H_∞ BASED FAULT DETECTION AND ISOLATION FOR MARKOVIAN JUMP SYSTEMS

Kaoutar Akouz, El Kébir Boukas

Mechanical Engineering Department, École Polytechnique de Montréal, PO Box 6079, Station Centreville, Montréal, PQ, H3C 3A7, Canada

Abstract: Most of industrial processes are a combination of continuous and discrete dynamics known as hybrid systems. Markov jump linear systems (MJLS) as a special class of this family are characterized by their switching from a mode to another. It is desirable that such systems be reliable and that their running be the most efficient and safe possible. Thus, the monitoring of MJLS is an interesting area to explore. This paper addresses the fault detection and isolation issue for markovian jump linear systems. Using an H_∞ filtering approach we take the fault estimate as being the residual. This generated residual is then evaluated to fault occurrence.

Keywords: fault detection and isolation, H_∞ filtering, linear matrix inequality (LMI), Markov jump linear systems

1. INTRODUCTION

Most of processes in nuclear, automotive, manufacturing, production engineering and agri-food industry are hybrid systems (Mahmoud, 2001). A particular class of this family is the Markov jump linear systems (MJLS) that can be defined as continuous time systems switching from a linear mode to another according to a finite state Markov chain. Thus, when in a given mode, the system evolves exactly like a linear deterministic one. Many researchers from the mathematic community and more recently from control community have tackled such systems. For instance, the main aspects explored by Boukas (Boukas, 2005) and references therein for MJLS are stochastic stability, stochastic stabilizability, H_∞ control problem and filtering problem.

An interesting concept we will consider in this paper for the MJL (Markov Jump Linear) systems is the fault detection and isolation one. Indeed, the issue of reliability, operating safety and environmental protection in industrial processes is a crucial one. Consequently, providing the system with the ability to detect and locate the occurrence of a fault in running modes is the corner stone of FDI (Fault Detection and Isolation) procedure.

Many methods exist in the literature and range principally in two classes: those that are not based on a mathematical model of the plant and those that are model based. For the last ones and depending on the system under study, we can find for deterministic processes: parity checks or analytical redundancy methods (Campa *et al.*, 2002), (Chow and Willsky, 2002), (Niemann *et al.*, 1995), (Staroswiecki and Comtet-Varga, 2001), detection filters (Bokor and Balas, 2004) and fault observers procedures (Hammouri *et al.*, 1999), (Shen and Hsu, 1998), (Tan and Edwards, 2003), mainly stochastic systems use statistical tests based on likelihood ratio (Basseville, 1998) and (Keller *et al.*, 1995).

As cited in the literature, some authors represented abrupt changes due to faults by random jumps (Tze-Thong and Milton, 1976), (Willsky and Jones, 1976). More recently, (Mahmoud, 2001) described active fault tolerant control systems by stochastic linear differential equation with randomly varying parameters. This variation due to faults has markovian transition characteristics.

The Fault Detection and Isolation problem for this class of systems has never been tackled. Thus, the main contribution of this paper is to employ the H_∞ approach to filter the fault vector in MJL systems. The resulting estimate is taken as a residual that is robust to perturbations and sensitive to faults. The obtained filter existence and stability conditions are expressed in LMI form.

The paper is organized as follows. The second section states the problem we want to investigate. The third section details the main results of the fault detection and isolation in markovian jump linear systems. In the forth section a numerical example is given to strengthen the theoretical concepts.

2. PROBLEM STATEMENT

Many authors have been interested in the application of H_∞ filtering approach to the detection and isolation of faults in the deterministic setting (Besançon, 2003), (Edelmayer and Bokor, 2002), (Markos *et al.*, 2004). In fact, considering the fault as an unknown entry of finite energy, detecting and locating it is equivalent to make the residual more sensitive to it than to other entries. One way to realize this is to estimate the fault vector using H_∞ theory and then to consider the estimate as the residual. Thus, the purpose is to minimize the H_∞ norm of the error between the fault and its estimate. Conditions to achieve this filtering objective for MJL systems are characterized via linear matrix inequalities (LMI).

Let us consider a stochastic system that switches between different modes N according to a continuous Markov process $\{s(t), t \geq 0\}$ taking values in the finite state space $S = \{1, 2, ..., N\}$ and having an infinitesimal generator:

$$\mathbf{\Delta} = (\lambda_{ij}), i, j \in S$$

with : $\lambda_{ij} \geq 0, \forall j \neq i, \lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$.

The dynamics of this system is supposed to be described by the following model:

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(s(t))\mathbf{x}(t) + \mathbf{B}(s(t))\tilde{\mathbf{w}}(t), \\ \mathbf{y}(t) = \mathbf{C_y}(s(t))\mathbf{x}(t) + \mathbf{B_r}(s(t))\tilde{\mathbf{w}}(t). \end{cases} \quad (1)$$

$\mathbf{x}(t)$ is the state variable of the system at time t, $\tilde{\mathbf{w}}(t) = \left[\mathbf{f}^T(t)\ \mathbf{w}^T(t) \right]^T$ is the generalized disturbance at time t, $\mathbf{f}(t)$ is the fault vector at time t, $\mathbf{w}(t)$ is the perturbation at time t, both are supposed to be of finite energy, $\mathbf{A}(i)$, $\mathbf{D}(i)$, $\mathbf{F}(i)$, $\mathbf{F_y}(i)$, $\mathbf{D_y}(i)$, $\mathbf{C_y}(i)$ are known matrices for each mode i in S.

$$\mathbf{B}(i) = \left[\mathbf{F}(i)\ \mathbf{D}(i) \right]$$
$$\mathbf{B_r}(i) = \left[\mathbf{D_y}(i)\ \mathbf{F_y}(i) \right]$$

A fault can occur at any time, resulting in system dysfunctionning. The main purpose is to detect and locate the fault as early as possible. To do so, we would have to deal with two issues:

- Residual generation: the fault vector estimation error $\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{f}(t)$ should be minimized such as:

$$sup_{\tilde{\mathbf{w}} \in \mathbf{L_2}} \frac{\|\mathbf{e}(t)\|_2}{\|\tilde{\mathbf{w}}(t)\|_2} < \gamma$$

  where: the fault vector estimate $\mathbf{r}(\mathbf{t})$ is the residual, $\|\mathbf{v}(t)\|_2 = \int_0^\infty (\mathbf{v})^T(t)(\mathbf{v})(t)dt$ and γ a positive constant. This is equivalent to minimizing the H_∞ norm of the influence of the generalized disturbance on the estimation error.
- Residual evaluation: the fault detection and isolation is accomplished by comparing the generated residual to a threshold J_{th} to see whether a fault has occurred or not. J_{th} denotes the minimum tolerable fault beyond which an alarm signal is set off.

3. MAIN RESULTS

In this section we will present the procedure for H_∞ based fault detection and isolation. As the fault is considered as an unknown input of the system, an inverse filtering procedure (Saberi *et al.*, 1999) is adopted to generate the residual. Then, we express the H_∞ filtering performance in LMI form.

3.1 Residual generator

$$\begin{cases} \dot{\mathbf{x}}_\mathbf{f}(t) = \mathbf{K}(s(t))\mathbf{x}_\mathbf{f}(t) + \mathbf{L}(s(t))\mathbf{y}(t), \\ \mathbf{r}(t) = \mathbf{M}(s(t))\mathbf{x}_\mathbf{f}(t) + \mathbf{N}(s(t))\mathbf{y}(t) \end{cases} \quad (2)$$

$\mathbf{r}(t)$ is the fault vector estimate at time t, $\mathbf{x}_\mathbf{f}(t)$ is the state variable of the filter at time t, $\mathbf{y}(\mathbf{t})$ is the output variable at time t, $\mathbf{K}(s(t))$, $\mathbf{L}(s(t))$, $\mathbf{M}(s(t))$ and $\mathbf{N}(s(t))$ are the design parameters of the filter.

Considering the fault estimation error:

$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{f}(t)$$

and setting :

$$\tilde{\mathbf{x}}(t) = \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}(t) - \mathbf{x_f}(t) \end{bmatrix}$$

After some algebraic developments we get the following extended model:

$$\begin{cases} \dot{\tilde{\mathbf{x}}}(t) = \tilde{\mathbf{A}}(s(t))\tilde{\mathbf{x}}(t) + \tilde{\mathbf{G}}(s(t))\tilde{\mathbf{w}}(t) \\ \mathbf{e}(t) = \tilde{\mathbf{C}}(s(t))\tilde{\mathbf{x}}(t) + \tilde{\mathbf{G}}_r(s(t))\tilde{\mathbf{w}}(t) \end{cases}$$

$$\tilde{\mathbf{A}}(i) = \begin{bmatrix} \mathbf{A}(i) & 0 \\ \mathbf{A}(i) - \mathbf{L}(i)\mathbf{C_y}(i) - \mathbf{K}(i) & \mathbf{K}(i) \end{bmatrix}$$

$$\tilde{\mathbf{C}}(i) = \begin{bmatrix} \mathbf{F}(i) & \mathbf{D}(i) \\ \mathbf{F}(i) - \mathbf{L}(i)\mathbf{F_y}(i) & \mathbf{D}(i) - \mathbf{L}(i)\mathbf{D_y}(i) \end{bmatrix}$$

$$\tilde{\mathbf{G}}(i) = \begin{bmatrix} \mathbf{M}(i) + \mathbf{N}(i)\mathbf{C_y}(\mathbf{i}) & -\mathbf{M}(i) \end{bmatrix}$$

$$\tilde{\mathbf{G}}_r(i) = \begin{bmatrix} \mathbf{N}(i)\mathbf{F_y}(i) - \mathbf{I} & \mathbf{N}(i)\mathbf{D_y}(i) \end{bmatrix}$$

i being the value of the mode $s(t)$.

3.2 H_∞ filtering approach

The aim is to minimize the effect of the perturbation $\tilde{\mathbf{w}}(t)$ on the estimation error of the fault vector. To do so, existence and stability conditions of such a filter are set in the following theorem:

Theorem 1:
Let γ be a positive constant and $\mathbf{R}$ a given symmetric and positive-definite matrix representing the weight of initial conditions. If there exists a set of symmetric and positive-definite matrices $\mathbf{P} = (\mathbf{P}(1), ..., \mathbf{P}(N))$ such that $\forall i \in S$:

$$\begin{bmatrix} \tilde{\mathbf{J}}_1(i) & \mathbf{P}(i)\tilde{\mathbf{G}}(i) & \tilde{\mathbf{C}}^T(i) \\ \tilde{\mathbf{G}}^T(i)\mathbf{P}(i) & -\gamma^2\mathbf{I} & \tilde{\mathbf{G}}_r^T(i) \\ \tilde{\mathbf{C}}(i) & \tilde{\mathbf{G}}_r(i) & -\mathbf{I} \end{bmatrix} < 0 \qquad (3)$$

and: $\begin{bmatrix} \mathbf{I} & \mathbf{I} \end{bmatrix} \mathbf{P}(i_0) \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \leq \gamma^2\mathbf{R}$
where: $\tilde{\mathbf{J}}_1(i) = \tilde{\mathbf{A}}^T(i)\mathbf{P}(i)+\mathbf{P}(i)\tilde{\mathbf{A}}(i)+\sum_{j=1}^N \lambda_{ij}\mathbf{P}(j)$ then the extended system is stochastically stable and the estimation error satisfies:

$$\|\mathbf{e}\|_2^2 \leq \gamma^2[\|\tilde{\mathbf{w}}\|_2^2 + \mathbf{x}^T(0)\mathbf{R}\mathbf{x}(0)]$$

To reach conditions for the design of the parameters of the filter we rearrange the LMI (3) considering that:

$$\mathbf{P}(i) = \begin{bmatrix} \mathbf{X}_1(i) & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2(i) \end{bmatrix}$$
$$\mathbf{Y}(i) = \mathbf{X}_2(i)\mathbf{L}(i)$$
$$\mathbf{Z}(i) = \mathbf{X}_2(i)\mathbf{K}(i)$$

this leads to the following LMI:

$$\begin{bmatrix} \mathbf{\Gamma}_0(i) & \mathbf{\Gamma}_1^T(i) & \mathbf{\Gamma}_2(i) & \mathbf{\Gamma}_3(i) & \mathbf{\Gamma}_4^T(i) \\ \mathbf{\Gamma}_1(i) & \mathbf{\Gamma}_5(i) & \mathbf{\Gamma}_6(i) & \mathbf{\Gamma}_7(i) & -\mathbf{M}^T(i) \\ \mathbf{\Gamma}_2^T(i) & \mathbf{\Gamma}_6^T(i) & -\gamma^2\mathbf{I} & 0 & \mathbf{\Gamma}_8^T(i) \\ \mathbf{\Gamma}_3^T(i) & \mathbf{\Gamma}_7^T(i) & \mathbf{0} & -\gamma^2\mathbf{I} & \mathbf{\Gamma}_9^T(i) \\ \mathbf{\Gamma}_4(i) & -\mathbf{M}(i) & \mathbf{\Gamma}_8(i) & \mathbf{\Gamma}_9(i) & -\mathbf{I} \end{bmatrix} < 0 \ (4)$$

where:

$$\mathbf{\Gamma}_0(i) = \mathbf{A}^T(i)\mathbf{X}_1(i) + \mathbf{X}_1(i)\mathbf{A}(i) + \sum_{j=1}^N \lambda_{ij}\mathbf{X}_1(j)$$
$$\mathbf{\Gamma}_1(i) = \mathbf{X}_2(i)\mathbf{A}(i) - \mathbf{Y}(i)\mathbf{C_y}(i) - \mathbf{Z}(i)$$
$$\mathbf{\Gamma}_2(i) = \mathbf{X}_1(i)\mathbf{F}(i), \mathbf{\Gamma}_3(i) = \mathbf{X}_1(i)\mathbf{D}(i)$$
$$\mathbf{\Gamma}_4(i) = \mathbf{M}(i) + \mathbf{N}(i)\mathbf{C_y}(i)$$
$$\mathbf{\Gamma}_5(i) = \mathbf{Z}^T(i) + \mathbf{Z}(i) + \sum_{j=1}^N \lambda_{ij}\mathbf{X}_2(j)$$
$$\mathbf{\Gamma}_6(i) = \mathbf{X}_2(i)\mathbf{F}(i) - \mathbf{Y}(i)\mathbf{F}_y(i)$$
$$\mathbf{\Gamma}_7(i) = \mathbf{X}_2(i)\mathbf{D}(i) - \mathbf{Y}(i)\mathbf{D}_y(i)$$
$$\mathbf{\Gamma}_8(i) = \mathbf{N}(i)\mathbf{F}_y(i) - \mathbf{I}, \mathbf{\Gamma}_9(i) = \mathbf{N}(i)\mathbf{D}_y(i)$$

The design of the filter parameters is summarized in the following theorem.

Theorem 2:
Let γ be a positive constant and $\mathbf{R}$ a given symmetric and positive-definite matrix representing the weight of initial conditions. If there exist sets of symmetric and positive-definite matrices: $\mathbf{X}_1 = (\mathbf{X}_1(1), ..., \mathbf{X}_1(N))$, $\mathbf{X}_2 = (\mathbf{X}_2(1), ..., \mathbf{X}_2(N))$ and a set of matrices $Y = (\mathbf{Y}(1), ..., \mathbf{Y}(N))$ satisfying the LMI (4) for all $i \in S$,

and $\mathbf{X}_1(i_0) + \mathbf{X}_2(i_0) < \gamma^2\mathbf{R}$

then, there exists a filter (or observer) in the form of (2) such that the estimation error is stochastically stable and bounded as:

$$\|\mathbf{e}\|_2^2 \leq \gamma^2[\|\tilde{\mathbf{w}}\|_2^2 - \mathbf{x}^T(0)\mathbf{R}\mathbf{x}(0)]$$

and the filter gains are given by:

$$\begin{cases} \mathbf{L}(i) = \mathbf{X}_2^{-1}(i)\mathbf{Y}(i), \\ \mathbf{K}(i) = \mathbf{X}_2^{-1}(i)\mathbf{Z}(i). \end{cases}$$

$\mathbf{M}(i)$ and $\mathbf{N}(i)$ are obtained straightforward from the LMI resolution.

3.3 Residual evaluation

Most of the time, the residual evaluation is done via the definition of a detection threshold. To determine this threshold, we solve the dynamical unfaulty system equation. When, dealing with a single fault the threshold should satisfy:

$$J_{th} = E(sup_{\mathbf{d},\mathbf{f}=\mathbf{0}} \|\mathbf{r}(t)\|_T)$$

if initial conditions and perturbation are zero: $\mathbf{r}(t) = 0$, which leads to: $J_{th} = 0$.

The fault detection is then realized during the faulty conditions over the time interval T:

- if $\|\mathbf{r}(t)\|_T > J_{th}$ then a fault has occurred
- Otherwise no fault is detected

with: $\|\mathbf{r}(t)\|_T = [E \int_{t_1}^{t_2} \mathbf{r}^T(t)\mathbf{r}(t)dt]^{1/2}$

4. NUMERICAL EXAMPLE

To show the effectiveness of the developed results let us consider a two-mode system with the following data:

$$\mathbf{\Delta} = \begin{bmatrix} -0.3 & 0.3 \\ 0.5 & -0.5 \end{bmatrix}$$

$$\mathbf{A}(1) = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \mathbf{A}(2) = \begin{bmatrix} 0 & 0.8 \\ -1 & -1 \end{bmatrix}.$$

$$\mathbf{F}(1) = \mathbf{F}(2) = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}$$

$$\mathbf{D}(1) = \mathbf{D}(2) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}$$

$$\mathbf{F}_y(1) = \mathbf{F}_y(2) = 1$$

$$\mathbf{C_y}(1) = \mathbf{C_y}(2) = \begin{bmatrix} 3 & 3 \end{bmatrix}$$

$$\mathbf{D}_y(1) = \mathbf{D}_y(2) = 0.05$$

Letting $\gamma = 0.2382$ and solving the LMI (4), we get the following gains:

$$\mathbf{M}(1) = \begin{bmatrix} -2.3802 & -2.2400 \end{bmatrix}$$

$$\mathbf{M}(2) = \begin{bmatrix} -2.3764 & -2.2118 \end{bmatrix}$$

$$\mathbf{N}(1) = 0.8041, \mathbf{N}(2) = 0.8024$$

$$\mathbf{L}(1) = \begin{bmatrix} -0.1400 \\ 1.2809 \end{bmatrix}, \mathbf{L}(2) = \begin{bmatrix} -0.1184 \\ 1.2566 \end{bmatrix}$$

$$\mathbf{K}(1) = \begin{bmatrix} 0.4452 & 1.5674 \\ -4.7824 & -4.6358 \end{bmatrix}$$

$$\mathbf{K}(2) = \begin{bmatrix} 0.3770 & 1.2971 \\ -4.7022 & -4.5126 \end{bmatrix}$$

To show the validness of the obtained results, let us simulate this system with the designed filter. For this purpose, we suppose that the system switches from mode 1 to mode 2 at $t = 40$. A single fault occurs at $t = 7$ in mode 1 and continues during mode 2 with a constant magnitude of 2 (Fig. 1).

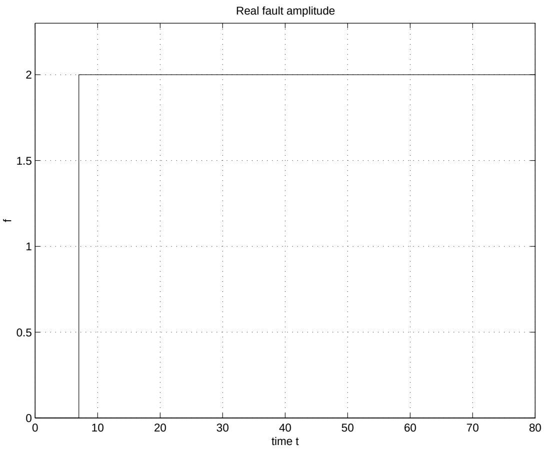

Fig. 1. Single fault detection (fault vs time)

A perturbation is induced in both modes such as $\mathbf{w}_1(t) = \mathbf{w}_2(t) = 1$. The objective is to see if the detection algorithm can assess the fault occurrence. At each time, we compute the residual norm and compare it to a threshold during a detection time interval T that is taken as $T = 5$ periods of time for both modes.

Thus, using the previous detection procedure, the detection results are displayed as:
A fault occurs during mode 1 at $t = 7$
A fault occurs during mode 2 at $t = 40$

The algorithm is able to detect the occurrence of the fault at $t = 7$ in mode 1 and its continuity during the switching to mode 2. The fault estimate evolution for the two modes is such that:

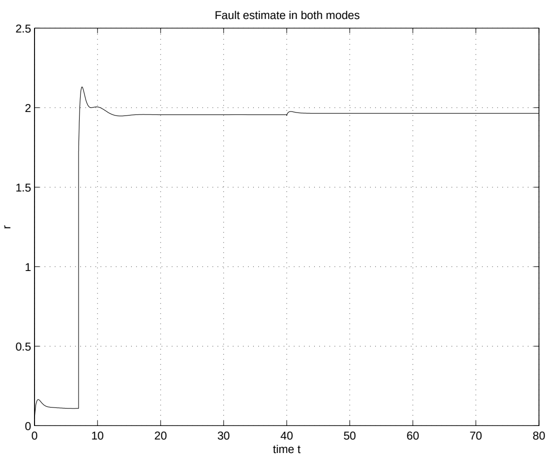

Fig. 2. Single fault detection (fault estimate vs time)

As shown on the curve, the fault is well estimated. Its detection at the exact time that it happens and its continuity during the mode switching are evident.

5. CONCLUSION

In this paper the application of H_∞ approach to the markovian jump linear systems was addressed. The conditions for fault vector estimator design were set in LMI form. The vector fault estimate is very close to the real one. Taking it as the residual, filtred fault vector is evaluated to detect its occurrence. The obtained results for the case of a single fault detection show that it is detected at the exact time it occurs.

REFERENCES

Basseville, M. (1998). On-board component fault detection and isolation using the statistical local approach. *Automatica* **34(11)**, 1391–1415.

Besançon, G. (2003). High-gain observation with disturbance attenuation and application to robust fault detection. *Automatica* **39**, 1095–1102.

Bokor, J. and G. Balas (2004). Detection filters for lpv systems - a geometric approach. *Automatica* **40**, 511–518.

Boukas, E.K. (2005). *Stochastic Switching Systems: Analysis and Design*. Birkhausser.

Campa, G., M.L. Fravolini, B. Seanor, M.R. Napolitano, D. Del Gobbo, G. Yu and S. Gururajan (2002). On-line learning neural networks for sensor validation for the flight control system of a b777 research scale model. *International Journal of Robust and Nonlinear Control* **12**, 987–1007.

Chow, E.Y. and A.S. Willsky (2002). Analytical redundancy and the design of robust failure detection systems. *IEEE Transactions on Automatic Control* **AC-29(7)**, 603–614.

Edelmayer, A. and J. Bokor (2002). Optimal h_∞ scaling for sensitivity optimization of detection filters. *International Journal of Robust and Nonlinear Control* **12**, 749–760.

Hammouri, H., M. Kinnaert and E.H. El Yaagoubi (1999). Observer-based approach to fault detection and isolation for nonlinear systems. *IEEE Transactions on Automatic Control* **44(10)**, 1879–1884.

Keller, J.Y., L. Summerer and M. Darouach (1995). Robust failure detection from the generalized likelihood ratio test. *Proceedings of the 34th Conference on Decision & Control.*

Mahmoud, M. M. (2001). PhD Thesis. University of Western Ontario.

Markos, M., G. Subhabrata and G.J. Balas (2004). An application of h_∞ fault detection and isolation to a transport aircraft. *Control Engineering Practice.*

Niemann, H., A. Saberi, A.A. Stoorvogel and P. Sannuti (1995). Exact, almost and delayed fault detection : An observer based approach. *International Journal of Robust and Nonlinear Control* **9**, 215–238.

Saberi, A., A.A. Stoorvogel and P. Sannuti (1999). Inverse filtering and deconvolution. *Proceedings of the American Control Conference* pp. 3270–3274.

Shen, L.C. and P.L. Hsu (1998). Robust design of fault isolation observers. *Automatica* **34(11)**, 1421–1429.

Staroswiecki, M. and G. Comtet-Varga (2001). Analytical redundancy relations for fault detection and isolation in algebraic dynamic systems. *Automatica* **37**, 687–699.

Tan, C.H. and C. Edwards (2003). Sliding mode observers for robust detection and reconstruction of actuator and sensor faults. *International Journal of Robust and Nonlinear Control* **13**, 443–463.

Tze-Thong, C. and B.A. Milton (1976). A sequential failure detection technique and its application. *IEEE Transactions on Automatic Control.*

Willsky, A.L. and H.L. Jones (1976). A generalized likelihood ratio approach to the detection and estimation of jumps in linear systems. *IEEE Transactions on Automatic Control.*