m \quad \leq T_{m,\max,1} \tag{28}$$

where $\theta_{d,\min,1}$, $\theta_{d,\max,1}$ are close to, but smaller than α and $\omega_{\text{diff},\min,1}$, $\omega_{\text{diff},\max,1}$, $T_{m,\min,1}$, $T_{m,\max,1}$ are close to zero.

Written on polytope form this becomes:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 1/i & -1 & 0 & 0 \\ -1/i & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix} x \leq \begin{bmatrix} \theta_{d,\max,1} \\ -\theta_{d,\min,1} \\ \omega_{\text{diff},\max,1} \\ -\omega_{\text{diff},\min,1} \\ T_{m,\max,1} \\ -T_{m,\min,1} \end{bmatrix} \tag{29}$$

5.2.2. Target set for phase 2 The final target set for a tip-in maneuver is chosen as achieving a minimum load acceleration, $a_l \geq a_{l,\min,2}$. For positive $a_{l,\min,2}$ and positive vehicle speed, ω_l, this can only be achieved in the $co+$-mode. Using the dynamics of this mode, the relation can be written as:

$$a_l = \dot{\omega}_l = \begin{bmatrix} 0 & -\frac{b_l}{J_l} & \frac{k}{J_l} & 0 \end{bmatrix} x - \frac{k\alpha}{J_l} \geq a_{l,\min,2} \quad (30)$$

or

$$\begin{bmatrix} 0 & \frac{b_l}{J_l} & -\frac{k}{J_l} & 0 \end{bmatrix} x \leq -a_{l,\min,2} - \frac{k\alpha}{J_l} \quad (31)$$

Additionally, the driveline oscillations should be reduced when the second target set is reached. This is accomplished by a speed difference limit similar to (27), but now in contact mode. In addition, an acceleration difference limit is required:

$$\dot{\omega}_{\text{diff,min,2}} \leq \dot{\omega}_m/i - \dot{\omega}_l \leq \dot{\omega}_{\text{diff,max,2}} \quad (32)$$

Since a positive acceleration is required as above, only the positive contact mode is active here. Using that dynamics, the inequality can be written:

$$\dot{\omega}_{\text{diff,min,2}} \leq H_C x + K_C \leq \dot{\omega}_{\text{diff,max,2}} \quad (33)$$

with

$$H_C = \begin{bmatrix} -\frac{b_m}{J_m i} & \frac{b_l}{J_l} & \left(-\frac{k}{J_m i^2} - \frac{k}{J_l}\right) & \frac{1}{J_m i} \end{bmatrix} \quad (34)$$

$$K_C = \left(\frac{k}{J_m i^2} + \frac{k}{J_l}\right)\alpha \quad (35)$$

The target set is defined as the combination of (31), (27) and (33):

$$\begin{bmatrix} 0 & \frac{b_l}{J_l} & -\frac{k}{J_l} & 0 \\ 1/i & -1 & 0 & 0 \\ -1/i & 1 & 0 & 0 \\ & H_C & & \\ & -H_C & & \end{bmatrix} x \leq \begin{bmatrix} -a_{l,\min,2} - \frac{k\alpha}{J_l} \\ \omega_{\text{diff,max,2}} \\ -\omega_{\text{diff,min,2}} \\ \dot{\omega}_{\text{diff,max,2}} - K_C \\ -\dot{\omega}_{\text{diff,min,2}} + K_C \end{bmatrix} \quad (36)$$

5.3 *Constraints*

The engine can deliver torque in a limited interval

$$T_{m,\min} \leq T_m \leq T_{m,\max} \quad (37)$$

In order to allow for faster decreases of the engine torque than dictated by the engine time constant (τ_{eng}), the control signal is constrained to the interval

$$u_{\min} \leq u \leq u_{\max} \quad (38)$$

where $u_{\max} = T_{m,\max}$ while $u_{\min} \leq T_{m,\min}$ (here, a factor 10 lower). This is motivated by the fact that it is possible (at least theoretically) to completely skip the firing of a cylinder in a spark-ignited engine.

In the first phase, the total shaft displacement, θ_d, is restricted to be lower than the backlash width α in order to avoid overshoot into the positive contact phase before the target set is reached:

$$\theta_d = \theta_1 - \theta_3 \leq \alpha \quad (39)$$

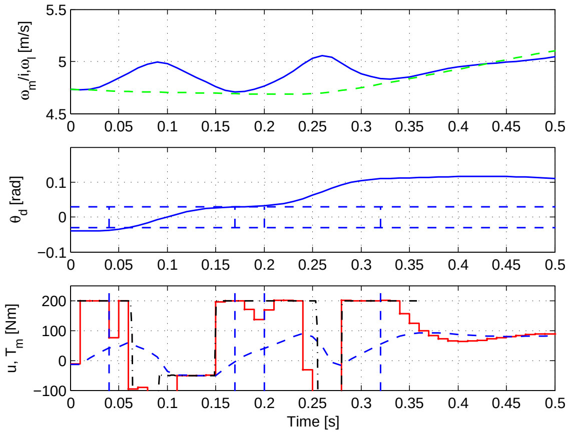

Fig. 3. Simulation of a backlash traverse. Upper plot: Engine speed, ω_m, (solid) and wheel speed, ω_l, (dashed), both scaled to vehicle speed. Middle plot: Total shaft displacement. The backlash limits ($\pm\alpha$) are indicated by dashed horizontal lines. Lower plot: Control signal, requested engine torque, u (solid), and engine torque, T_m, (dashed). The open-loop optimal control signal is also shown (dash-dotted). Dashed vertical lines represent switches between different controller phases.

5.4 *Controller synthesis*

For the two phases described above, MPC controllers are synthesized using the MPT Toolbox. Care must be taken that the target set of the first controller is enclosed in the set of controllable states for the second controller, $\mathcal{X}_{set,1} \subset \mathcal{K}_{N,2}^{\text{PWA}}$, so that a switch between the controllers is possible.

Due to the extensive off-line computation times, the $\mathcal{K}_N^{\text{PWA}}$ sets may not be large enough to incorporate all possible initial values for the transients the controllers are designed for. In these cases, the acceleration mode (LQ) controller is used until $\mathcal{K}_N^{\text{PWA}}$ is entered. This will be seen in the simulation results below.

6. SIMULATION RESULTS

The simulations start with the system in constant retardation and in negative contact. At t=0 s, the acceleration setpoint is changed to $a_l = 1.5$ m/s^2, which approximately corresponds to $\theta_d = 0.1$ rad. The controllers have a sampling time of 10 ms.

In Figure 3 is seen that at t=0.17 s, the system has reached the first target set (almost contact), and switches to the second phase. The final target set is reached at t=0.32 s. After this point, the acceleration (LQ) controller takes over.

As described above, the LQ controller is used as a "back-up" controller until the controllable region for phase 1 is reached. In the figure, phase 1 is reached at t=0.04 s. During the LQ period, the control signal is taking on its maximum value. This is due to the tuning of the LQ controller and that it has the driver's requested acceleration as setpoint. A maximum control signal during the initial time steps is intuitively correct, and according to the results in (Lagerberg, 2004), this is in fact optimal. Therefore, the back-up strategy does not seem to reduce the performance significantly.

Similarly to phase 1, the LQ controller is used as back up in the beginning of phase 2, between t=0.17 and t=0.20 s, after which the second phase controller takes over. Since the second phase MPC controller starts with maximum control signal for a number of samples, it would probably be better to use the maximum control signal during the back up period.

7. CONCLUSIONS

The presented controller designs are examples of the use of recently developed MPC control algorithms to a realistic application. The experience from this study is that off-line computed MPC gives promising results. In comparison with the open-loop optimal results in (Lagerberg, 2004), the performance of the presented controllers are similar, see Figure 3. For the powertrain model at hand, it is possible to come close to the theoretically optimal open-loop performance with a feedback controller.

However, more investigations are needed. For example, the model complexity has to be very low in order to achieve a controller within reasonable time. In the powertrain application, the major simplification made is that delays are ignored. The effects of these simplifications need to be investigated.

Robustness of the controller to disturbances and model uncertainties is not considered here, something that is of great importance for a successful implementation of this control strategy in a real application. The MPT toolbox provides options to include disturbance models in the problem formulation. This may be used to improve robustness also to model uncertainties such as the neglected delay.

The controllers in this paper assume that all state variables are measured. A state observer for the powertrain system is presented e.g. in (Lagerberg and Egardt, 2003). A combination of observer and controller is a suggested future research direction.

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