EXPERIMENTS ON STABILIZING RECEDING HORIZON CONTROL OF A DIRECT DRIVE MANIPULATOR

Yasunori Kawai * **Yujiro Nakaso** * **Takehito Azuma** *
Masayuki Fujita *

* *Department of Electrical and Electronic Engineering, Kanazawa University, 2-40-20 Kodatsuno, Kanazawa 920-8667, JAPAN*

Abstract: In this paper, the application of receding horizon control to a two-link direct drive robot arm is demonstrated. Instead of the terminal constraints, a terminal cost on receding horizon control is used to guarantee the stability, because of the computational demand. The key idea of this paper is to apply the receding horizon control with the terminal cost which is derived from the energy function of the robot system. The energy function is given as the control Lyapunov function by considering the inverse optimality. The experimental results are compared with respect to stability, performance by applying the receding horizon control and the inverse optimal control to the robot arm.

Keywords: Robot control, Predictive control, Model-based control, Stabilization

1. INTRODUCTION

Receding horizon control, known as model predictive control, has been a popular control strategy (D. Q. Mayne and Scokaert, 2000). It is a form of control where the current control action is obtained by solving a finite horizon optimal control problem on-line. Recently, several researchers have attempted to address the problem of the stability for receding horizon control to allow its application. Because of the computational demand, the optimization problem with a terminal cost may be used instead of the terminal constraints. Specially, by utilizing a suitable control Lyapunov function as the terminal cost, the stability of the receding horizon control is guaranteed. The application of this method have been done on the Caltech Ducted Fan (J. Yu and Huang, 2001) and on the F-16 Aircraft (R. Bhattacharya and Packard, 2002). In those works, the control Lyapunov function is derived using the quasi-LPV method.

On the other hand, in the robot control, the passivity-based approach has gained much attention which tackles the robot control problem by exploiting the robot system's physical structure (Berghuis and Nijmeijer, 1993; Jaritz and Spong, 1996). The idea of this design approach philosophy is to use the natural energy of the robot system such that the control objective is achieved. Recently, the inverse optimal control approach is considered by using the natural energy (Maruyama and Fujita, 1999; Krstic and Li, 1998).

In this paper, it is proposed that the natural energy of the robot system can be used as the terminal cost on the receding horizon control. In order to guarantee the stability of the receding horizon control with the energy functions, it is shown that the energy functions are control Lyapunov functions under some conditions. In the main theorems, the methods which arise from the previous works (Slotine and Li, 1987) in the passivity-based

Fig. 1. Two-link direct drive manipulator

approach, are applied to the receding horizon control by considering the inverse optimality. The focus of this paper is on the application of the receding horizon control techniques to the robot systems. For the regulation control and the tracking control, this paper demonstrates the stability with the receding horizon control and summarizes the results of the comparison between the inverse optimal control and the receding horizon control on the two-link direct drive robot manipulator shown in Fig. 1

The organization of this paper is as follows. The problem formulation is described in Section 2. In Section 3, the stability of the receding horizon control is derived. In Section 4, the results of the experimental studies are illustrated. Finally, our conclusions are presented.

2. PROBLEM FORMULATION

Consider the standard equations describing the dynamics of an n-DOF rigid robot system

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \tag{1}$$

where $M(q) \in R^{n\times n}$ is the positive definite inertia matrix, and q, $\dot{q}$ and $\ddot{q}$ are the joint angles, velocities, and accelerations, respectively. The vector $C(q,\dot{q})\dot{q} \in R^n$ represents the Coriolis and centrifugal torques, $G(q)$ is the gravitational torques, and τ is the control input. The motion equation possesses the property that the matrix

$$\dot{M}(q) - 2C(q,\dot{q}) \tag{2}$$

is skew symmetric (Ortega and Spong, 1989). This implies that the robot dynamics define a passive mapping between joint torque and joint velocity.

In our case, $n = 2$ and, referring to Fig. 2, the terms in the dynamic equations are given by

$$M(q) = \begin{bmatrix} M_1 + M_2 + 2R\cos q_2 & M_2 + R\cos q_2 \\ M_2 + R\cos q_2 & M_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -R\dot{q}_2 \sin q_2 & -R(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ R\dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$M_1 = m_1 r_1^2 + m_2 l_1^2 + I_1 = 6.5240 \times 10^{-1}$$

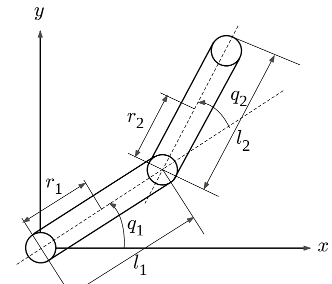

Fig. 2. Schematic diagram of the two-link direct drive manipulator

$$M_2 = m_2 r_2^2 + I_2 = 3.7900 \times 10^{-2}$$

$$R = m_2 l_1 r_2 = 4.1400 \times 10^{-3}.$$

For the system (1), we apply the receding horizon control. In the receding horizon control, every δ seconds, an optimal control problem which minimizes the following objective function, is solved over a T second horizon, starting from the current state

$$J = \int_t^{t+T} l(x,u)d\tau + V(x(t+T)) \tag{3}$$

where t is the current time, T is the predictive horizon, $l(x,u)$ is a positive definite function, $V(x(t+T))$ represents the terminal cost. Here, x is the state of the system, which depends on the problem. The first δ seconds of the optimal control u^* is then applied to the system, driving the system from $x(t)$ at current time t to $x^*(t+\delta)$ at the next sample time $t+\delta$. Repeating these calculations yields a feedback control law.

3. STABILITY OF RECEDING HORIZON CONTROL

In previous researches, the stability of receding horizon control is considered. The first method imposes a terminal boundary condition on state as $x(t+T) = 0$ (Mayne and Michalska, 1990) and terminal inequality constraints (Michalska and Mayne, 1993). Since the nonlinear optimization problem with some terminal constraints is computationally demanding, the receding horizon controller is obtained by solving the optimization problem with a terminal cost. In the work (G. D. Nicolao and Scattolini, 1998), the stability of the receding horizon control is guaranteed by using a possible non quadratic terminal penalty. In another method, first a globally stabilizing control law is achieved by finding a global control Lyapunov function, the control Lyapunov function is used as the terminal cost (J. Yu and Huang, 2001; A. Jadbabaie and Hauser, 2001). Here, the control Lyapunov function is a C^1, proper, positive definite function $V : R^n \to R_+$ such that

$$\inf_u \left[\dot{V}(x,u) + l(x,u)\right] \leq 0. \quad (4)$$

However, the natural energy of the robot system have been proposed (Slotine and Li, 1987). Therefore, this paper indicates that the natural energy is the control Lyapunov function. In this paper, the regulation problem and the tracking problem are considered. First, the inverse optimal control law is derived. Next, the conditions that the natural energy is the control Lyapunov function, are proposed. It is proved that the stability of the receding horizon control with the energy functions is guaranteed.

3.1 Regulation Problem

The control objective consists of positioning the robot at some desired position q_d, where q_d is constant. The control law τ is defined as

$$\tau = g(q) + u. \quad (5)$$

Substituting (5) into (1) yields

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = u. \quad (6)$$

The definition of the vector e, s_r are given as follows

$$e := q - q_d, \quad s_r := \dot{q} + \Lambda e \quad (7)$$

where Λ is the constant positive definite matrix. The equations of the robot system can be written as

$$\frac{d}{dt}\begin{bmatrix} e \\ s_r \end{bmatrix} = \begin{bmatrix} s_r - \Lambda e \\ \begin{pmatrix} \left(\Lambda - M^{-1}(q)C(q,\dot{q})\right) s_r \\ + \left(M^{-1}(q)C(q,\dot{q})\Lambda - \Lambda^2\right) e \end{pmatrix} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(q) \end{bmatrix} u. \quad (8)$$

The energy function for the system (8) is provided as (Slotine and Li, 1987)

$$V_r = \frac{1}{2}s_r' M(q) s_r + \frac{1}{2}e' K_e e. \quad (9)$$

The time derivative of (9) becomes

$$\begin{aligned}\dot{V}_r &= s_r' M(q)\dot{s}_r + \frac{1}{2}s_r'\dot{M}(q)s_r + e'K_e\dot{q} \\ &= \frac{1}{2}s_r'\left(\dot{M}(q) - 2C(q,\dot{q})\right)s_r + s_r'M(q)\Lambda s_r \\ &\quad + s_r'C(q,\dot{q})\Lambda e - s_r'M(q)\Lambda^2 e + s_r'u \\ &\quad + e'K_e s_r - e'K_e\Lambda e \\ &= e'\left(-K_e\Lambda\right)e + s_r'\left(M(q)\Lambda\right)s_r + s_r'u \\ &\quad + s_r'\left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)e. \quad (10)\end{aligned}$$

Here, the the property of the skew symmetric (2) is used. Assuming that the control law is given as $u = -Rs_r$, $\dot{V}_r$ can be written in quadratic form

$$\dot{V}_r = \begin{bmatrix} e \\ s_r \end{bmatrix}' \begin{bmatrix} -K_e\Lambda & \left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)'/2 \\ \left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)/2 & M(q)\Lambda - R \end{bmatrix} \begin{bmatrix} e \\ s_r \end{bmatrix}. \quad (11)$$

Then, the following relation is derived

$$\begin{aligned}-4x_r'Qx_r &= -\left(u + 2Rs_r\right)' R^{-1}\left(u + 2Rs_r\right) \\ &\quad + u'R^{-1}u + 4\dot{V}_r \quad (12)\end{aligned}$$

where the vector x and matrix Q are given by

$$x_r := [e,\ s_r]' \quad (13)$$

$$-Q := \begin{bmatrix} -K_e\Lambda & \left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)'/2 \\ \left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)/2 & M(q)\Lambda - R \end{bmatrix}. \quad (14)$$

Thus, the optimal control law is derived by $u = -2Rs_r$. Furthermore, the condition that the energy function (9) is the control Lyapunov function, is derived from the definition (4)

$$4x_r'Qx_r + u'R^{-1}u + 4\dot{V}_r \leq 0. \quad (15)$$

To satisfy the condition (15), the matrix R is given

$$\begin{aligned}R &= M(q)\Lambda + \frac{1}{4}\left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right) \\ &\quad \cdot \left(K_e\Lambda\right)^{-1}\left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)' + K \quad (16)\end{aligned}$$

where the matrix K is a positive definite matrix. Therefore, the following theorem is proposed.

Theorem 1. For the system (8), receding horizon control is considered. If the performance index $l(x,u)$ and the terminal cost $V(x(t+T))$ are given as

$$l(e, s_r, u) = 4x_r'Qx + u'R^{-1}u \quad (17)$$

$$V(e, s_r) = 4\left(\frac{1}{2}s_r'M(q)s_r + \frac{1}{2}e'K_e e\right) \quad (18)$$

then the stability of the system which consists of (8) and receding horizon control is guaranteed.

PROOF. Let J^* be the cost associated with the solution of (3). Next, the stability is proved in terms of J^* qualifies as a Lyapunov function

(A. Jadbabaie and Hauser, 2001). Construct the following suboptimal control strategy for the time interval $[t, t+T+\delta]$

$$\tilde{u} := u^*_{[t,t+T]} + u^k_{[t+T,t+T+\delta]}. \tag{19}$$

The following relation between J^* and J is given

$$\begin{aligned} J^*(t) &= \int_t^{t+T} l(e^*, s_r^*, u^*)d\tau + V(e^*(t+T), s_r^*(t+T)) \\ &= \int_t^{t+\delta} l(e^*, s_r^*, u^*)d\tau - \int_{t+T}^{t+T+\delta} l(e^k, s_r^k, u^k)d\tau \\ &\quad + J(t+\delta) + V(e^*(t+T), s_r^*(t+T)) \\ &\quad - V(e^k(t+T+\delta), s_r^k(t+T+\delta)). \end{aligned} \tag{20}$$

From the property $J^* \le J$, replacing J by the optimal value, and dividing both sides by δ and letting $\delta \to 0$, the following result are obtained

$$\begin{aligned} &\lim_{\delta \to 0} \frac{J^*(t+\delta) - J^*(t)}{\delta} \\ &\quad \le \dot{V}(e^*(t+T), s_r^*(t+T)) + l(e^k, s_r^k, u^k)d\tau \\ &\quad\quad - l(e^*, s_r^*, u^*). \end{aligned} \tag{21}$$

If J^* is C^1, then the above inequality can be written as

$$\begin{aligned} \dot{J}^*(t) &\le -l(e^*, s_r^*, u^*) \\ &\quad + \left(l + \dot{V}\right)(e^*(t+T), s_r^*(t+T), u^k)d\tau. \end{aligned} \tag{22}$$

Then, J^* will be a control Lyapunov function. Therefore, the stability is guaranteed.

3.2 Tracking Problem

The control objective is to track the desired position q_d, where q_d is time varying parameter. The control law τ is chosen as

$$\tau = M(q)\ddot{q}_d + C(q, \dot{q})\dot{q}_d + g(q) + u. \tag{23}$$

The closed-loop error system (1) and (23) becomes

$$M(q)\ddot{e} + C(q, \dot{q})\dot{e} = u \tag{24}$$

where e is defined as $e := q - q_d$. The robot dynamics is derived

$$\frac{d}{dt}\begin{bmatrix} e \\ s_t \end{bmatrix} = \begin{bmatrix} s_t - \Lambda e \\ \begin{pmatrix} \left(\Lambda - M^{-1}(q)C(q,\dot{q})\right)s_t \\ + \left(M^{-1}(q)C(q,\dot{q})\Lambda - \Lambda^2\right)e \end{pmatrix} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1}(q) \end{bmatrix} u \tag{25}$$

where s_t is defined by using the positive definite matrix Λ

$$s_t := \dot{e} + \Lambda e. \tag{26}$$

The energy function for the system (25) has been proposed in (Slotine and Li, 1987)

$$V_t = \frac{1}{2}s_t' M(q) s_t + \frac{1}{2}e' K_e e. \tag{27}$$

The time derivative of (27) along the system (25) is obtained

$$\begin{aligned} \dot{V}_t &= e'\left(-K_e\Lambda\right)e + s_t'\left(M(q)\Lambda\right)s_t + s_t' u \\ &\quad + s_t'\left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)e. \end{aligned} \tag{28}$$

If the control input is chosen as $u = -Rs_t$, then the function (28) is given by

$$\dot{V}_t = \begin{bmatrix} e \\ s_t \end{bmatrix}' \begin{bmatrix} -K_e\Lambda & \left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)'/2 \\ \left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)/2 & M(q)\Lambda - R \end{bmatrix} \begin{bmatrix} e \\ s_t \end{bmatrix}. \tag{29}$$

The following relation is obtained

$$\begin{aligned} -4x_t' Q x_t &= -\left(u + 2Rs_t\right)' R^{-1}\left(u + 2Rs_t\right) \\ &\quad + u' R^{-1} u + 4\dot{V}_t \end{aligned}$$

where the vector $x_t := [e,\ s_t]'$. Then, $u = -2Rs_t$ is the optimal control law. If the energy function V_t satisfy the following condition, then V_t is the control Lyapunov function

$$4x_t' Q x_t + u' R^{-1} u + 4\dot{V}_t \le 0. \tag{30}$$

Therefore, the matrix R is given

$$\begin{aligned} R &= M(q)\Lambda + \frac{1}{4}\left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right) \\ &\quad \cdot \left(K_e\Lambda\right)^{-1}\left(C(q,\dot{q})\Lambda - M(q)\Lambda^2 + K_e\right)' + K \end{aligned} \tag{31}$$

where matrix K are positive definite matrix. Therefore, the following theorem are provided.

Theorem 2. For the system (25), receding horizon control is considered. If the performance index $l(x, u)$ and the terminal cost $V(x(t+T))$ are given as

$$l(e, s_t, u) = 4x_t' Q_1 x_t + u' R_1 u \tag{32}$$

$$V(e, s_t) = 4\left(\frac{1}{2}s_t' M(q) s_t + \frac{1}{2}e' K_e e\right) \tag{33}$$

then the stability of the system which consists of (25) and receding horizon control is guaranteed.

PROOF.

The proof is proved by the same way as Theorem 1.

4. EXPERIMENTAL RESULTS

In this section, the comparison between the receding horizon control and the inverse optimal control is shown by using the two-link robot manipulator shown in Fig. 1. The manipulator is actuated with dc motors and is controlled by the digital signal processor (DSP) which is the product of dSPACE. The regulation and the tracking control problem are considered. To solve the real time optimization, the software C/GMRES (Ohtsuka, 2004) is used.

4.1 Regulation Control

The regulation problem is to control the angle of the robot arm from the initial condition $q_1(0) = q_2(0) = 0$ rad, $\dot{q}_1(0) = \dot{q}_2(0) = 0$ rad/s to the desired condition $q_{d1} = q_{d2} = \pi/4$ rad, $\dot{q}_{d1} = \dot{q}_{d2} = 0$ rad/s. The design parameters are set up as follows

$$\Lambda = \begin{bmatrix} 4.5 & 0 \\ 0 & 9 \end{bmatrix}, \quad K_e = \begin{bmatrix} 20 & 0 \\ 0 & 50 \end{bmatrix}$$
$$K = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}. \tag{34}$$

Fig. 3 illustrates the time responses and the trajectories, where the solid line, the dashed line, the dot-dashed line and the dot line represent the receding horizon controller with horizon length $T = 0.05$ s, $T = 0.1$ s, $T = 0.5$ s and the inverse optimal controller, respectively.

From these figures, it can be shown that the stability is guaranteed. However, the convergence to the equilibrium point is not completely achieved. It seems that the receding horizon controllers have a good performance for the link 1. On the contrary, the performance of the inverse optimal controller is useful for the link 2. The following cost $\int_0^2 l(e, s_r, u)dt$ is calculated in Table 1. A review of Table 1 leads that the cost of the inverse optimal controller is larger than the receding horizon control. In the simulations which is not shown in this paper, as the horizon length is increased, the cost steadily decreased. However, the same results are not obtained in this experiment.

Table 1. Values of the cost function

Inverse Optimal	Horizon $T = 0.05$[s]	Horizon $T = 0.1$[s]	Horizon $T = 0.3$[s]
142.0	128.5	128.0	128.5

4.2 Tracking Control

The purpose of the problem is to track the desired trajectory $q_{d1} = q_{d2} = \frac{\pi}{4}\sin(\pi t)$ rad, $\dot{q}_{d1} = \dot{q}_{d2} = \frac{\pi^2}{4}\cos(\pi t)$ rad/s where the initial condition is $q_1(0) = q_2(0) = 0$ rad, $\dot{q}_1(0) = \dot{q}_2(0) = 0$ rad/s. Each parameters are designed as follows

$$\Lambda = \begin{bmatrix} 4.5 & 0 \\ 0 & 9 \end{bmatrix}, \quad K_e = \begin{bmatrix} 20 & 0 \\ 0 & 50 \end{bmatrix}$$
$$K = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}. \tag{35}$$

The results are illustrated in Fig. 4 and Table 2. The solid line, the dashed line, the dot-dashed line and dot line show the receding horizon controller with horizon length $T = 0.05$ s, $T = 0.1$ s, $T = 0.5$ s and the inverse optimal control, respectively. Fig. 4 indicates that the stability is guaranteed. Table 2 represents the following cost $\int_0^4 l(e, s_t, u)dt$. The cost is increased as the horizon length gets longer. As the same as the regulation control, the same results as the simulation are not given in this experiment.

Table 2. Values of the cost function

Inverse Optimal	Horizon $T = 0.05$[s]	Horizon $T = 0.1$[s]	Horizon $T = 0.3$[s]
53.5	53.0	54.1	63.3

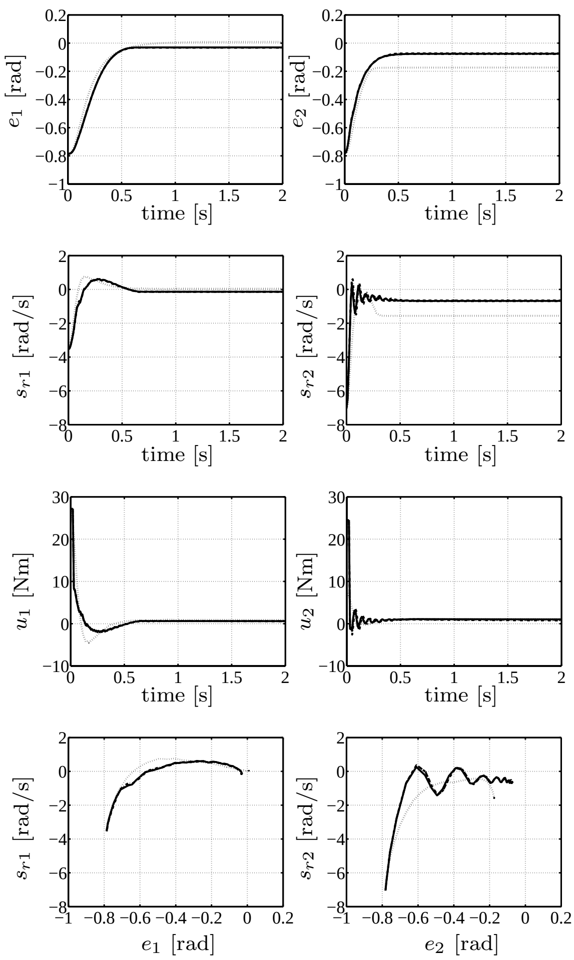

Fig. 3. Experimental Result (solid: $T = 0.05$ [s], dashed: $T = 0.1$ [s], dot-dashed: $T = 0.3$ [s]), dot: Inverse optimal control)

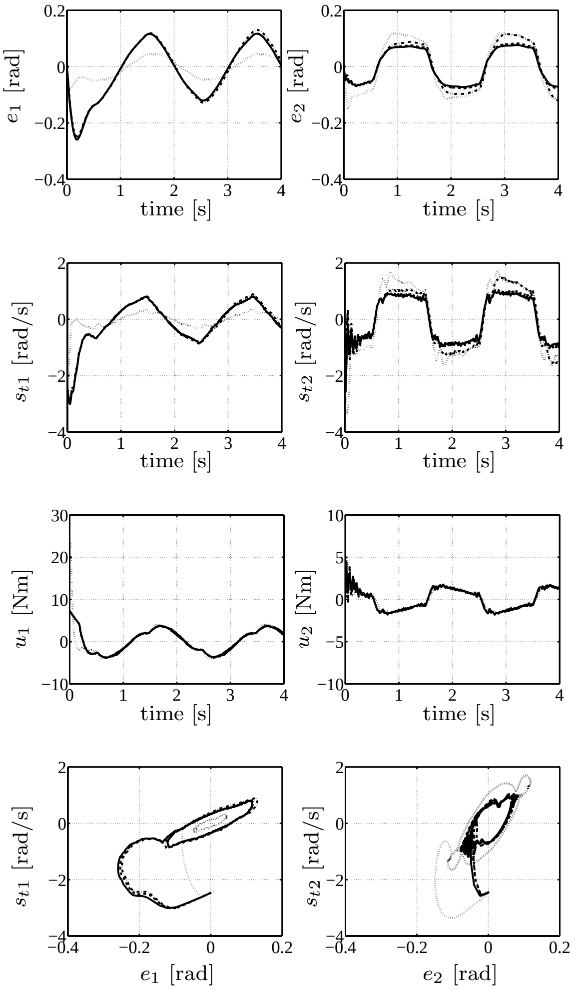

Fig. 4. Experimental Result (solid: $T = 0.05$ [s], dashed: $T = 0.1$ [s], dot-dashed: $T = 0.3$ [s]), dot: Inverse optimal control)

5. CONCLUSION

The purpose of this paper was to apply the receding horizon control to the two-link direct drive manipulator. The main idea of this paper is to use the energy function of the robot system as a terminal cost, which is derived from the passivity-based control. Furthermore, the inverse optimal control by using the energy function is considered. The conditions that the energy function is given as the control Lyapunov function are derived. It was shown that the proposed receding horizon controller can stabilize the system via the experiment. From the experimental results, it is shown that the horizon length plays an important role for the performance of the cost. Finally, the comparison between the receding horizon control and the inverse optimal control are obtained. It is expected the proposed receding horizon control can be applied for other system which have the property of the passivity.

REFERENCES

A. Jadbabaie, J. Yu and J. Hauser (2001). Unconstrained receding-horizon control of nonlinear systems. *IEEE Transactions on Automatic Control* **46**, 776–783.

Berghuis, H. and H. Nijmeijer (1993). A passivity approach to controller-observer design for robots. *IEEE Transactions on Robotics and Automation* **9**, 740–754.

D. Q. Mayne, J. B. Rawlings, C. V. Rao and P. O. M. Scokaert (2000). Constrained model predictive control: Stability and optimality. *Automatica* **36**, 789–814.

G. D. Nicolao, L. Magni and R. Scattolini (1998). Stabilizing receding-horizon control of nonlinear time-varying systems. *IEEE Transactions on Automatic Control* **43**, 1030–1036.

J. Yu, A. Jadbabaie, J. Primbs and Y. Huang (2001). Comparison of nonlinear control design techniques on a model of the caltech ducted fan. *Automatica* **37**, 1971–1978.

Jaritz, A. and M. W. Spong (1996). An experimental comparison of robust control algorithms on a direct drive manipulator. *IEEE Transactions on Robotics and Automation* **4**, 627–640.

Krstic, M. and Z. Li (1998). Inverse optimal design of input-to-state stabilizing nonlinear controllers. *IEEE Transactions on Automatic Control* **43**, 336–350.

Maruyama, A. and M. Fujita (1999). Inverse optimal h_∞ disturbance attenuation of robotic manipulators. In *Proc. of 1999 European Control Conference,* Paper Number F806.

Mayne, D. Q. and H. Michalska (1990). Receding horizon control of nonlinear systems. *IEEE Transactions on Automatic Control* **35**, 814–824.

Michalska, H. and D. Q. Mayne (1993). Robust receding horizon control of constrained nonlinear systems. *IEEE Transactions on Automatic Control* **38**, 1623–1633.

Ohtsuka, T. (2004). A continuation/gmres method for fast computation of nonlinear receding horizon control. *Automatica* **40**, 563–574.

Ortega, R. and M. W. Spong (1989). Adaptive motion control of rigid robots: a tutorial. *Automatica* **25**, 877–888.

R. Bhattacharya, G. J. Balas, M. A. Kaya and A. Packard (2002). Nonlinear receding horizon control of an f-16 aircraft. *Journal of Guidance, Control, and Dynamics* **25**, 924–931.

Slotine, J. J. and W. Li (1987). On the adaptive control of robot manipulators. *International Journal of Robotics Research* **6**, 49–59.