PID CONTROL DESIGN FOR REHABILITATION BY QUASI-ISOMETRIC TRAINING IN PARAPLEGIA: A SIMULATION STUDY

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Abstract: This paper reports simulation results about the design of a PID controller of Functional Electrical Stimulation (FES) to be used for quasi-isometric exercises in rehabilitation of paraplegics. The simulations refer to a specific experimental device developed at the Fondazione Don Gnocchi, mainly used in standing up and sitting down using FES. This is a seesaw, with the patient on one side and a weight on the other side. The patient is seated so that its posture can be fully known in real-time by continuously monitoring the knee joint angle. The design of feedback controllers for such a device is currently based on nonlinear strategies. These are necessary when aiming to standing and sitting control. However, there is a strong demand by rehabilitation physiotherapists for easy-to-tune controllers, even though these could be used only in specific conditions. So, a PID linear controller has been designed and tested by means of simulations. The controller tuning is performed by using four different methods specifically designed for unstable systems with delay. Comparison of the performnce obtained with each controller is provided.

Keywords: Rehabilitation engineering, Functional Electrical Stimulation

1. INTRODUCTION

Functional Electrical Stimulation (FES) is a technique that uses electrical pulses to induce skeletal muscle contraction and limb movements. In a paraplegic patient who suffered a *Spinal Cord Injury* (SCI) at the thoracic level, partial motion activity of the lower limbs can be induced by means of FES. In fact, an injury causing a complete lesion of the spinal cord, results in an interruption of the neural pathways and thus the impossibility for the physiological stimulus to reach the muscles innervated below the level of the lesion. However, the muscles preserve their ability to contract themselves. Therefore, movements can be induced by proper electrical stimulation, allowing a partial restoration of functionality of the patient legs. The generation of appropriate electrical stimulation patterns to induce functional movements can be achieved by the application of feedback control theory (Schauer and Hunt, 2000; Ferrarin *et al.*, 2002; Previdi and

Carpanzano, 2003; Previdi *et al.*, 2004a; Previdi *et al.*, 2004b).

In the present work, only complete spinal lesions are considered, where the control of muscles distal to the lesion site is completely lost. The topic of this paper is a simulation study about the design of a feedback PID linear controller of Functional Electrical Stimulation to be used in rehabilitation training of paraplegics. The experimental set up, called *FES - Weight Reliever* (FES-WR), developed at the Fondazione Don Gnocchi Onlus and described in Ferrarin *et al.* (2002) is at the basis of the simulations carried out in the present paper. The FES-WR is a seesaw, with the patient on one side and a weight on the other side. The patient is seated so that its posture can be fully known in real-time by continuously monitoring the knee joint angle. By delivering a suitable electrical stimulation to the quadriceps muscles group, the patient can be raised and sit with smooth movements. The main use of this device is sit to stand to sit exercises and it has been clearly experienced that nonlinear feedback control strategies are necessary to achieve smooth movements of the knee joint from the sit position to full extension (Ferrarin *et al.*, 2002; Previdi *et al.*, 2004a; Previdi *et al.*, 2004b). However, this device can be used also to perform different exercises in rehabilitation requiring small amplitude knee joint movements. For instance, quasi-isometric exercises can be done, where the patient must stand still with an assigned knee angle and only small movement about a given position are requested. To perform such exercises it is not necessary to use complex control strategies, which could be difficult to tune for a user not specialist in control and require a big computational effort. So, we investigated the possibility of using a linear PID controller, also to fulfill the requirements of physiotherapists to have simple and easy to tune control algorithms.

Previous research in standing-up and sitting-down has evidenced that linear closed-loop controllers provide poor tracking performances (Riener and Fuhr, 1998), if complete standing must be achieved. In the present work, PID controller is designed to work for small amplitude knee joint movements. The controller parameter tuning is performed on the basis of a linear model which is estimated on the basis of small movement I/O data. The model results to be unstable with time delay. So, tuning rules for PID controllers designed for unstable systems with delay are used. Specifically, four tuning rules will be analyzed and compared on the basis of tracking performances and difficulty in computing the controller parameters.

The paper has the following structure: in Sect. 2 the experimental setup at the basis of the simulations of this work is described. In Sect. 3 frequency domain identification of model is performed, to represent the patient and seesaw dynamics. In Sect. 4 the tuning rules for PID controllers with the simulation results are shown.

2. DESCRIPTION OF THE SYSTEM

The plant of the control system considered in this work is composed by the patient and a Weight-Reliever (WR) device together. This system has been mathematically represented by means of a simulation model described in (Riener *et al.*, 1998). The model refers to the experimental set up developed at the Fondazione Don Gnocchi Onlus (Ferrarin *et al.*, 2002), which is aimed to train the paralysed quadriceps muscles of patients with spinal cord lesion, for the increase of muscle strength and endurance, in view of functional applications of FES like walking or cycling. The WR device provides the following functions: to partially support the weight of the patient during the standing up movement; to prevent patient from falling; to reduce the degrees of freedom of the multi-link human musculo-skeletal system. It consists of a seesaw construction, equipped with a saddle at one extremity and with a counterweight at the other. There is a four-bar linkage which imposes the saddle to maintain a horizontal position during the rotation of the main beam. The amount of counterweight is defined on the basis of patient weight and strength of quadriceps contraction elicited by electrical stimulation. The human body is described by a three-segmental model consisting of shanks, thighs, and upper body, since feet are flat on the ground. Nine mono and bi-articular muscle groups are modeled in the sagittal plane inducing moments about the ankle, knee, and hip joints. Each muscle group has its own activation and contraction dynamics, taking into consideration the effect of spatial and temporal summation, a linear second order calcium dynamics, muscle fatigue and recovery. In addition, a nonlinear relationship has been used to capture the force-frequency characteristic of an artificially stimulated muscle (see Riener and Quintern, 1997 for further details). The active moment developed by a single muscle group is calculated from its nonlinear moment arm and the muscle force, which is a function of maximum isometric muscle force, muscle activation, nonlinear force-length, and nonlinear force-velocity relations (see Riener and Fuhr, 1998 for further details). In the body-segmental dynamics, total joint moment is the sum of active, passive elastic and passive viscous joint moments. The latter has been modeled as a linear relation between the angular joint velocity and the damping coefficient, while passive elastic joint moments have been modeled by double exponential equations which account for the influence of the adjacent joint angles. Active joint moment is the sum of the joint moments produced by each muscle group. It is assumed that the patient's hip is rigidly connected with the WR beam and that the trunk remains vertical during movement, yielding a close chain consisting of shanks, thighs and WR beam, characterized by only one degree of freedom, since all angles can be expressed as a function of the knee joint angle. This hypothesis is certainly true for small movements, as in the case of the present work.

3. FREQUENCY DOMAIN IDENTIFICATION

In this section, a linear model of the plant dynamics is estimated. Black-box identification is used, i.e. the plant model is derived by I/O measurements on the plant. Identification will be performed by using "small signal" data, i.e. the model is estimated to approximate the plant dynamics only about a given operating point of the plant, corresponding to half of the knee joint extension (about 1 rad). The identification procedure must be performed in closed loop, due to the plant instability. This means that a stabilizing PI feedback controller is applied to the plant. It is worth noting that this controller is not designed to achieve any specific tracking performance: its aim is only to provide external stability when closed in loop with the plant, so that data acquisition for identification can be done (Previdi et al., 2004a).

The I/O data for the model identification has been generated as follows: the closed loop system has been fed with a set of 27 pure-tone reference signals, i.e. $w(t) = A\sin(\Omega_i t) + 1$ with frequencies $\Omega_i = n \cdot 10^m$ rad/s, where $n = 1,2,3,...,9$ and $m = -1,0,1$. The amplitude A has been kept small enough (about 5°) to reduce nonlinear distortion in the measured signals. Then a direct method for closed loop identification has been used (see Van Den Hof and Schrama, 1995; Forssell and Ljung, 1999). Specifically, the control action $u(t)$, i.e. the pulsewidth modulation of the electrical stimulation, and the system output $y(t)$, i.e. the knee joint angle, being pure-tone signals at the same frequency of the reference, have been directly recorded and used for identification. Using the estimated magnitude and phase of both u and y, a set of points $\left\{G\left(e^{j\Omega_i}\right)\right\}_{i=1,\ldots,27}$ of the frequency response of the open-loop plant is available. Then, the following third order parametric model with delay has been chosen:

$$G(s) = \frac{\rho e^{-s\tau}}{(s-p_1)(s-p_2)(s-p_3)} \qquad (1)$$

where: ρ is the Evans gain; τ is delay of the system; p_i with $i = 1,\ldots,3$ are the poles. At least one pole must be positive, to force the system to be unstable. The choice of this structure has been done by looking at the measured frequency response (see Fig. 1). From this figure it is quite evident that the plant has a delay and a relative degree equal to 3. The estimated parameter values are: $\rho = -1.135; p_1 = 2.27; p_2 = p_3 = -8.26; \tau = -0.043$.

In Fig. 1 the frequency response magnitude and phase values computed from the sinusoidal test input are shown (star plot) together with the frequency response values of the estimated model.

In Fig. 2 a comparison between the plant and the model output in time domain is shown. The reference signal is a square wave with amplitude 0.1 rad, period 40 s and mean value 1 rad, i.e. the operating point about which the model has been estimated. This square wave reference has been filtered with a first order filter with unit gain and time constant 1 s. From this figure it is possible to argue some outcomes. First of all, the plant shows nonlinear behaviour also for small amplitude excitations. In fact, the plant response to the upward step is different from that to the downward one. Secondly, the estimated model is slightly more damped than the plant. Finally, the performance of the PI controller used for closed loop identification are definitely poor (small damping, long settling time), even though the integral action is able to reduce to zero the steady state error.

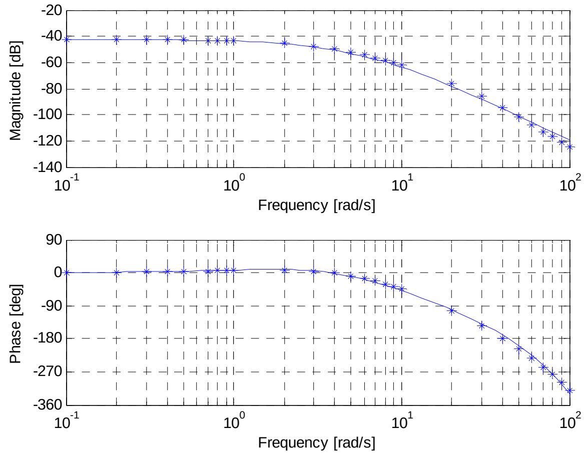

Figure 1. Bode plots of the frequency response values estimated from the I/O sinusoidal measurements about the operating point u^0=236; y^0=1.0 (*). The solid lines represent the magnitude and phase of the frequency response of the model (Eq. 1) with the estimated parameter values.

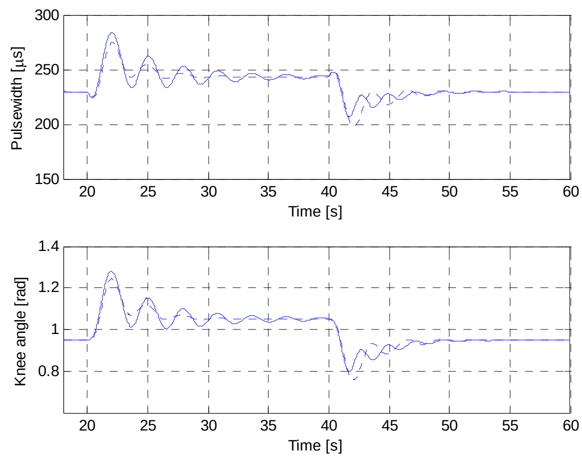

Figure 2. Comparison between plant response (solid line) and model simulated response (dotted line). In the upper plot the pulsewidth modulation signal of the electrical stimulation (the control action); in the lower plot the knee joint angle (the output).

4. PID CONTROLLER TUNING AND SIMULATION RESULTS

In this work, a PID controller is used. The tuning of the controller has been performed using four different tuning rules (O'Dwyer, 2004; Rotstein and Lewin, 1991; Lee *et al.*, 2000; McMillan, 1984; Poulin and Pomerleau, 1996; Poulin and Pomerleau, 1997) and the performances of the controllers are compared by means of simulations. The tuning rules used in the following are specifically designed for closed loop control of unstable systems with delay. The controller parameters are directly computed on the basis of the estimated model parameters (See Sect. 3) and, in some cases, fine tuning of additional parameters must be done.

The controller parameters are tuned on the basis of a plant second order model with delay:

$$G(s)=\frac{K_m e^{-s\tau}}{(T_1 s-1)(1+sT_2)} \qquad (2)$$

As shown in Section 3, the estimated model is a third order model. So, it must be reduced to be put in the form of Eq. (2). To this aim, the highest frequency pole has been neglected, taking care to preserve the value of the model gain. So, with reference to Eq. (2), the following model parameters must be used for the controller tuning: $K_m=-7.322\cdot 10^{-3};\tau=0.150;T_1=0.4406;T_2=0.121$.

Notice that, in order to keep a good approximation of the model phase, the delay value has been slightly increased.

In the following, two different PID controller structures are considered: the so called *controller with filtered derivative* (Eq. 3) and the *classical controller* (Eq. 4):

$$G_c(s)=K_c\left(1+\frac{1}{sT_i}+\frac{sT_d}{1+s\frac{T_d}{N}}\right) \qquad (3)$$

$$G_c(s)=K_c\left(1+\frac{1}{sT_i}\right)\left(\frac{1+sT_d}{1+s\frac{T_d}{N}}\right) \qquad (4)$$

The controller performances will be tested using a reference square wave signal $w(t)$ with period 20 s and amplitude 0.15 rad, starting in equilibrium conditions with knee joint value 1 rad. This signal represents the situation of a patient during quasi-isometric exercise. The performance will be evaluated considering the quality of the tracking and the following quadratic performance index:

$$J=\frac{1}{N}\sum_{t=1}^{N}\left(y(t)-w(t)\right)^2 \qquad (5)$$

The first tuning rule is due to Rotstein and Lewin (1991) and it is referred to the controller structure of Eq. (3). The regulator parameters are determined by the following equations:

$$K_c=\frac{T_1\left[\lambda\left(\frac{\lambda}{T_1}+2\right)+T_2\right]}{\lambda^2 K_m} \qquad (6.1)$$

$$T_i=\lambda\left(\frac{\lambda}{T_1}+2\right)+T_2 \qquad (6.2)$$

$$T_d=\frac{\lambda\left(\frac{\lambda}{T_1}+2\right)T_2}{\lambda\left(\frac{\lambda}{T_1}+2\right)+T_2} \qquad (6.3)$$

where λ is a further parameter to be used for fine tuning. Its value must be chosen on the basis of the ratio between the delay value τ and the largest time constant of the poles, which is T_1 in the present case. So, in the present case, $\lambda\in[0.39,0.83]$. By means of simulations, the value $\lambda=0.45$ has been easily chosen. So, the resulting controller parameters are: $K_c=-440;T_i=1.48;T_d=0.111$, with $N=20$.

In Fig. 3 simulation results are shown. The value of the performance index of Eq. (5) is $J=4.69\cdot 10^{-4}$.

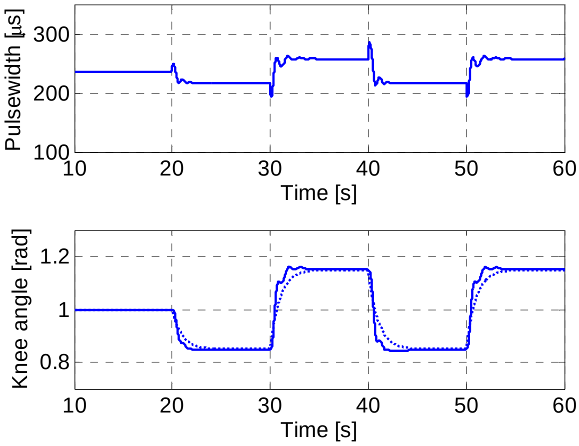

Figure 3. Closed-loop simulation test using a PID controller with the structure of Eq. (3) and parameters tuned by Eqs. (6). The upper plot represents the control action. The lower plot the reference signal (dotted) and the plant simulated output (solid).

The second tuning rule is due to Lee *et al.* (2000) and it is again referred to the controller structure of Eq. (3). This tuning rule can be applied only under the hypothesis that the ratio between the plant delay and the highest time constant of the poles is less than 2 (in the present case it is about 1.24). The proposed regulator parameters are computed using the following equations:

$$K_c=\frac{T_i}{-K_m(2\lambda+\tau_m-\alpha)} \qquad (7.1)$$

$$T_i=-T_1+T_2+\alpha-\frac{\lambda^2+\alpha\tau_m-0.5\tau_m^2}{2\lambda+\tau_m-\alpha} \qquad (7.2)$$

$$T_d=\frac{-T_1\alpha+T_2\alpha-T_1T_2-\frac{\tau_m^2(0.167\tau_m-0.5\alpha)}{2\lambda+\tau_m-\alpha}}{T_i}+ -\frac{\lambda^2+\alpha\tau_m-0.5\tau_m^2}{2\lambda+\tau_m-\alpha} \qquad (7.3)$$

with:

$\alpha = T_1\left[\left(\frac{\lambda}{T_1}+1\right)^2 e^{\tau_m/T_1}-1\right]$ and λ is the desired closed loop time constant. By choosing $\lambda = 0.2$, which corresponds to a closed loop settling time of about 1 s, the following controller parameters are obtained: $K_c = -449; T_i = 1.048; T_d = 0.156$, with $N = 20$. Notice that, although this tuning procedure is based on a completely different theoretical analysis, the obtained parameter values are very similar to those provided by Eqs. (6).

In Fig. 4 simulation results are shown. The value of the performance index of Eq. (5) is $J = 2.83 \cdot 10^{-4}$.

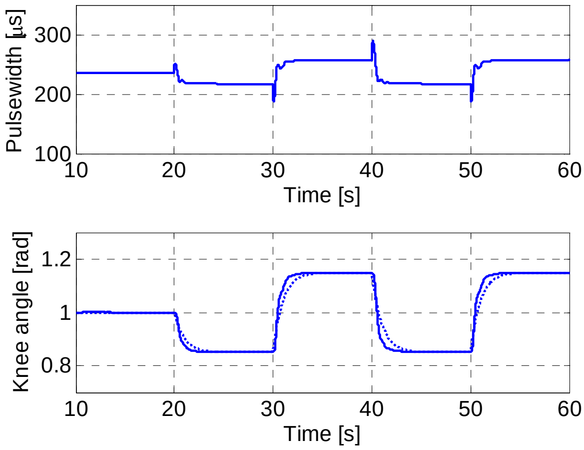

Figure 4. Closed-loop simulation test using a PID controller with the structure of Eq. (3) and parameters tuned by Eqs. (7). The upper plot represents the control action. The lower plot the reference signal (dotted) and the plant simulated output (solid).

The last tuning rule tested in the present work for the controller structure of Eq. (3) is based on the work by McMillan (1984):

$$K_c = \frac{1.111 T_1 T_2}{K_m \tau_m^2}\left\{\frac{1}{1+\left[\frac{(T_1+T_2)T_1T_2}{(T_1-T_2)(T_1-\tau_m)\tau_m}\right]^{0.65}}\right\}^2 \tag{8.1}$$

$$T_i = 2\tau_m\left\{1+\left[\frac{(T_1+T_2)T_1T_2}{(T_1-T_2)(T_1-\tau_m)\tau_m}\right]^{0.65}\right\} \tag{8.2}$$

$$T_d = 0.25 T_i \tag{8.3}$$

Notice that, in Eqs. (8), there are no other parameters but those of the model of Eq. (2). So, the application of these tuning rules is straightforward, but no fine tuning is allowed to the user. Finally, observe that the choice of $T_d = 0.25T_i$ is done in order to obtain a controller with two real coincident zeros. The controller parameters are: $K_c = -514; T_i = 0.7933; T_d = 0.1983$, with $N = 20$.

In Fig. 5 simulation results are shown. The value of the performance index of Eq. (5) is $J = 1.44 \cdot 10^{-4}$.

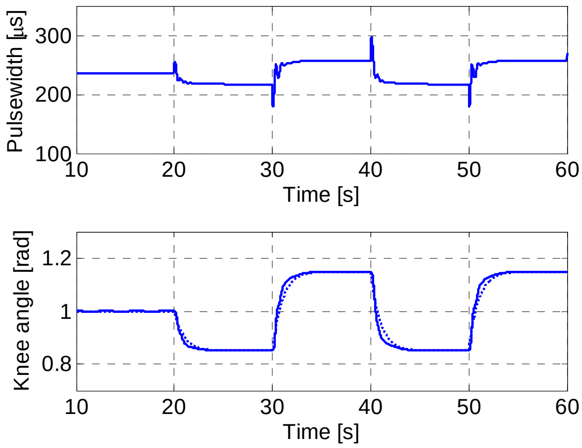

Figure 5. Closed-loop simulation test using a PID controller with the structure of Eq. (3) and parameters tuned by Eqs. (8). The upper plot represents the control action. The lower plot the reference signal (dotted) and the plant simulated output (solid).

Finally, also a tuning rule for the classical controller (Eq. 4) has been tested based on the work of Poulin and Pomerleau (1996, 1997). It is based on a minimum ITAE criterion and the regulator parameters are obtained by the following equations:

$$K_c = \frac{bT_1\sqrt{1+\frac{aT_2^2}{4(\tau_m+T_2)^2}}}{K_m\left[aT_1-4(\tau_m+T_2)\right]} \tag{9.1}$$

$$T_i = \frac{4T_1(\tau_m+T_2)}{aT_1-4(\tau_m+T_2)} \tag{9.2}$$

$$T_d = T_2 \tag{9.3}$$

where a and b are two parameters for fine tuning. In the present application case, we experienced that the choice of these parameters is not an easy task. As a guideline for their use, the following criteria must be considered. First, the parameter a could not take any value, in order not to have infinite values for the integral time and the controller gain. Similarly, it has a lower bound value to avoid negative integral time values. The parameter b (proportionally) influences only the controller gain. Finally, notice that the present controller exploits the filtered derivative action to cancel the stable pole of the plant moving it at higher frequency.

By choosing $a = 3.2$, $b = 2.5$ the following controller parameters are obtained:

$K_c = -497; T_i = 1.466; T_d = 0.121$, with $N = 20$.

In Fig. 6 simulation results are shown. The value of the performance index of Eq. (5) is $J = 3.22 \cdot 10^{-4}$.

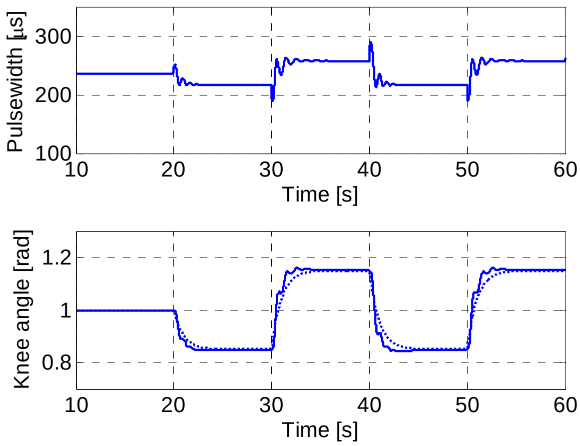

Figure 6. Closed-loop simulation test using a PID controller with the structure of Eq. (4) and parameters tuned by Eqs. (9). The upper plot represents the control action. The lower plot the reference signal (dotted) and the plant simulated output (solid).

5. CONCLUSIONS

In this work, the problem of control of Functional Electrical Stimulation in paraplegic patients has been considered. Specifically, control of small amplitude movements on a weight reliever device is considered. To this aim, a linear PID controller has been used and tested by means of simulations based on a complete simulation model. Four different tuning rules for the PID controller have been considered. These are specifically designed for unstable plants with delay. All the proposed tuning methods show good performances while doing a typical rehabilitation exercises, but they need different efforts in tuning. The main limitations of the present study are about some specific clinical aspects. Specifically, the presence of spasms, typical of paraplegic subjects, has not been considered. The motivation is that these issues are very specific to each subject and difficult to be modelled. So, their evaluation will be demanded to the experimental work.

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