VIBRATION SUPPRESSION SHAPE FILTER GENERATION FROM WINDOW FUNCTIONS

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Abstract: A control profile is generated which suppresses one or more resonant dynamics in a flexible dynamic system. This control profile can be used as a velocity profile, or as a shape filter to an arbitrary control command. The robustness can be arbitrarily improved, which brings about a smoother profile. The technique can be applied to both open-loop and closed-loop systems. *Copyright ©2005 IFAC*

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1. INTRODUCTION

Control of flexible structures has been extensively studied in recent years. Flexible structures such as high-speed disk drive actuators require extremely precise positioning under very tight time constraints. Whenever a fast motion is commanded, residual vibration in the flexible structure is induced, which increases the settling time. One solution is to design a closed-loop controller to damp out vibrations caused by the command inputs and disturbances to the plant. However, the resulting closed-loop response may still be too slow to provide an acceptable settling time, and the closed-loop control is not able to compensate for high frequency residual vibration which occurs beyond the closed-loop bandwidth. An alternative approach is to develop an appropriate reference trajectory that is able to minimize the excitation energy imparted to the system at its natural frequencies.

Fig. 1. A typical mechanical flexible system.

Fig. 1 shows a typical mechanical flexible system, where $\frac{1}{s}$ is an integrator, K_v is a velocity constant

gain, and K_p is a position constant gain. The high frequency modes can be described as a transfer function $R(s) = \lim_{n\to\infty} \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s + 1}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + 1}$ in which an infinite number of lightly damped resonant structures is possible. The goal of vibration suppression trajectory generation is to find a fast input trajectory, under some physical constraint, with minimum possible residual vibration. The position reference input can be generated from a step movement command $s(t) = S \cdot 1(t)$, through a finite support filter, $f(t)$, $0 \le t \le T$, where T is the time duration of the finite support filter. To guarantee that the filtered command reaches the same set point as the step movement command, the integral of $f(t)$ must be equal to 1, i.e., $\int_0^T f(t)dt = 1$. This finite support filter $f(t)$, $0 \le t \le T$, which generates a vibration suppression position reference profile is called a "vibration suppression shape filter", or simply a "shape filter."

To suppress all the high frequency resonant dynamics in a flexible system, Zhou and Misawa (Zhou and Misawa, 2005*b*) have proposed vibration suppression shape filter and control profile generation based on optimal energy concentration functions. In practical system, a lower resonance frequency mode may exist which is located far from the high frequency resonance modes. If the low frequency is chosen to be a bandwidth in (Zhou and Misawa, 2005*b*) for the vibra-

tion suppression control profile generation, the time duration of the profile is inefficiently increased. In this case, Zhou and Misawa (Zhou and Misawa, 2005a) have proposed vibration suppression shape filter and control profile generation for a specific resonance mode. In (Zhou and Misawa, 2005a), a vibration suppression shape filter and a vibration suppression control profile were generated based on a rectangle window and the continuous-time Input Shaping® (Singer and Seering, 1990) was shown to be a special case of the discrete-time rectangle window based shape filter with a particular sampling period. In this paper, vibration suppression shape filter and control profile generation based on other window functions is studied.

2. VIBRATION SUPPRESSION SHAPE FILTER FOR A SPECIFIC RESONANCE MODE

From the analysis of the relationship between control input and residual vibration of a damped resonant mode with the natural frequency ω_i and damping ratio ζ_i, the following conclusion is drawn (Zhou and Misawa, 2005a).

Conclusion 2.1. If there exists a finite support base function $h(t)$, $0 \leq t \leq T_0$, such that $H(\omega_{di}) = H(\sqrt{1 - \zeta_i^2}\omega_i) = 0$, then, $h(t)$ may have two possible properties:

(1) The function $\frac{h(t)}{e^{\zeta_i\omega_i t}}$ is a control profile candidate to eliminate the residual vibration caused by the resonant mode with the natural frequency ω_i and the damping ratio ζ_i.

(2) The function $\frac{h(t)}{e^{\zeta_i\omega_i t}}$ with a constraint $\int_0^{T_0} \frac{h(t)}{e^{\zeta_i\omega_i t}} dt = 1$, is a vibration suppression shape filter that can be used to filter out an arbitrary control profile, and the shaped control profile eliminates the residual vibration caused by the resonant mode with the natural frequency ω_i and the damping ratio ζ_i.

The proof of Conclusion 2.1 is given in (Zhou, 2005). The philosophy of Conclusion 2.1 advocates that a shape filter should be able to be both a traditional shaping filter, as well as a possible command signal in its own right. Let $f_1(t) = \frac{h(t)}{e^{\zeta_i\omega_i t}}$, then robustness of the properties in Conclusion 2.1 can be improved by the following filter operation,

$$f_n(t) = \int_0^t f_{n-1}(t-\tau)f_1(\tau)d\tau, \quad n \geq 2, \quad (1)$$

and the resultant spectrum of $f_n(t)$ is $F_n(\omega) = F_1^n(\omega)$. Here, the control profile or shape filter $f_1(t)$ in (1) is said to have the robustness of order 1. The control profile or shape filter $f_n(t)$ generated from the filter operation in (1) is said to have the robustness of order n. In the case that $f(t)$ is a non-continuous

impulse function, the robustness improvement method (1) becomes the idea advocated by Singer and Seering (Singer and Seering, 1990).

In (Zhou and Misawa, 2005a), a vibration suppression shape filter and a vibration suppression control profile were generated based on a rectangle window. In the following sections, smooth shape filter generation based on other window functions is studied. Instead of analyzing the continuous-time case, the discrete-time case is directly generated.

3. SHAPE FILTER GENERATION USING HANNING WINDOW

If the sampling period is T_s sec and the total discrete-time sequence has $M + 1$ impulses, the Hanning window function is

$$h[k] = \begin{cases} 0.5 - 0.5\cos(2\pi k/M), & \text{if } 0 \leq k \leq M, \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

If $H(\omega_{di}T_s) = H(\sqrt{1 - \zeta_i^2}\omega_i T_s) = 0$, then $\omega_{di}T_s = \frac{4\pi}{M}$ and $M = \frac{4\pi}{\omega_{di}T_s}$. If M is a positive integer, a smooth shape filter can be generated as $f[k] = \frac{h[k]/e^{\zeta_i\omega_i kT_s}}{\sum_{m=0}^{M} h[m]/e^{\zeta_i\omega_i mT_s}}$. To improve robustness, the filter operation in (1) is performed. Let $f_1[k] = f[k]$, then robust shape filters $f_2[k], f_3[k], \ldots$ can be generated following the filter operation in (1). Since the sequence $h[k]$ is known, the sequence $f[k]$ can be generated through a simple numerical calculation.

For a second-order harmonic oscillator of the natural frequency ω_i rad/sec and the damping ratio ζ_i, i.e., $\frac{\omega_i^2}{s^2 + 2\zeta_i\omega_i s + \omega_i^2}$, the magnitude of the total response immediately after the Nth impulse is given by (Singer and Seering, 1990)

$$V_{amp}(\omega_i, \zeta_i) = e^{-\zeta_i\omega_i t_N} \frac{\omega_i}{\sqrt{1 - \zeta_i^2}} \sqrt{(AC(\omega_i, \zeta_i))^2 + (AS(\omega_i, \zeta_i))^2}, \quad (3)$$

where

$$AC(\omega_i, \zeta_i) = \sum_{k=1}^{N} A_k e^{\zeta_i\omega_i t_k} \cos\left(\omega_i\sqrt{1 - \zeta_i^2} t_k\right),$$

$$AS(\omega_i, \zeta_i) = \sum_{k=1}^{N} A_k e^{\zeta_i\omega_i t_k} \sin\left(\omega_i\sqrt{1 - \zeta_i^2} t_k\right).$$

The variables A_k and t_k are the amplitude and time location at which the impulse occurs, N is the total number of impulses, and t_N is the time of the last impulse. The sensitivity of the impulse shape filter can be displayed graphically by a sensitivity curve: a plot of residual vibration amplitude versus frequency error. Let $q = \omega_{actual}/\omega_{model}$, (3) becomes

$$V_{amp}(q\omega_i, \zeta_i) = e^{-\zeta_i q\omega_i t_N} \frac{q\omega_i}{\sqrt{1 - \zeta^2}} \sqrt{(AC(q\omega_i, \zeta_i))^2 + (AS(q\omega_i, \zeta_i))^2}, \quad (4)$$

where $\omega_i = \omega_{model}$, $\omega_{actual} = q\omega_i$, $AC(q\omega_i, \zeta_i) = \sum_{k=1}^{N} A_k e^{\zeta_i q\omega_i t_k} \cos\left(q\omega_i \sqrt{1-\zeta_i^2} t_k\right)$, and $AS(q\omega_i, \zeta_i) = \sum_{k=1}^{N} A_k e^{\zeta_i q\omega_i t_k} \sin\left(q\omega_i \sqrt{1-\zeta_i^2} t_k\right)$. Since for any finite impulse shape filter $f(t)$, $0 \leq t \leq T_0$, the integral of $f(t)$ is $\int_0^{T_0} f(t)dt = 1$ and the rigid body movement amplitude can be assumed as $\int_0^{T_0} f(t)dt = 1$. The residual vibration level can be defined as a percentage of the rigid body motion amplitude, i.e., $\frac{V_{amp}(q\omega_i, \zeta_i)}{\int_0^{T_0} f(t)dt} = V_{amp}(q\omega_i, \zeta_i)$. For the impulse shape filter case, $\int_0^{T_0} f(t)dt = \sum_{k=1}^{N} A_k = 1$. The residual vibration level (4) can be plotted for the Hanning based shape filter $f_1[k]$ and rectangle based shape filter $f_1[k]$. The sampling period T_s is chosen to be $\frac{\pi}{100\omega_{di}}$. Fig. 2 shows the sensitivity curve of the rectangle based shape filter (Zhou and Misawa, 2005a) with $\omega_{model} = 1$ rad/sec and different damping $\zeta_i = 0, 0.05, 0.2$. Fig. 3 shows the sensitivity curve of the Hanning based shape filter $f_1[k]$ with $\omega_{model} = 1$ rad/sec and different damping $\zeta_i = 0, 0.05, 0.2$. Clearly, the robustness of the Hanning based shape filter is increased both at the model natural frequency and the unmodeled high frequency.
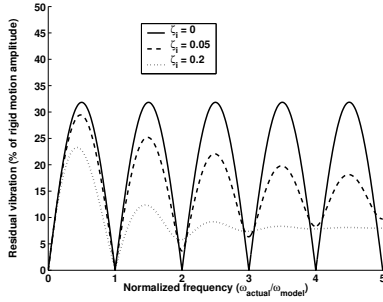

Fig. 2. Rectangle based shape filter $f_1[k]$ sensitivity plot versus actual natural frequency.
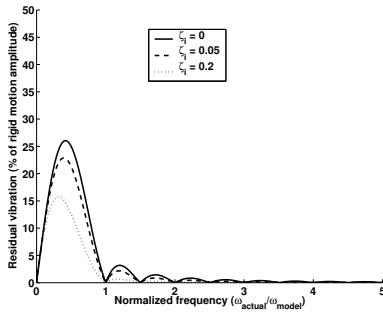

Fig. 3. Hanning based shape filter $f_1[k]$ sensitivity plot versus actual natural frequency.

The definition in (4) is different from the sensitivity concept of Singer (Singer and Seering, 1990) and Singhose (Singhose, 1997). In their definition, the sensitivity curve is expressed as the magnitude of the total response immediately after the Nth impulse divided by the magnitude of the response with unit impulse occurring at time 0. Since the magnitude of the response with unit impulse occurring at time 0 is given by $\frac{\omega_i}{\sqrt{1-\zeta_i^2}}$, their definition of residual vibration level is $\frac{V_{amp}(q\omega_i, \zeta_i)}{\frac{\omega_i}{\sqrt{1-\zeta_i^2}} \sum_{k=1}^{N} A_k}$ or simply

$$e^{-\zeta_i q\omega_i t_N} \sqrt{(AC(q\omega_i, \zeta_i))^2 + (AS(q\omega_i, \zeta_i))^2}, \quad (5)$$

because $V_{amp}(q\omega_i, \zeta_i)$ exactly has the term $\frac{\omega_i}{\sqrt{1-\zeta_i^2}}$ and $\sum_{k=1}^{N} A_k = 1$. It is known that the magnitude of the response with unit impulse occurring at time 0, which is $\frac{\omega_i}{\sqrt{1-\zeta_i^2}}$, linearly increases with respect to actual undamped natural frequency ω_i if ζ_i is assumed to be a constant. Therefore, the definition (5) does not express the absolute residual vibration magnitude. For example, the definition (5) shows that the residual vibration level is 100% when the actual undamped natural frequency is 0. However, the true magnitude of the residual vibration given in (4) is 0 when the actual undamped natural frequency is 0.

Since the Hanning function starts and ends at zero, the Hanning based shape filter $f_1[k]$ can be used as a velocity candidate. Fig. 4 shows the rigid body acceleration, velocity, and position profiles generated from the Hanning based shape filter $f_1[k]$, with one resonant mode with parameters $\omega_i = 1$ rad/sec and damping ratio $\zeta_i = 0.2$. Discrete-time shape filter generation for an arbitrary sampling period can be implemented following the procedure in (Zhou, 2005).
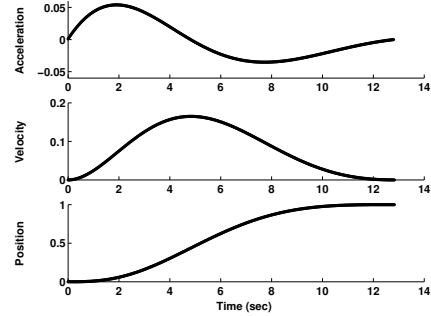

Fig. 4. Acceleration, velocity and position profiles from the Hanning based shape filter $f_1[k]$ with $\omega_i = 1$ rad/sec and $\zeta_i = 0.2$.

4. SHAPE FILTER GENERATION USING BLACKMAN WINDOW

If the sampling period is T_s sec and the total discrete-time sequence has $M + 1$ impulses, the Blackman window function is

$$h[k] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) \\ \quad + 0.08\cos(4\pi k/M), & \text{if } 0 \leq k \leq M, \\ 0, & \text{otherwise.} \end{cases}$$

If $H(\omega_{di}T_s) = H(\sqrt{1-\zeta_i^2}\ \omega_i T_s) = 0$, then $\omega_{di}T_s = \frac{6\pi}{M}$ and $M = \frac{6\pi}{\omega_{di}T_s}$. If M is a positive integer, a smooth shape filter can be generated as $f[k] =$

$\frac{h[k]/e^{\zeta_i \omega_i k T_s}}{\sum_{m=0}^{M} h[m]/e^{\zeta_i \omega_i m T_s}}$. Again, it is not necessary to derive the explicit form for $f[k]$. Since sequence $h[k]$ is known, the sequence $f[k]$ can be generated through a simple numerical calculation. Let $f_1[k] = f[k]$, then robust shape filters $f_2[k], f_3[k], \ldots$ can be generated following the filter operation in (1).

5. CAN HAMMING WINDOW BE USED TO SHAPE FILTER GENERATION?

If the sampling period is T_s sec and the total discrete-time sequence has $M + 1$ impulses, the Hamming window function is

$$h[k] = \begin{cases} \dfrac{25}{46} - \dfrac{21}{46}\cos{(2\pi k/M)}, & \text{if } 0 \le k \le M, \\ 0, & \text{otherwise.} \end{cases}$$

By calculation (Zhou, 2005), equation $H(\omega) = 0$ has no solution and the shape filter cannot be generated from Hamming window in theory by following Conclusion 2.1. However, approximately when $\omega \approx \frac{4\pi}{M}$, $|H(\omega)|$ approaches an extremely small number (Oppenheim, 1989). If $M = \frac{4\pi}{\omega_{di} T_s}$ and M is a positive integer, a smooth shape filter can be generated as $f[k] = \frac{h[k]/e^{\zeta_i \omega_i k T_s}}{\sum_{m=0}^{M} h[m]/e^{\zeta_i \omega_i m T_s}}$. In this case, since $|H(\omega_{di} T_s)| \approx 0$, the residual vibration cannot be eliminated in theoretical sense. But an extremely small $|H(\omega_{di} T_s)|$ implies the residual vibration is sufficiently suppressed. Again, it is not necessary to derive the explicit form for $f[k]$. Since the sequence $h[k]$ is known, the sequence $f[k]$ can be generated through a simple numerical calculation. Let $f_1[k] = f[k]$, then robust shape filters $f_2[k], f_3[k], \ldots$ can be generated following the filter operation in (1).

6. SHAPE FILTER GENERATION USING OTHER CONTINUOUS FUNCTIONS

From Conclusion 2.1, a smooth shape filter can be generated from a finite support smooth function $h(t)$, such that $H(\omega_{di}) = 0$. Some simple window functions have been used as base functions to generate the shape filters in the previous sections. Numerous smooth functions $h(t)$ may have the property of $H(\omega_{di}) = 0$ or $H(\omega_{di}) \approx 0$. So all of them can be used as base functions to generate the shape filters. Here, some window functions that possess the property are listed: $\cos^\alpha{(X)}$ window, Riesz window, Riemann window, de la Vallé-Poussin window, Tukey window, Bohman window, Poisson window, Hanning-Poisson window, Cauchy window, Gaussian window, Dolph-Chebeshev window, Kaiser-Bessel window, Barcilon-Temes window, Nuttall window, Modified Bartlett-Hanning window, and Others. These window functions were originally used for harmonic analysis (Harris, 1978), (Nuttall, 1981), (Ha and Pearce, 1989).

7. SHAPE FILTER GENERATION USING SEVERAL SMOOTH FUNCTIONS

A section of a base function $h[k]$ can be constructed from other smooth functions through products, sums, convolutions, integral, or other mathematical operations. If the constructed base function has the property of $H(\omega_{di} T_s) = 0$, then it can be used to generate the shape filters through Conclusion 2.1. Here, a simple example is given to show the basic idea.

Here, a base function $h[k]$ is generated from three Hanning window functions $h[k] = h_{s1}[k] + h_{s2}[k] + h_{s3}[k], 0 \le k \le 2M$, where

$$h_{s1}[k] = \begin{cases} \dfrac{1}{2} - \dfrac{1}{2}\cos{(\dfrac{\pi k}{M})}, & \text{if } 0 \le k \le 2M, \\ 0, & \text{otherwise,} \end{cases}$$

$$h_{s2}[k] = \begin{cases} C\left[\dfrac{1}{2} - \dfrac{1}{2}\cos{(\dfrac{2\pi k}{M})}\right], & \text{if } 0 \le k \le M, \\ 0, & \text{otherwise,} \end{cases}$$

$$h_{s3}[k] = \begin{cases} C\left[\dfrac{1}{2} - \dfrac{1}{2}\cos{(\dfrac{2\pi k}{M})}\right], & \text{if } M \le k \le 2M, \\ 0, & \text{otherwise,} \end{cases}$$

and C is a constant number. Fig. 5 shows a typical combination of functions $h_{s1}[k]$, $h_{s2}[k]$, $h_{s3}[k]$, and the resultant base function $h[k]$.
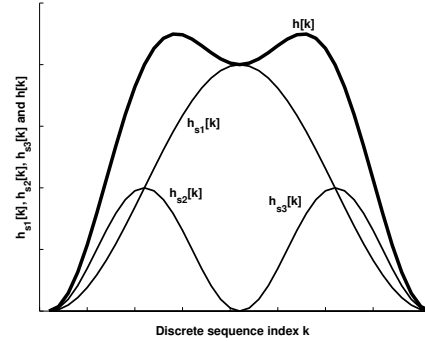

Fig. 5. A typical combination of functions $h_{s1}[k]$, $h_{s2}[k]$, $h_{s3}[k]$, and the resultant base function $h[k]$.

If $H(\omega_{di} T_s) = H(\sqrt{1 - \zeta_i^2}\,\omega_i T_s) = 0$, then $\omega_{di} T_s = \frac{2[\cos{(\pi/M)}]^2(C+1)+2C\cos{(\pi/M)}-1}{2C\cos{(\pi/M)}+2C+1}$. So M can be derived to be

$$M = \frac{\pi}{\arccos{\dfrac{C\cos{(\omega_{di} T_s)}-C+\sqrt{B}}{2C+2}}}, \qquad (6)$$

where $B = C^2[\cos{(\omega_{di} T_s)}]^2 + 2C^2\cos{(\omega_{di} T_s)} + C^2 + 2C + 6C\cos{(\omega_{di} T_s)} + 2 + 2\cos{(\omega_{di} T_s)}$. If M is a positive integer, a smooth shape filter can be generated as $f[k] = \frac{h[k]/e^{\zeta_i \omega_i k T_s}}{\sum_{m=0}^{2M} h[m]/e^{\zeta_i \omega_i m T_s}}, 0 \le k \le 2M$.

Normalized base functions $h[k]$ and shape filters are generated based on the same natural frequency $\omega_i = 1$ rad/sec and the damping ratio $\zeta_i = 0.2$ with different values of parameter C. Fig. 6 shows the normalized base function $h[k]$ and the shape filter $f[k]$ with $C = 0.25$.
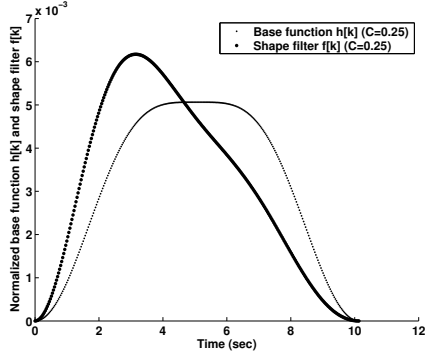
Fig. 6. Normalized base function $h[k]$ and the shape filter $f[k]$ with $C = 0.25$.

The value of M in Fig. 6 is made to be a positive integer by choosing appropriate sampling periods. In the derivation, the total impulses of the base function $h[k]$ are assumed to be an odd number $2M + 1$, however, the total impulses of the base function can also be an even number $2M$.

8. NON-SYMMETRIC CONTINUOUS FUNCTION BASED SHAPE FILTER GENERATION

Fig. 7 shows the architecture of the vibration suppression shape filter. From the previous study (Zhou and Misawa, 2005a), the Input Shaping® impulse filters (Singer and Seering, 1990) belong to the impulse function based shape filters. In the previous vibration shape filter generation, all the base functions have a symmetric waveform. In this section, non-symmetric function based shape filter generation method is studied. Two methods to generate a non-symmetric base function are studied.
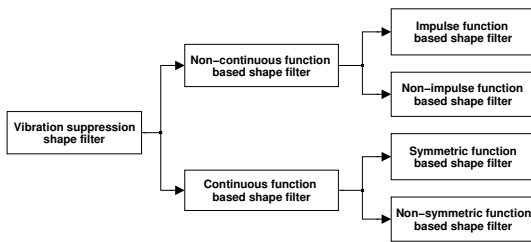

Fig. 7. The architecture of the vibration suppression shape filter.

8.1 Non-Symmetric Base Function Generation from the Derivative of a Base Function

If a discrete-time signal $g[k]$, $0 \leq k \leq M$, is a base function such that $G(\omega_{di}T_s) = 0$, here T_s is the sampling period in sec and ω_{di} is the damped natural frequency in rad/sec, the differencing in time of $g[k]$ is

$$dg[k] = \begin{cases} g[k] - g[k-1], & \text{if } 0 \leq k \leq M+1, \\ 0, & \text{otherwise.} \end{cases}$$

A non-symmetric base function $h[k]$ can be generated by the linear combination of $g[k]$ and the differencing signal $dg[k]$ by $h[k] = k_1 \cdot g[k] + k_2 \cdot dg[k], 0 \leq k \leq M + 1$, where k_1 and k_2 are two constants. The discrete-time Fourier transform of $h[k]$ is given by $H(\omega) = k_1 \cdot G(\omega) + k_2 \cdot (1 - e^{-j\omega})G(\omega)$. So the new function $h[k]$ can be used as a base function because the spectrum $H(\omega)$ at $\omega = \omega_{di}T_s$ is exactly zero as given by $H(\omega_{di}T_s) = k_1 \cdot G(\omega_{di}T_s) + k_2 \cdot (1 - e^{-j\omega_{di}T_s})G(\omega_{di}T_s) = k_1 \cdot 0 + k_2 \cdot (1 - e^{-j\omega_{di}T_s}) \cdot 0 = 0$.

A simple example generating a non-symmetric base function is demonstrated here. If $g[k]$ is a Hanning function given in (2), the new generated base function is $h[k] = k_1 \cdot g[k] + k_2 \cdot dg[k]$. If $H(\omega_{di}T_s) = H(\sqrt{1 - \zeta_i^2}\omega_i T_s) = 0$, then $\omega_{di}T_s = \frac{4\pi}{M}$ and $M = \frac{4\pi}{\omega_{di}T_s}$. If M is a positive integer, a shape filter can be generated as $f[k] = \frac{h[k]/e^{\zeta_i \omega_i k T_s}}{\sum_{m=0}^{M} h[m]/e^{\zeta_i \omega_i m T_s}}$.

Fig. 8 shows a construction of a non-symmetric base function $h[k]$ with the undamped natural frequency $\omega_i = 1$ rad/sec and the damping ratio $\zeta_i = 0.1$. The sampling period is chosen to be $T_s = \frac{\pi}{100\omega_{di}}$. The constants k_1 and k_2 are chosen to be $k_1 = 1$ and $k_2 = -25$. Fig. 9 shows the normalized base function $h[k]$ and the shape filter $f[k]$.
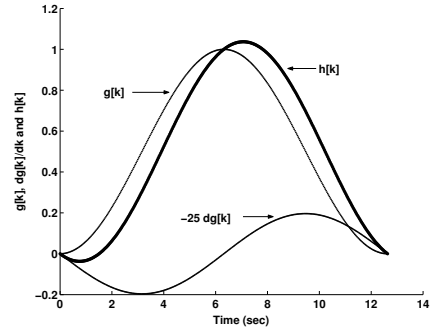

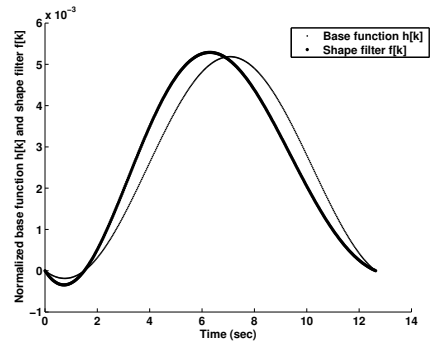Fig. 8. Construction of a non-symmetric base function h[k].

Fig. 9. Normalized base function $h[k]$ and shape filter $f[k]$.

8.2 Non-Symmetric Base Function Generation from the Self Convolution of a Base Function

If a discrete-time signal $g[k]$, $0 \leq k \leq M$, is a base function such that $G(\omega_{di}T_s) = 0$, here T_s is the

sampling period in sec and ω_{di} is the damped natural frequency in rad/sec, the self convolution of $g[k]$ is

$$g * g[k] = \begin{cases} \sum_{m=0}^{k} g[k-m]g[m], & \text{if } 0 \le k \le 2M, \\ 0, & \text{otherwise.} \end{cases}$$

A non-symmetric base function can be generated by the linear combination of $g[k]$ and the self convolution signal $g * g[k]$ by $h[k] = k_1 \cdot g[k - k_0] + k_2 \cdot g * g[k]$, where k_0 is a non-negative integer. The discrete-time Fourier transform of $h[k]$ is given by $H(\omega) = k_1 \cdot e^{-j\omega k_0}G(\omega) + k_2 \cdot G(\omega)^2$. So the new function $h[k]$ can be used as a base function because the spectrum $H(\omega)$ at $\omega = \omega_{di}T_s$ is exactly zero as given by $H(\omega_{di}T_s) = k_1 \cdot e^{-j\omega_{di}T_s k_0}G(\omega_{di}T_s) + k_2 \cdot G(\omega_{di}T_s)^2 = k_1 \cdot e^{-j\omega_{di}T_s k_0} \cdot 0 + k_2 \cdot 0 = 0$.

A simple example generating a non-symmetric base function is demonstrated here. If $g[k]$ is a Hanning function given in (2) and $k_0 = M$, the new generated base function is $h[k] = k_1 \cdot g[k - M] + k_2 \cdot g * g[k]$. If $H(\omega_{di}T_s) = H(\sqrt{1 - \zeta_i^2}\omega_i T_s) = 0$, then $\omega_{di}T_s = \frac{4\pi}{M}$ and $M = \frac{4\pi}{\omega_{di}T_s}$. If M is a positive integer, a shape filter can be generated as $f[k] = \frac{h[k]/e^{\zeta_i \omega_i k T_s}}{\sum_{m=0}^{M} h[m]/e^{\zeta_i \omega_i m T_s}}$.

Fig. 10 shows a construction of a non-symmetric base function $h[k]$ with the undamped natural frequency $\omega_i = 1$ rad/sec and the damping ratio $\zeta_i = 0.1$. The sampling period is chosen to be $T_s = \frac{\pi}{100\omega_{di}}$. The constants k_1 and k_2 are chosen to be $k_1 = 1$ and $k_2 = 1/125$. Fig. 11 shows the normalized base function $h[k]$ and the shape filter $f[k]$.
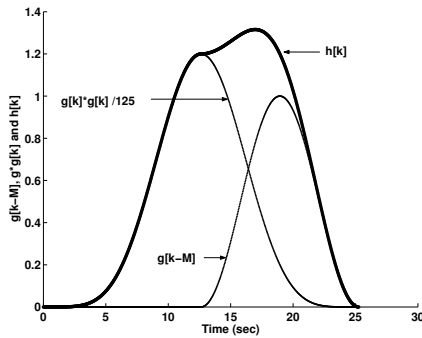

Fig. 10. Construction of a non-symmetric base function $h[k]$.

It must be noted that there are a number of methods to generate a non-symmetric base function. The methods described in this section are only possible methods and not exhaustive ones.

9. CONCLUSIONS

A vibration suppression shape filter is generated from a continuous window function. The robustness can be arbitrarily improved and the robustness brings about a smoother profile. The shape filter can also be used as a velocity profile in the case of zero initial and final values. Since the profile is generated from a
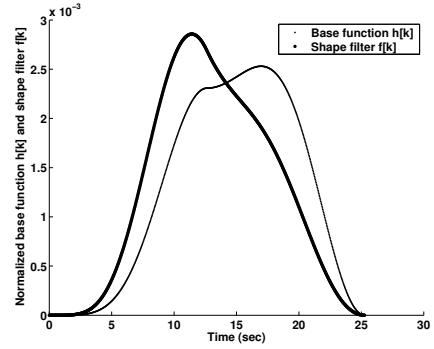

Fig. 11. Normalized base function $h[k]$ and shape filter $f[k]$.

continuous function, a smoothly changing discrete-time shape filter can be generated. The methods in this paper were tested on hard disk drive actuator position control at the Oklahoma State University Advanced Controls Laboratory. The experimental results of both Input Shaping[R] and rectangle based shape filter are reported in (Zhou and Misawa, 2005b). The methods in this paper are patented (pending). Commercial use of these methods requires written permission from the Oklahoma State University.

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