LINEAR PARAMETER-VARYING ANTI-WINDUP CONTROL FOR ACTIVE MICROGRAVITY ISOLATION

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Abstract: In this paper a linear parameter-varying (LPV) anti-windup approach is applied to provide anti-windup compensation for adaptive active microgravity vibration isolation. For such systems, anti-windup protection is required because of actuator saturation in response to inertially based forces acting on the isolated platform. For the example presented in the paper, a static anti-windup compensator scheduled based on the measurement of the rack displacement is designed so that it applies a correcting signal only when the control is saturated. The LPV anti-windup scheme is combined with an LPV controller that shifts its focus from a "soft" setting to a "stiff" setting depending on the need for acceleration minimization or relative displacement reduction to prevent bumping. The performance of the overall closed-loop system is demonstrated by simulations.

Keywords: Saturation control, Nonlinear control, Time varying system, Parametric variation, Aerospace Control

1. INTRODUCTION

Actuator saturation exists in almost all engineering control systems. Due to saturation, the actual plant input is different from the controller output. This discrepancy is called controller windup (Aström and Wittenmark, 1994). Since actuator saturation is ignored in linear control design, controller windup could result in degradation from expected linear performance, large overshoot or possible instability (Campo and Morari, 1990). As a result, actuator saturation has received increasing attention from the research community (Bernstein and Michel, 1995; Hu and Lin, 2001; Kapila and Grigoriadis, 2001). However, due to the complexity of the anti-windup problem, early anti-windup schemes were mostly heuristic in nature. Only in the last decade has the problem been addressed in a more systematic way with stability guarantees and clear performance specifications. A popular approach to saturating control is the anti-windup method that employs a two-step design procedure. The idea here is to first design the linear controller by ignoring the saturation nonlinearity and then add anti-windup compensation to minimize the adverse effects of saturation on closed-loop performance. In (Teel and Kapoor, 1997), a rigorous definition of anti-windup compensation was provided in terms of $\mathcal{L}_2$ stability and performance. The performance of the anti-windup augmentation is characterized by the $\mathcal{L}_2$ norm of the deviation between the actual response of the augmented closed-loop system and the ideal response of the unconstrained

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system. In (Grimm *et al.*, 2003), LMI-based anti-windup compensator synthesis with input-output quadratic stability and performance guarantees for a stable system is presented. Most previous anti-windup compensator designs are only applicable to open-loop stable LTI systems, limiting their usefulness in practical problem. When the system is nonlinear and open-loop unstable, the control synthesis problem becomes very difficult to solve and therefore, global stabilization cannot be achieved (Hu and Lin, 2001; Teel, 1999). In order to solve the anti-windup problem for such systems, the result in (Grimm *et al.*, 2003) are extended to unstable systems by restricting the bound on input nonlinearity to a small conic sector, thereby leading to regional stability in (Wu *et al.*, 2000). Moreover, the anti-windup control scheme for LTI plants in (Wu and Lu, 2003) is generalized to linear parameter varying (LPV) systems in (Lu and Wu, 2003) because of the relevance of LPV control to nonlinear systems. An alternative systematic anti-windup approach which can be brought to bear on nonlinear and open-loop unstable system is to embed it within a larger linear parameter-varying (LPV) problem; see (Kapila and Grigoriadis, 2001; Wu *et al.*, 2000) for details. This allows standard LPV stability and performance methods to be applied to the problem. However, this single step approach may result in conservative designs.

In this paper, the anti-windup techniques presented in (Lu and Wu, 2003) are applied to a linear parameter-varying (LPV) controller, designed to provide adaptive microgravity vibration isolation performance; see (Mehendale *et al.*, 2003) for details. The designed adaptive LPV controller provides improved isolation and position control over the full range of operating conditions via the use of parameter-dependent weighting functions. However, because the controller is linear and aggressive, disturbances such as inertially-based forces applied to the isolated platform, cause the controller to command values to the actuator that exceed its saturation limits. During such events, isolation performance is degraded and modifications of the nominal control algorithm are necessary to keep the system well-behaved.

For the isolation problem considered here, the plant is marginally stable LTI, whereas the controller is LPV. Hence, A static, i.e., no dynamic state, LPV anti-windup controller is designed to augment the nominal LPV controller. A static anti-windup compensator is more practical than full or reduced-order dynamic ones because it is easier to implement. Also, a static compensator stops influencing the nominal system as soon as the controller comes out of saturation. Numerical simulations are used to demonstrate the effectiveness of the designed anti-windup compensation

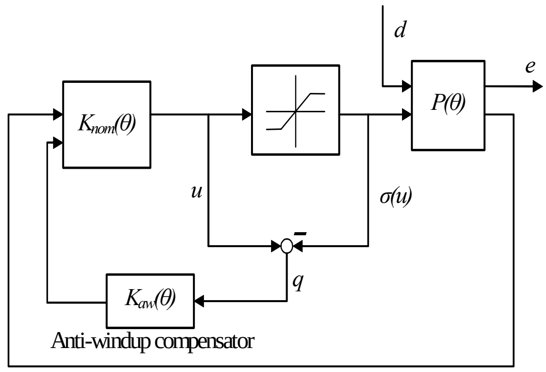

Fig. 1. Anti-windup controller structure

scheme in an adaptive active microgravity isolation problem.

The organization of the paper is as follows. Section 2 introduces the anti-windup compensator design method. Anti-windup compensator synthesis for the adaptive active microgravity isolation problem is presented in Section 3. Finally, concluding remarks are found in Section 4.

2. ANTI-WINDUP SYNTHESIS FOR LPV SYSTEMS WITH INPUT SATURATION

The anti-windup synthesis approach utilized in this paper is based on robust stability and performance results for sector-bounded uncertainties. It has been systematically presented in (Lu and Wu, 2003). The anti-windup control structure is shown in Fig. 1, where $P(\theta)$ is the linear parameter-varying plant and $\sigma(u)$ is the actuator nonlinearity under consideration as follows.

$$\sigma(u_i) = \begin{cases} u_i & |u_i| < u_i^{\max} \\ sign(u_i)u_i^{\max} & |u_i| \geqslant u_i^{\max} \end{cases}, \; i = 1, ..., n_u.$$

Following standard anti-windup procedures, a nominal practical (rate independent) LPV controller $K_{nom}(\theta)$ will be designed first, ignoring the input nonlinearity. The nominal LPV controller $K_{nom}(\theta)$ is designed to stabilize the open-loop system when no input saturation exists, and its design determines the nominal performance of the closed-loop system. The structure of this nominal controller is

$$\begin{bmatrix} \dot{x}_k \\ u \end{bmatrix} = \begin{bmatrix} A_k(\theta) & B_k(\theta) \\ C_k(\theta) & D_k(\theta) \end{bmatrix} \begin{bmatrix} x_k \\ y \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \qquad (1)$$

where $x_k \in R^{n_k}$ is the controller state. The inputs v_1 and v_2 are additional inputs added after the nominal design and are used for antiwindup augmentation. The LPV anti-windup compensator is of the form

$$\begin{bmatrix} \dot{x}_{aw} \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A_{aw}(\theta, \dot{\theta}) & B_{aw}(\theta) \\ C_{aw}(\theta, \dot{\theta}) & D_{aw}(\theta) \end{bmatrix} \begin{bmatrix} x_{aw} \\ q \end{bmatrix}, \qquad (2)$$

where $x_{aw} \in R^{n_{aw}}$ is the state. The adverse effect of input saturation is minimized in an $\mathcal{H}_\infty$ norm sense by using the LPV antiwindup compensator. Theorem 1 in (Lu and Wu, 2003) provides synthesis conditions in terms of LMIs for such a antiwindup compensator and the antiwindup compensator can be constructed explicitly using Theorem 2 in (Lu and Wu, 2003). It can be seen that the anti-windup compensator (2) is essentially a rate dependent compensator which is not a practical compensator, when parameter dependent Lyapunov functions are applied. However, the rate dependence only enter the A_{aw} and C_{aw} so that the corresponding static compensator is a rate independent compensator. Such a compensator is preferable since it is easier to implement. The synthesis LMIs for the static antiwindup compensator can be founded in Remark 1 of (Lu and Wu, 2003).

Note that the synthesis condition for the antiwindup compensator is obtained by reducing the deadband nonlinearity $\Delta_i = 1 - \frac{\sigma(u_i)}{u_i}$ to sect $[0, k_i]$ with $0 < k_i < 1$ and then obtaining regional stability for open-loop unstable systems. Hence, it will restrict the magnitude of the control input signal u_i to be less than $\left(\frac{1}{1-k_i}\right) u_i^{\max}$. It is usually hard to verify the condition $|u_i| \leqslant (\frac{1}{1-k_i}) u_i^{\max}$ because u_i are the controller's outputs. Using the results in (Hindi and Boyd, 1998), when there is no direct feedthrough from disturbance to system output, a domain of attraction and maximum size of disturbance can be estimated. However, the estimated stability region could be conservative.

3. ANTI-WINDUP COMPENSATOR DESIGN FOR ACTIVE MICROGRAVITY ISOLATION CONTROL

In (Mehendale *et al.*, 2003), an adaptive LPV controller with parameter dependent performance requirements is designed for an active microgravity isolation system. Due to space limitations, only a brief overview of the system description is provided here and the reader is referred to (Mehendale *et al.*, 2003) for details. A schematic of the system is shown in Figure (2). The goal of the control design is to achieve a level of isolation between the base acceleration $\ddot{x}_{off}$ and the inertial acceleration $\ddot{x}_{on}$ of the isolated platform. The isolated platform must operate in a limited rattlespace of 0.5 inches, and hence an additional design constraint is that the relative displacement $x_{on} - x_{off}$ does not exceed the 0.5 inch rattlespace limit in order to prevent the platform from bumping into its hardstops. In order to achieve these objectives the LPV controller is scheduled on the relative displacement θ_d. By scheduling on relative displacement, the LPV controller is able to shift its focus from a "soft" setting to a "stiff" setting depending on the need for acceleration minimization or relative displacement reduction to prevent

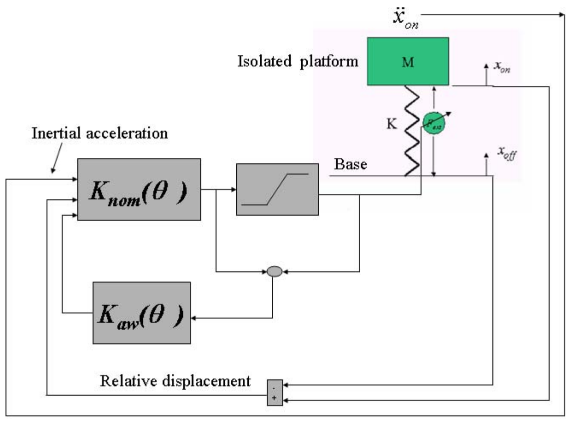

Fig. 2. Schematic of the isolation system

bumping. In order to achieve the design objectives, weights that depend parametrically on the scheduling variables θ_d are appended to the basic LTI plant to create the control design generalized LPV plant. The parametric uncertainty in the spring constant K is taken into account in the LPV design by modeling it as an input divisive uncertainty. The LPV controller is then designed for this generalized interconnection. This approach allows the controller to achieve excellent isolation performance over the range of base motion environments, while at the same time preventing the isolated platform from exceeding its hard rattlespace limits. However, since the controller is linear and aggressive, there are disturbances, such as inertially-based forces acting on the isolated platform that cause actuator saturation leading to significant degradation in isolation performance. Hence, adding an anti-windup protection loop is necessary to ensure good isolation performance. The anti-windup method presented in Section 2 is used for the anti-windup compensator design.

In this paper, both the LPV controller and antiwindup compensator designs are carried out for a simplified model of a microgravity isolation system. The mass of the payload is assumed to be $M = 15$ slug and the spring constant K is assumed to lie between 0 and 20 lbf/ft. The nominal controller design follows the same line in (Mehendale *et al.*, 2003) and will not be repeated here.

3.1 Anti-windup Compensator Synthesis

Based on the above discussion, the system "seen" by the anti-windup controller is a LTI plant with the control loops closed by an LPV controller. The LPV controller designed in (Mehendale *et al.*, 2003) is implemented as a linear combination of 11 grid-point controllers. These associated infinite dimensional LMIs for antiwindup compensator design can be reduced to a set of finite dimensional LMIs, by using standard griding techniques in (Apkarian and Adams, 1998) and solving the inequalities at the grid points. However, for validation the constraints have to be checked later with a much denser grid. The rate bound of parameter is chosen to be the same as the LPV controller design, $[-0.025\ 0.025]$. In order to maximize the guaranteed stabilization region, the K value of sector-bounded input nonlinearity $sect[0, K]$ is chosen as close to 1 as possible with reasonable closed-loop performance level. For the antiwindup design at hand K was chosen as 0.9999. The LMIs are solved over the same parameter grid as was used in the original LPV controller design, and the solutions are then verified to satisfy the constraints over a 101 point dense grid. The corresponding closed-loop $\mathcal{L}_2$ performance level is 9.58.

4. ANTI-WINDUP DESIGN RESULTS

The anti-windup compensator designed as discussed above was verified through time-domain simulations. A nonlinear hysteresis model of the spring was used to validate the performance of the controller and anti-windup compensator. The spring constant varies between 0 and 20 lbf/ft. The saturation limit for the actuator force is set to be [-3 3] lbs. Recall that the goal of the adaptive LPV controller designed in (Mehendale *et al.*, 2003) is to minimize the acceleration of the isolated platform, subject to limited available rattlespace. The goal of the antiwindup compensator is to minimize isolation performance degradation in the presence of actuator saturation.

An inertial force profile (see Figure 3) was applied to the isolated payload when the payload is located at the center of its rattlespace to test the isolation performance when displacement is small. In this regime the LPV controller is focusing on minimizing acceleration. The following three cases were simulated to demonstrate the effectiveness of the designed anti-windup compensator. The dashdot lines (Case 1) in Fig 4, 5 and 6 show the acceleration, actuator force and relative displacement responses of the nominal closed-loop system in the absence of saturation, i.e., the actuator can provide unlimited control force. They serve as the benchmark nominal performance for comparison purposes. The dashed lines (Case 2) in the figures show the same variables with the actuator force limited but with no antiwindup compensation. The solid lines (Case 3) in the figures are the responses of the closed loop system with the designed anti-windup protection under actuator saturation. It can be seen that the acceleration for Case 2 is much larger than the acceleration for Case 1, while for both cases, the relative displacements are under the displacement limit. Clearly, actuator saturation has degraded the isolation performance significantly so that it is necessary to add anti-windup protection to the system. The acceleration for Case 3 is much smaller than the acceleration for Case 2 which means the isolation performance has been improved significantly by introducing the designed anti-windup compensator. It is also clear that the Case 3 and Case 1 acceleration responses are close, which implies that the antiwindup compensator has successfully minimized performance deviation in the presence of saturation. Figure 5 shows that the anti-windup compensator helps the controller get out of saturation early, and also avoids large over shoot. This is the reason why anti-windup compensator improves the isolation performance. Fig 6, shows that the relative displacement for Case 3 is larger than in Case 2. This is expected since the payload is centered in its rattlespace and hence the controller is focusing on minimizing acceleration. However, it is still significantly below the displacement limit. Hence, it can be concluded that the anti-windup compensator improves the performance of the nominal controller under actuator saturation when the relative displacement is small.

In order to test the anti-windup performance when the relative displacement is large, a representative base motion displacement profile (Fig 7) was applied to cause a large relative displacement. The same inertial force profile as before was then applied at 30 sec and the same three Cases were simulated, for the above base motion displacement profile. Once again it is seen that the acceleration for Case 3 is much smaller than Case 2, while the difference of the displacements is very small; see Figures 8, 9 and 10. Clearly, the isolation performance is significantly improved by using the anti-windup compensator.

The maximum commanded control inputs to the saturation block corresponding to the anti-windup situation (Case 3) were $u = 1.5334 \times 10^4\ lbs$ for the small displacement simulation and $u = 2.0977 \times 10^4\ lbs$ for the large displacement simulation. It can be verified that the condition $|u| \leq (1/(1-K))u^{\max} = 3 \times 10^4\ lbs$ is satisfied. Because there is a direct feedthrough from the disturbance to the output in the system considered in this paper, the estimation of the maximum inertial force to guarantee the condition $|u| \leq (1/(1-K))u^{\max}$ cannot be achieved (Hindi and Boyd, 1998). Therefore, simulation results were used to determine the maximum size of the inertial force. Indeed, simulation results have shown that the maximum commanded control input increases as the size of the inertial force increases, and the condition $|u| \leq (1/(1-K))u^{\max}$ holds until the inertial force reaches 4.3 lbs.

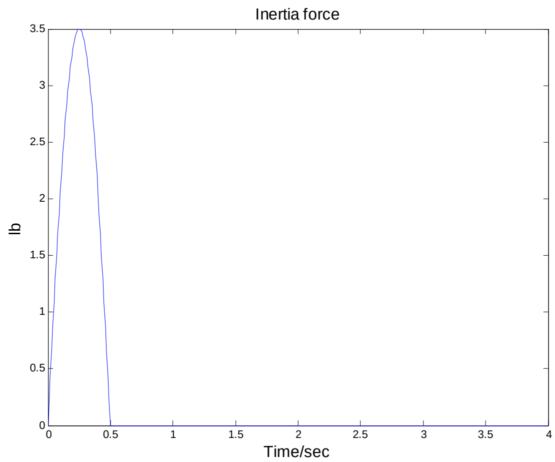

Fig. 3. Inertial force profile

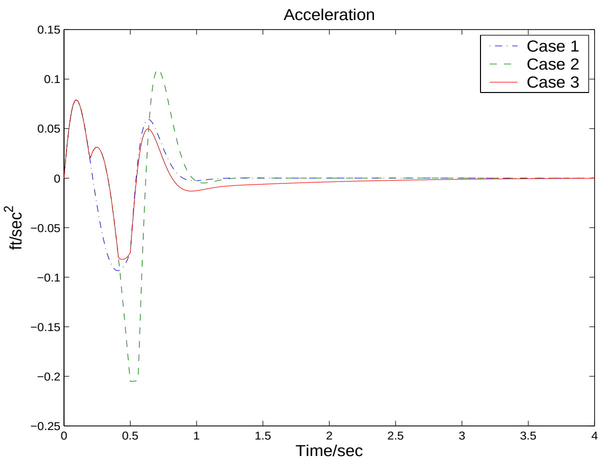

Fig. 4. Acceleration Performance, (Case 1) Unconstrained contorl; (Case 2) Constrainted control; (Case 3) Constrainted control with anti-windup

It is noted that application of the single step LPV anti-windup scheme of (Wu *et al.*, 2000) to this problem results in a conservative design where the target isolation curve (see (Mehendale *et al.*, 2003)) and the antiwindup objectives cannot be achieved simultaneously.

5. CONCLUSION

In this paper, an LPV anti-windup scheme proposed in (Lu and Wu, 2003) has been successfully applied to provide anti-windup protection for an adaptive LPV controller for active microgravity isolation. Both the LPV controller and the anti-windup gain are scheduled based on the measurment of the rack displacement. The design approach followed a classical two-step anti-windup paradigm. Numerical simulations have been used to illustrate the effectiveness of the anti-windup design.

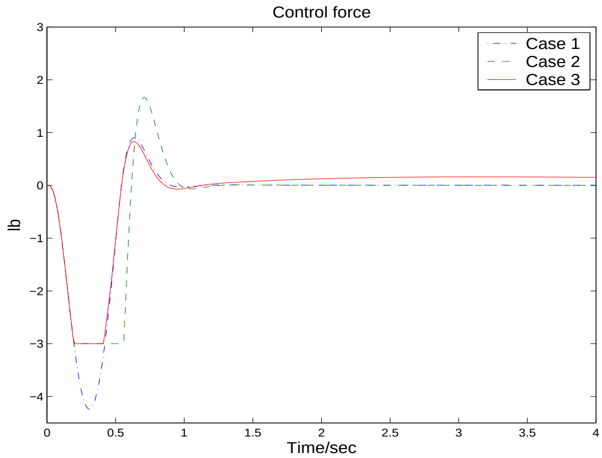

Fig. 5. Control force, (Case 1) Unconstrainted contorl; (Case 2) Constrainted control; (Case 3) Constrainted control with anti-windup

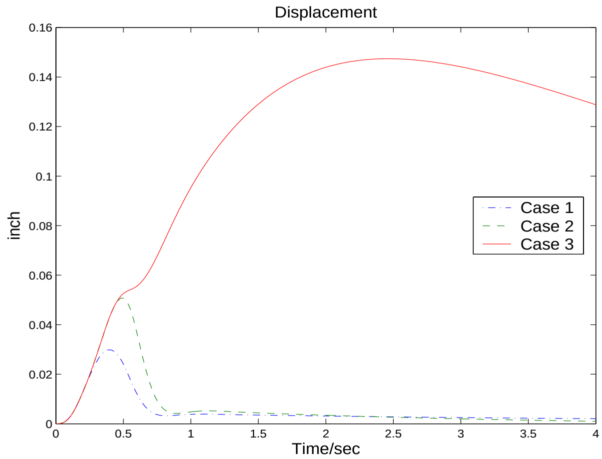

Fig. 6. Relative Displacement, (Case 1) Unconstrainted contorl; (Case 2) Constrainted control; (Case 3) Constrainted control with anti-windup

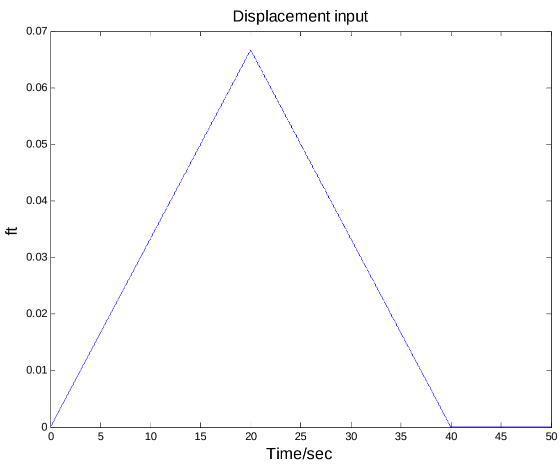

Fig. 7. Base motion disturbance profile

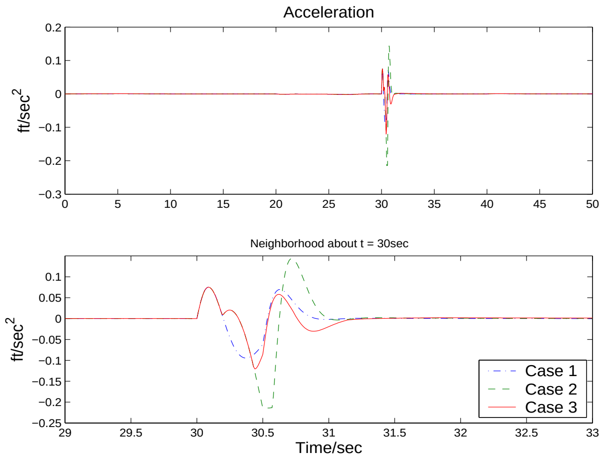

Fig. 8. Acceleration Performance, (Case 1) Unconstrained contorl; (Case 2) Constrainted control; (Case 3) Constrainted control with anti-windup

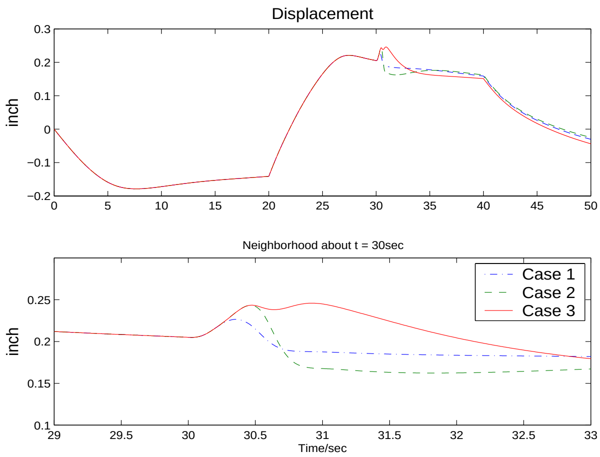

Fig. 9. Relative displacement, (Case 1) Unconstrainted contorl; (Case 2) Constrainted control; (Case 3) Constrainted control with anti-windup

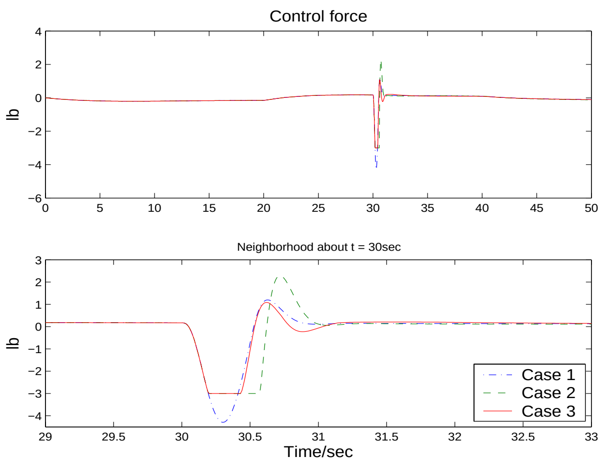

Fig. 10. Control force, (Case 1) Unconstrainted contorl; (Case 2) Constrainted control; (Case 3) Constrainted control with anti-windup

REFERENCES

Apkarian, P. and R. Adams (1998). Advanced gain-scheduling techniques for uncertain system. *IEEE Transactions on Control Systems Technology* **6**, 21–32.

Aström, K. J. and B. Wittenmark (1994). *Computer Controlled Systems: Theory and Design*. Prentice-Hall. Englewood Cliffs, NJ.

Bernstein, D.S. and A.N. Michel (1995). A chronological bibliography on saturating actuators. *Int. J. Robust Nonlinear Control* **5**, 375–380.

Campo, P.J. and M. Morari (1990). Robust control of processes subject to input saturation nonlinearities. *Computers and Chemical Engineering* **14**, 343–358.

Grimm, G., J. Hatfield, I. Postlehwaite, A.R. Teel, M.C. Turner and L. Zaccarian (2003). Anti-windup for stable linear systems with input saturation: An LMI-based synthesis. *IEEE Transactions on Automatic Control* **48**(9), 1509–1524.

Hindi, H. and S. Boyd (1998). Analysis of linear systems with saturation using convex optimization. *Proc. 37th IEEE Conf. Dec. Contr.* pp. 903–908.

Hu, T. and Lin, Z., Eds.) (2001). *Control Systems with Actuator Saturation: Analysis and Design*. Birkhauser. Boston, MA.

Kapila, V. and Grigoriadis, K.M., Eds.) (2001). *Actuator Saturation Control*. Marcel Dekker. New York.

Lu, B. and F. Wu (2003). LPV antiwindup compensation for enhanced flight control performance. *American Institute of Aeronautics and Astronautics Paper* (5495), 1–11.

Mehendale, C. S., I. J. Fialho and K. M. Grigoriadis (2003). A linear parameter-varying framework for adaptive active microgravity isolation. In: *Proceedings of the American Control Conference*. Denver, Colorado. pp. 1452–1457.

Teel, A.R. (1999). Anti-windup for exponentially unstable linear system. *Int. J. Robust Nonlinear Control* **9**, 701–716.

Teel, A.R. and N. Kapoor (1997). The l_2 anti-windup problem: Its definition and solution. In: *Proceedings of the Fourth European Control Conference*. Brussels, Belgium.

Wu, F. and B. Lu (2003). Anti-windup control for unstable LTI systems. In: *Proceedings of the American Control Conference*. Denver, Colorado. pp. 1224–1229.

Wu, F., K.M. Grigoriadis and A. Packard (2000). Anti-windup controller design using linear parameter-varying control methods. *International Journal of Control* **73**(12), 1104–1114.