OPTIMAL UCAV PATH PLANNING UNDER MISSILE THREATS

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Abstract: The problem of path planning for unmanned combat aerial vehicles (UCAVs) in the presence of radar-guided surface-to-air missiles (SAMs) is treated. The problem is formulated in the framework of the model, which includes three subsystems: the aircraft, the radar, and the missile. Based on this model, the problem of UCAV path planning is formulated as a minimax optimal control problem, with the aircraft lateral acceleration serving as control. Necessary conditions of optimality for this minimax problem are derived and utilized as a basis for an efficient numerical solution. Illustrative examples are considered that confirm standard flying tactics of "denying range, aspect, and aim," by yielding flight paths that "weave" to avoid long exposures.

Keywords: Path planning, Aircraft control, Minimax techniques, Time lag optimal control

1. INTRODUCTION

This paper is devoted to the problem of automated path planning for Unmanned Combat Aerial Vehicles (UCAVs) in the presence of radar-guided Surface-to-Air Missiles (SAMs). This problem features the interaction between three subsystems: the aircraft and its characteristics, the radar and its capabilities, and the missile and its lethality. Therefore, the solution of the UCAV path planning problem requires realistic models of these three subsystems. Although the current literature offers models for each of them separately, there is no approach that integrates models of the three subsystems in a unified framework. The purpose of this paper is to propose such an integrated model, formulate path planning as a minimax optimization problem, derive necessary conditions for its solution, and use them in an efficient numerical procedure.

The literature related to UAV path planning can be divided in two groups. The first group treats the problem under the assumption of isotropic (i.e., independent of aspect and bank angles) Radar Cross Section (RCS), while the second assumes non-isotropic RCS. Due to space limitations, only representative publications are cited here, while a more complete review can be found in (Kabamba *et al.*, 2004).

In the case of isotropic RCS, the path planning problem has been addressed based on minimizing the total reflected energy received by the radar (Pachter and Hebert, 2001). Other methods include the use of Voronoi diagrams (Chandler *et al.*, 2000), singular perturbations (Rao *et al.*, 1990), and wavelets (Godbole *et al.*, 2000).

For the case of non-isotropic RCS, the problem has been addressed using potential field methods (McFarland *et al.*, 1999) and virtual forces (Bortoff, 2000). The work of (Moore, 2002) optimized the aircraft bank and yaw angles within aerodynamic restrictions for a given route. Finally, (Misovec *et al.*, 2003) models the detection probability as being dependent on azimuth, elevation, and slant range. With the exception of (Moore, 2002) and (Bortoff, 2000), none of the above references account for the coupling between the RCS and dynamics through the aspect and bank angles. Moreover, only (Misovec *et al.*, 2003) accounts for the probabilistic nature of aircraft detection by a radar.

The current paper contributes to the literature an integrated model with the following original features:

- The aircraft RCS depends on both the aspect and bank angles. Moreover, the turn rate of the aircraft is determined by its bank angle. Hence, the RCS and aircraft dynamics are coupled through the aspect and bank angles.
- The probabilistic nature of radar tracking is accounted for. Specifically, an estimate of the probability of tracking is derived as an explicit function of the aircraft RCS and range.
- The decision process for launching a SAM is also modeled. Specifically, the probability of missile launch is the probability of tracking, averaged over a time interval of length T_{resp}. Here, T_{resp} represents the response time of the radar.
- The requirement to maintain tracking during missile flyout is also included. Specifically, once a SAM is launched, the probability of being downed is the probability of tracking, averaged over the flyout time.

Based on this integrated model, the problem of UCAV path planning is formulated as a minimax optimal control problem, with moving average functional, where the aircraft lateral acceleration serves as control. The current paper treats this optimization problem and provides the following original contributions.

- Necessary conditions of optimality for the minimax optimal control problem with moving average functional are formulated.
- Properties of the optimal paths are derived, based on the above necessary conditions.
- An efficient numerical optimization procedure is proposed that utilizes the above properties of optimal paths.

Numerous results obtained from this research have been omitted from this paper, and can be found in (Kabamba *et al.*, 2004).

2. MODELING

2.1 Aircraft Model

The bank-to-turn aircraft is assumed to move in a horizontal plane at a constant altitude z according to the equations

$$\begin{aligned} \dot{x} &= v\cos\psi, \\ \dot{y} &= v\sin\psi, \\ \dot{\psi} &= \frac{u}{v}, \\ |u| &< U, \end{aligned} \tag{1}$$

where x and y are the Cartesian coordinates of the aircraft, ψ is the heading angle, v is the speed, u is the input signal and is the acceleration normal to the flight path vector, while U represents the maximum allowable lateral acceleration.

Let

$$\begin{aligned} \theta &= \arctan(y/x), \\ \alpha &= \theta - \psi + \pi, \\ \phi &= \arctan\frac{z}{\sqrt{x^2+y^2}}, \end{aligned} \tag{2}$$

be the azimuth, aspect, and elevation angles, respectively, and let the bank angle μ be given by

$$\mu = \arctan(u/g), \tag{3}$$

where g is the acceleration of gravity.

The aircraft RCS is modeled as a function of the aspect angle α, the elevation angle ϕ, and the bank angle μ, so that

$$RCS = \sigma(\alpha, \phi, \mu). \tag{4}$$

2.2 Radar Model

The radar model will be presented in terms of its inputs (aircraft range and RCS) and output (an estimate of the probability that an aircraft can be tracked for an interval of time).

For the sake of simplicity, assume that the radar is located at the origin of the Cartesian coordinate system (x, y, z). Let $R = \sqrt{x^2+y^2+z^2}$ be the slant range from the radar to the aircraft (i.e., the aircraft range).

Let $P_t(\tau)$ be the instantaneous probability that the radar tracks the aircraft at time τ. This probability is given by

$$P_t = \frac{1}{1 + c_1^* \exp(\frac{c_2^*\sigma}{R^4})}, \tag{5}$$

where c_1^* and c_2^* depend on the radar, and σ is defined in (4) (see (Kabamba *et al.*, 2004) for details). Based on (5), the probability that the radar tracks the aircraft over an interval $[t-\Delta T,\ t]$ is

$$\frac{1}{\Delta T}\int_{t-\Delta T}^{t} P_t(\tau)d\tau. \tag{6}$$

2.3 Missile Launch Model

Prior to missile launch, the radar must continuously track the aircraft during some response time T_{resp}. To model this phenomenon, equate the probability of missile launch with the probability that the aircraft has been tracked over the interval T_{resp}. Thus, by (6), the probability of missile launch at time t is

$$\frac{1}{T_{resp}} \int_{t-T_{resp}}^{t} P_t(\tau) d\tau. \tag{7}$$

2.4 Missile Lethality Model

Accurate missile guidance requires that the radar system maintain track on the aircraft during the time of flight of the missile (Ben-Asher and Yaesh, 1998). Let R_0 be the aircraft range at the time the missile is fired and v_m be the average missile speed. Assuming that the aircraft range does not change significantly while the missile is in flight, the missile flyout time is given by

$$T_{fo} = \frac{R_0}{v_m}. \tag{8}$$

The probability that the aircraft is shot down at time t, given that the missile was fired, is

$$\frac{1}{T_{fo}} \int_{t-T_{fo}}^{t} P_t(\tau) d\tau. \tag{9}$$

3. PROBLEM FORMULATION

3.1 Mission Description

The missions considered in this paper are to fly from a given initial location to a given destination, in a given mission time, T_M, while avoiding being downed by radar-guided SAMs. Our formulation allows the specification of a sequence of waypoints that the UCAV must fly over. The heading angles of the UCAV at the waypoints and the flight times between consecutive waypoints may be free or given. Hence, the mission is specified by initial and final conditions, mission time, waypoints, and flight times between waypoints.

3.2 Dynamic Optimization Problem

Define the threat window as $T \triangleq T_{resp} + T_{fo}$. Combining (7) and (9) yields the probability of the aircraft being shot down at time t as

$$\frac{1}{T} \int_{t-T}^{t} P_t(\tau) d\tau. \tag{10}$$

This means that the probability of the aircraft being shot down is equivalent to the probability that *the aircraft is tracked during a window of time whose length equals the sum of response time and missile flyout time.*

Based on the model introduced in Section 2, the following dynamic optimization problem is formulated: minimize the maximum value of (10) subject to the aircraft dynamics and boundary conditions. In other words,

$$\min_u \max_{t\in[0,T_M]} \frac{1}{T} \int_{t-T}^{t} P_t(\tau) d\tau, \tag{11}$$

$$\begin{aligned} \text{subject to : } & \bullet\ (1) - (5), \\ & \bullet\ \text{boundary conditions, including} \\ & \qquad - \text{initial conditions,} \\ & \qquad - \text{final conditions,} \\ & \qquad - \text{waypoints.} \end{aligned}$$

4. OPTIMAL CONTROL

The necessary conditions of optimality for a general minimax optimization problem with moving average are derived in (Kabamba *et al.*, 2004). Here, these necessary conditions are applied to the problem (11). To accomplish this, augment (1) with

$$\dot{\xi}(t) = \frac{P_t(t) - P_t(t-T)}{T}. \tag{12}$$

Then (11) becomes

$$\min_u \max_{t\in[0,T_M]} \xi(t), \tag{13}$$

$$\begin{aligned} \text{subject to : } & \bullet\ (1) - (5), (12) \\ & \bullet\ \text{boundary conditions, including} \\ & \qquad - \text{initial conditions,} \\ & \qquad - \text{final conditions,} \\ & \qquad - \text{waypoints.} \end{aligned}$$

The resulting system is the same as that of (1), with the addition of a state described by the time-delay differential equation (12). The Hamiltonian for this problem is

$$\begin{aligned} H = & p_x(t) v \cos\psi(t) + p_y(t) v \sin\psi(t) \\ & + p_\psi(t) \frac{u(t)}{v} + p_\xi \left[P_t(t) - P_t(t-T)\right], \end{aligned} \tag{14}$$

where p_x, p_y, p_ψ, and p_ξ are the costates.

Consider the case that the maximum of (11) occurs at a finite number of isolated points $t_1, t_2, \ldots, t_k$. For this case, the necessary conditions of optimality define two separate optimization problems. Each will be addressed in turn.

4.1 Optimization Problem 1

Let t_i be a time at which $\xi(t)$ achieves a maximum. Consider now the optimization problem during the interval $[t_i - T, t_i]$, which will be referred to as Optimization Problem 1. For this problem, the costate equations become

$$\begin{aligned}
\dot{p}_x &= \mu_i \frac{\partial P_t(t)}{\partial x}, \\
\dot{p}_y &= \mu_i \frac{\partial P_t(t)}{\partial y}, \\
\dot{p}_\psi &= p_x(t) v \sin\psi(t) - p_y(t) v \cos\psi(t) \\
&\quad + \mu_i \frac{\partial P_t(t)}{\partial \psi}, \\
\dot{p}_\xi &= 0,
\end{aligned} \tag{15}$$

and, if $\partial P_t/\partial u \neq 0$, the necessary conditions of optimality give the optimal control as a solution to

$$\frac{p_\psi}{v} - \mu_i \frac{\partial P_t}{\partial u} = 0, \tag{16}$$

where $\mu_i > 0$ and $\sum_{i=1}^{k} \mu_i = 1$.

When $\partial P_t/\partial u = 0$, the necessary conditions yield the optimal control as

$$u = U \ \mathrm{sign}(\mathrm{p}_\psi). \tag{17}$$

In addition, a singular solution is possible when $p_\psi = 0$ and $\dot{p}_\psi = 0$. Thus, when $\partial P_t/\partial u = 0$, the optimum control is either "bang-bang," as given by (17), or "bang-singular-bang" when a singular condition holds (Bryson and Ho, 1975).

4.2 Optimization Problem 2

Let $[t_j, t'_j]$ be an interval that is disjoint from any interval $[t_i - T, t_i]$ described in Optimization Problem 1. The optimization problem during any such interval $[t_j, t'_j]$ will be referred to as Optimization Problem 2. During this interval, the costate equations are

$$\begin{aligned}
\dot{p}_x &= 0, \\
\dot{p}_y &= 0, \\
\dot{p}_\psi &= p_x(t) v \sin\psi(t) - p_y(t) v \cos\psi(t), \\
\dot{p}_\xi &= 0.
\end{aligned} \tag{18}$$

The optimal control is either bang-bang as in (17) or singular if $p_\psi = 0$ and $\dot{p}_\psi = 0$. But $\dot{p}_\psi = 0$ is just equivalent to

$$\tan\psi = \frac{p_y}{p_x}. \tag{19}$$

Thus, during any interval $[t_j, t'_j]$ described in this optimization problem, the optimal control is always "bang-bang" or "bang-singular-bang."

5. PROPERTIES OF OPTIMAL PATHS

The exact solution of (11) requires that the times t_i when the isolated maxima occur, as well as the boundary conditions at the beginning and end of each interval $[t_i - T, t_i]$, be selected optimally. Unfortunately, the necessary conditions do not indicate how these times or boundary conditions are to be chosen. Thus, the necessary conditions of optimality do not define a unique candidate for the solution of (11). Additionally, an exact solution requires knowledge of the aircraft RCS, since P_t depends on this function. In spite of these limitations, it is possible to characterize the qualitative properties of optimal controls for the two optimization problems, and use these properties to develop an efficient numerical optimization method.

5.1 Properties for Optimization Problem 1

When Optimization Problem 1 applies, consider the following assumptions, which simplify the problem enough to determine the qualitative nature of the trajectory.

Assumption 1: $\partial P_t/\partial u = 0$ from the nose or tail aspect.

Assumption 2: From any aspect angle other than the nose or tail, the aircraft RCS increases monotonically as the magnitude of bank angle increases. Additionally, $\partial\sigma/\partial\mu = 0$ at $\mu = 0$.

Assumption 3: P_t is a saturation function of RCS. (see Figure 1 where the saturation function and the exact expression (5) are shown). The effect of this approximation is to set $\partial P_t/\partial\sigma \equiv 0$, hence $\partial P_t/\partial u \equiv 0$, for all but a small range of values of σ.

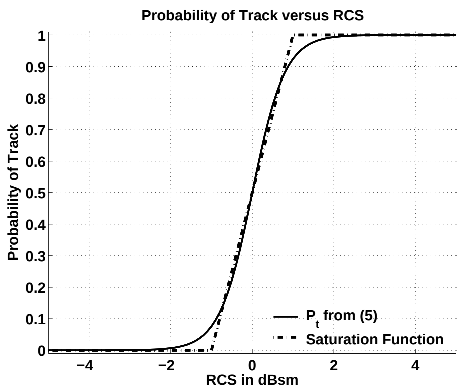


Fig. 1. P_t versus RCS

Under these conditions, the optimum control necessarily has the following properties:

Property 1: If $p_\psi \neq 0$ when flying directly towards or away from a radar, then a turn must be initiated using maximum bank. Indeed, since $\partial P_t/\partial u = 0$, and $p_\psi \neq 0$, the control is "bang-bang" by (17).

Property 2: If $p_\psi \neq 0$ and $P_t = 1$ when flying straight, then a turn must be initiated using maximum bank. Indeed, the fact that $P_t = 1$

during straight flight indicates $\partial P_t/\partial u = 0$ for all μ. Since $p_\psi \neq 0$, the control is "bang-bang" by (17).

Property 3: When $p_\psi = 0$, no turn is required. Indeed, if $p_\psi = 0$ then $\partial P_t/\partial u = 0$ by (16). Two conditions that satisfy this requirement are when $\mu = 0$ or when P_t, as a function of RCS, is saturated. In either case, $\mu = 0$ satisfies the necessary conditions.

5.2 Properties for Optimization Problem 2

When Optimization Problem 2 applies, the control is either "bang-bang" or "bang-singular-bang." Because, in this case, the cost functional is independent of the control input, there is no unique solution. Therefore, this problem is treated as a minimum time problem, which has the same necessary conditions, and results in the minimum path length. Waypoints are treated as interior-point state constraints that necessitate jump discontinuities in p_x, p_y, and p_ψ (Bryson and Ho, 1975).

6. NUMERICAL PROCEDURE

The properties of optimal paths outlined in Section 5 suggest that a reasonable suboptimal trajectory might be achieved by using only zero bank or maximum bank. As a result, the proposed numerical method for solving the dynamic optimization problem (11) is as follows. An initial path is determined by requiring overflight of a sequence of movable control points and fixed waypoints. Overflight of control points is followed by a maximum bank turn to roll out on a heading towards the next point. Turns at waypoints are not required. The coordinates of the control points serve as inputs to a finite-dimensional, nonlinear programming problem, which is solved numerically using the MATLAB Optimization Toolbox. Fixed arrival times at designated waypoints are treated as equality constraints. The requirement to complete the mission within the interval $[0, T_m]$ is met by establishing an inequality constraint on the time at the last waypoint.

7. EXAMPLE

7.1 RCS Model

A simplified example of an RCS model for an aircraft is that of an ellipsoid. Although this model does not represent the RCS of a particular aircraft, it captures three important characteristics: a) relatively small frontal RCS, b) larger beam aspect RCS, and c) relatively large RCS when viewed from above or below. Additionally, this RCS model satisfies Assumptions 1 and 2 of Section 5. Note that the numerical procedure of this section also applies to RCS models that may better approximate the actual aircraft RCS.

Following (Mahafza, 2000), the RCS of an ellipsoid is given by

$$\sigma(\alpha,\phi,\mu) = \frac{\pi a^2 b^2 c^2}{(a^2 \sin^2 \alpha_e \cos^2 \mu_e + b^2 \sin^2 \alpha_e \sin^2 \mu_e + c^2 \cos^2 \alpha_e)^2}, \tag{20}$$

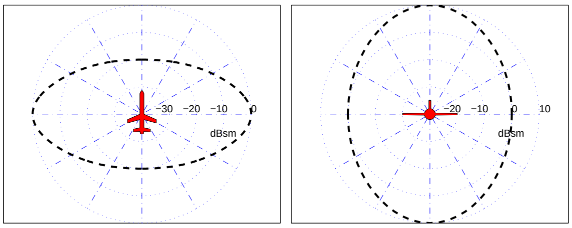
$$\alpha_e = \arccos\left(\cos(\phi)\cos(\alpha)\right), \tag{21}$$

$$\mu_e = \mu - \arctan\left(\frac{\tan(\phi)}{\sin(\alpha)}\right), \tag{22}$$

where the parameters a, b, and c represent the principal axes of the ellipsoid.

7.2 UCAV and Radar Parameters

The aircraft model assumes that the UCAV has a speed of $v = 252$ m/sec (0.8 M) and its RCS, depicted in Figure 2, is characterized by the parameters $(a, b, c) = (0.3172, 0.1784, 1.003)$. The maximum bank angle is $U = 78.5°$ and corresponds to a 5g level turn. The radar model



a) UAV RCS as a Function of Aspect Angle   b) UAV RCS as a Function of Bank Angle

Fig. 2. Dependence of UCAV RCS on Aspect and Bank Angles

assumes that $T_{resp} = 30$ sec, and the missile flyout time is fixed at $T_{fo} = 30$ sec. Hence, the threat window of the radar-missile subsystems is 1 min. The aircraft altitude is $z = 15.1\,km$ and the mission time, T_M, is 750 sec.

7.3 Mission Specifications

This mission is specified in terms of the radar position, initial UCAV position and velocity vector, waypoints, destination, and the arrival time at each waypoint. Distances are in kilometers, time is in seconds, and speed is in m/sec. Specifically:
- Radar position=(0,0).
- Initial position A=(-100, -12), $t = 0$.
- Initial velocity $(v_x, v_y) = (252, 0)$.
- Waypoint B=(-74.1, 0), $t = 135.0$.
- Waypoint C=(-51.9, 52.3), $t = 522.0$.
- Destination D=(-31.3, 96.9), $t = 750.0$.

7.4 Optimal path

Using these mission specifications and the optimization approach outlined in Section 6, the optimal path is computed and depicted in Figure 3. In this figure the following conventions are used:

- Initial position, waypoints, and destination are depicted as alphabetically labeled large circles while control points are depicted as small circles.
- Radar position is shown by a diamond.
- The range at which $P_t = 0.5$ for a target with $\sigma = 1m^2$ is shown by a dashed arc of circle.
- The UCAV ground track is a solid line.
- The instantaneous P_t is indicated by the darkness of a line-of-sight from the radar towards the aircraft as shown in the legend of Figure 3.

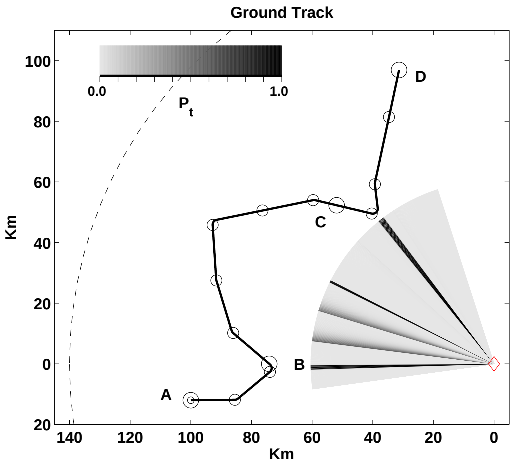


Fig. 3. Optimal Path Based on Model (1)-(10)

On the optimal path, the maximum values of (10) is 0.159. This is especially noteworthy since the entire flight path is well within the range at which a target with RCS $\sigma = 1m^2$ is likely to be tracked (i.e., $P_t > 0.5$). This optimal path corroborates the tactical recommendation given to fighter pilots facing SAM threats: "deny range, aspect, and aim" (Shaw, 1985). Indeed, consider the BC leg; range is denied by flying as far from the radar as the time constraint allows; aspect is denied by avoiding, as much as possible, to show the larger RCS beam aspect; and aim is denied by making the periods of time, during which the UCAV is continuously tracked, short. More importantly, this path exhibits a characteristic property of optimal paths for aircraft with non-uniform RCS under threat: *The optimal trajectory exploits "weaving" maneuvers to avoid long continuous exposure of aspects with large RCS.* This property is observed in all scenarios considered to-date (Kabamba *et al.*, 2004).

8. CONCLUSIONS

In this paper, the problem of optimal path planning for UCAVs in the presence of radar-guided missile threats is treated. A new, integrated model is introduced that accounts for the interaction between the aircraft, the radar, and the missile.

Within the framework of this model, the problem of UCAV path planning is formulated as a minimax optimal control problem, with the aircraft lateral acceleration serving as control. Numerical results on UCAV path planning appear to confirm standard flying tactics of "denying range, aspect, and aim," by yielding flight paths that "weave" to avoid long exposures of aspects with large RCS.

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