IDENTIFICATION OF NOISY INPUT-OUTPUT SYSTEM USING BIAS-COMPENSATED LEAST-SQUARES METHOD

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Abstract: In this paper, a new bias-compensated least-squares (BCLS) based algorithm is proposed for identification of noisy input-output system. It is well known that BCLS method is based on compensation of asymptotic bias on the least-squares (LS) estimates by making use of noise variances estimates. The main feature of the proposed algorithm is to introduce a generalized least-squares type estimator in order to obtain the good estimates of noise variances. The results of a simulated example indicate that the proposed algorithm provides good estimates.

Keywords: Estimation, Identification, Least-squares method

1. INTRODUCTION

Many identification methods are based on the assumption that input measurement is noise-free. However, this condition is not satisfied in most practical situations. In the presence of input noise, those methods have been shown to give erroneous results. Several methods have been proposed to estimate unknown parameters of linear discrete-time system in the presence of input and output noises. Joint Output (JO) method (Söderström, 1981) and Koopmans-Levin (KL) method (Fernando and Nicholson, 1985) require *a priori* knowledge about the values of variances or the ratio to measurements noises.

Bias-compensated least-squares (BCLS) method is proposed by Sagara *et al.* (Sagara and Wada, 1977) and it has been extended by Wada *et al.* (Wada *et al.*, 1990) to the input-output noise case without any *a priori* knowledge of noise variances. BCLS method based on compensation of asymptotic bias on the least-squares (LS) estimates by making use of noise variances estimates is very efficient method for estimation of noisy input-output system parameters. In recent years, BCLS method has been developed to improve the estimation accuracy and several recursive algorithms have been proposed (Eguchi *et al.*, 1992; Jia *et al.*, 2001).

On the other hand, another method named bias-eliminated least-squares (BELS) method has been proposed by Zheng *et al.* (Zheng and Feng, 1989) in which the different estimation method of asymptotic bias is used and further developed to be the efficient method (Zheng, 1999; Zheng, 2000; Zheng, 2001; Zheng, 2002) to treat bias problem in noisy input-output system identification.

In this paper, a new BCLS based algorithm is proposed for identification of linear discrete-time system in the case where input and output measurements are corrupted by white noise. Since the unknown noise variances estimates are required for compensation of asymptotic bias of LS estimates, the estimation of these noise variances plays an important role in BCLS method. If the good estimates of noise variances are obtained, the estimation accuracy of the resulting BCLS estimates can be improved. For this purpose, a generalized least-squares type estimator is introduced in order to obtain the good estimates of noise variances. It is demonstrated that the improvement in the estimate accuracy of noise variances estimates (and the resulting BCLS estimates) is achieved.

This paper is organized as follows. In section 2 and section 3, the problem statement is presented and the asymptotic bias of LS estimator is described. In section 4, the BCLS algorithm is derived for estimating unknown parameters of linear discrete-time system in the presence of input and output noises and it can be learned that the unknown noise variances must be estimated in order to obtain consistent estimates of parameters. In section 5, the recursive algorithm of unknown noise variances are derived by introducing a generalized least-squares type estimator and in section 6, Jia *et al.*'s BCLS method (Jia *et al.*, 2001) and Zheng's BELS method (Zheng, 2001) are briefly described. The simulation results are presented in section 7 and finally section 8 gives the conclusion.

2. PROBLEM STATEMENT

Consider the parameter estimation problem of single-input single-output linear discrete-time system described as follows:

$$A(q^{-1})y_t = B(q^{-1})u_t \tag{1}$$

where u_t and y_t are the true input and output, q^{-1} is shift operator, $q^{-1}u_t = u_{t-1}$, and the polynomials $A(q^{-1})$ and $B(q^{-1})$ are defined by

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_n q^{-n} \tag{2}$$
$$B(q^{-1}) = \qquad b_1 q^{-1} + \cdots + b_n q^{-n} \ . \tag{3}$$

Let z_t and w_t be the noise-corrupted measurements of y_t and u_t, respectively, i.e.

$$z_t = y_t + e_t, \ w_t = u_t + d_t \tag{4}$$

where e_t and d_t are the output and input measurement noises, respectively. The measurement noises e_t and d_t have the following statistical properties

$$E[d_t] = 0, \ E[e_t] = 0 \tag{5}$$
$$E[d_i d_j] = \sigma_d^2 \delta_{i,j}, \ E[e_i e_j] = \sigma_e^2 \delta_{i,j} \tag{6}$$
$$E[d_i e_j] = 0 \tag{7}$$

where $\delta_{i,j}$ is Kronecker's delta. The true input u_t is zero-mean stationary random process with finite variance, and u_t, d_t and e_t are assumed to be statistically independent of each other.

Substituting (4) into (1) yields

$$A(q^{-1})z_t = B(q^{-1})w_t + v_t \tag{8}$$

where v_t is a composite noise defined by

$$v_t = A(q^{-1})e_t - B(q^{-1})d_t \ . \tag{9}$$

Define some vectors as

$$\boldsymbol{\theta}^T = [\boldsymbol{a}^T, \ \boldsymbol{b}^T] = [a_1 \cdots a_n, \ b_1 \cdots b_n] \tag{10}$$
$$\begin{aligned} \boldsymbol{p}_t^T &= [-\boldsymbol{z}_t^T, \ \boldsymbol{w}_t^T] \\ &= [-z_{t-1} \cdots - z_{t-n}, \ w_{t-1} \cdots w_{t-n}] \end{aligned} \tag{11}$$
$$\begin{aligned} \boldsymbol{q}_t^T &= [-\boldsymbol{y}_t^T, \ \boldsymbol{u}_t^T] \\ &= [-y_{t-1} \cdots - y_{t-n}, \ u_{t-1} \cdots u_{t-n}] \end{aligned} \tag{12}$$
$$\begin{aligned} \boldsymbol{r}_t^T &= [-\boldsymbol{e}_t^T, \ \boldsymbol{d}_t^T] \\ &= [-e_{t-1} \cdots - e_{t-n}, \ d_{t-1} \cdots d_{t-n}] \end{aligned} \tag{13}$$

then (4), (8) and (9) can be written as

$$\boldsymbol{p}_t = \boldsymbol{q}_t + \boldsymbol{r}_t \tag{14}$$
$$z_t = \boldsymbol{p}_t^T \boldsymbol{\theta} + v_t \tag{15}$$
$$v_t = e_t - \boldsymbol{r}_t^T \boldsymbol{\theta} \ . \tag{16}$$

Let the equation error ξ_t for an estimate $\widehat{\boldsymbol{\theta}}$ of $\boldsymbol{\theta}$ be defined as

$$\begin{aligned} \xi_t &= \widehat{A}(q^{-1})z_t - \widehat{B}(q^{-1})w_t \\ &= z_t - \boldsymbol{p}_t^T \widehat{\boldsymbol{\theta}} \end{aligned} \tag{17}$$

where the polynomials $\widehat{A}(q^{-1})$ and $\widehat{B}(q^{-1})$ are defined by

$$\widehat{A}(q^{-1}) = 1 + \widehat{a}_1 q^{-1} + \cdots + \widehat{a}_n q^{-n} \tag{18}$$
$$\widehat{B}(q^{-1}) = \qquad \widehat{b}_1 q^{-1} + \cdots + \widehat{b}_n q^{-n} \tag{19}$$

and

$$\widehat{\boldsymbol{\theta}}^T = [\widehat{\boldsymbol{a}}^T, \ \widehat{\boldsymbol{b}}^T] = [\widehat{a}_1 \cdots \widehat{a}_n, \ \widehat{b}_1 \cdots \widehat{b}_n] \ . \tag{20}$$

The least-squares estimate is given by

$$\widehat{\boldsymbol{\theta}}_{LS,N} = \left(\sum_{t=1}^{N} \boldsymbol{p}_t \boldsymbol{p}_t^T \right)^{-1} \sum_{t=1}^{N} \boldsymbol{p}_t z_t \ . \tag{21}$$

From the assumption of e_t and d_t, the composite noise v_t defined by (9) is not white. Hence the least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$ has a bias asymptotically. In the next section, the asymptotic bias induced by least-squares estimator is derived.

3. ASYMPTOTIC BIAS OF LEAST-SQUARES ESTIMATOR

Substituting (15) into (21) yields

$$\hat{\boldsymbol{\theta}}_{LS,N} = \boldsymbol{\theta} + \boldsymbol{P}_N \sum_{t=1}^{N} \boldsymbol{p}_t v_t \tag{22}$$

where $\boldsymbol{P}_N$ is the product moment matrix as follows:

$$\boldsymbol{P}_N = \left(\sum_{t=1}^{N} \boldsymbol{p}_t \boldsymbol{p}_t^T \right)^{-1} . \tag{23}$$

Taking probability limit of above equation yields

$$\underset{N\to\infty}{p\lim}\, \widehat{\boldsymbol{\theta}}_{LS,N} = \boldsymbol{\theta} + \boldsymbol{h} \tag{24}$$

where $\boldsymbol{h}$ is the asymptotic bias of the least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$ defined as

$$\boldsymbol{h} = \boldsymbol{R}_{pp}^{-1} \underset{N\to\infty}{p\lim} \frac{1}{N} \sum_{t=1}^{N} \boldsymbol{p}_t v_t \tag{25}$$

where $\boldsymbol{R}_{pp} = E[\boldsymbol{p}_t \boldsymbol{p}_t^T]$. Using the assumption of e_t, d_t and (14), (16), it is easily shown that

$$\begin{aligned} \underset{N\to\infty}{p\lim} \frac{1}{N} \sum_{t=1}^{N} \boldsymbol{p}_t v_t &= E[\boldsymbol{p}_t v_t] \\ &= E[(\boldsymbol{q}_t + \boldsymbol{r}_t)(e_t - \boldsymbol{r}_t^T \boldsymbol{\theta})] \\ &= -E[\boldsymbol{r}_t \boldsymbol{r}_t^T]\boldsymbol{\theta} \\ &= -\boldsymbol{D}\boldsymbol{\theta} \end{aligned} \tag{26}$$

where $\boldsymbol{D} = \mathrm{diag}\{\sigma_e^2 \boldsymbol{I}_n;\ \sigma_d^2 \boldsymbol{I}_n\}$ and $\boldsymbol{I}_n$ is $n \times n$ identity matrix. From (24), (25) and (26), the asymptotic bias $\boldsymbol{h}$ can be expressed as follows:

$$\begin{aligned} \boldsymbol{h} &= \underset{N\to\infty}{p\lim}\, \widehat{\boldsymbol{\theta}}_{LS,N} - \boldsymbol{\theta} \\ &= \boldsymbol{R}_{pp}^{-1} \{-\boldsymbol{D}\boldsymbol{\theta}\} \ . \end{aligned} \tag{27}$$

4. BIAS-COMPENSATED LEAST-SQUARES METHOD

From (24), it can be expected that a consistent estimate of $\boldsymbol{\theta}$ can be obtained by compensating for the asymptotic bias $\boldsymbol{h}$ in the least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$. From (27), estimate of the asymptotic bias $\boldsymbol{h}$ becomes

$$\widehat{\boldsymbol{h}}_N = -N\boldsymbol{P}_N \boldsymbol{D}\boldsymbol{\theta} \ . \tag{28}$$

Hence bias-compensated least-squares estimate $\widehat{\boldsymbol{\theta}}_{BC,N}$ is given by

$$\begin{aligned} \widehat{\boldsymbol{\theta}}_{BC,N} &= \widehat{\boldsymbol{\theta}}_{LS,N} - \widehat{\boldsymbol{h}}_N \\ &= \widehat{\boldsymbol{\theta}}_{LS,N} + N\boldsymbol{P}_N \boldsymbol{D} \widehat{\boldsymbol{\theta}}_{BC,N-1} \ . \end{aligned} \tag{29}$$

The recursive algorithm of $\widehat{\boldsymbol{\theta}}_{LS,N}$ and $\boldsymbol{P}_N$ are obtained by the ordinary recursive least-squares algorithm

$$\widehat{\boldsymbol{\theta}}_{LS,N} = \widehat{\boldsymbol{\theta}}_{LS,N-1} + \frac{\boldsymbol{P}_{N-1}\boldsymbol{p}_N (z_N - \boldsymbol{p}_N^T \widehat{\boldsymbol{\theta}}_{LS,N-1})}{1 + \boldsymbol{p}_N^T \boldsymbol{P}_{N-1} \boldsymbol{p}_N} \tag{30}$$

$$\boldsymbol{P}_N = \boldsymbol{P}_{N-1} - \frac{\boldsymbol{P}_{N-1}\boldsymbol{p}_N \boldsymbol{p}_N^T \boldsymbol{P}_{N-1}}{1 + \boldsymbol{p}_N^T \boldsymbol{P}_{N-1} \boldsymbol{p}_N} \ . \tag{31}$$

Practically the variances of input and output noises σ_e^2 and σ_d^2 in (29) are unknown, it is necessary to estimate them.

5. ESTIMATION OF NOISE VARIANCES

To estimate the noise variances of input and output noises σ_e^2 and σ_d^2, a filter $\alpha(q^{-1})$ defined by the following equation is introduced

$$\alpha(q^{-1}) = \sum_{i=0}^{l} \alpha_i q^{-i},\ (l \geq n) \ . \tag{32}$$

Now, let the filtered signal for equation error ξ_t be defined as

$$\widetilde{\xi}_t = \alpha(q^{-1})\xi_t = \sum_{i=0}^{l} \alpha_i \xi_{t-i} \ . \tag{33}$$

Minimizing the sum of squared $\widetilde{\xi}_t$ yields the estimator $\widehat{\boldsymbol{\varphi}}_N$ of $\boldsymbol{\theta}$

$$\widehat{\boldsymbol{\varphi}}_N = \left(\sum_{t=1}^{N} \widetilde{\boldsymbol{p}}_t \widetilde{\boldsymbol{p}}_t^T \right)^{-1} \left(\sum_{t=1}^{N} \widetilde{\boldsymbol{p}}_t \widetilde{z}_t \right) \tag{34}$$

where $\widetilde{\boldsymbol{p}}_t$ is the filtered signal for $\boldsymbol{p}_t$ and $\widetilde{z}_t$ is the filtered signal for z_t as

$$\widetilde{\boldsymbol{p}}_t = \alpha(q^{-1})\boldsymbol{p}_t = \sum_{i=0}^{l} \alpha_i \boldsymbol{p}_{t-i} \tag{35}$$

$$\widetilde{z}_t = \alpha(q^{-1})z_t = \sum_{i=0}^{l} \alpha_i z_{t-i} \ . \tag{36}$$

The estimator $\widehat{\boldsymbol{\varphi}}_N$ can be considered as generalized least-squares type estimator, and the recursive algorithm of $\widehat{\boldsymbol{\varphi}}_N$ is obtained by

$$\widehat{\boldsymbol{\varphi}}_N = \widehat{\boldsymbol{\varphi}}_{N-1} + \frac{\widetilde{\boldsymbol{P}}_{N-1}\widetilde{\boldsymbol{p}}_N(\widetilde{z}_N - \widetilde{\boldsymbol{p}}_N^T\widehat{\boldsymbol{\varphi}}_{N-1})}{1+\widetilde{\boldsymbol{p}}_N^T\widetilde{\boldsymbol{P}}_{N-1}\widetilde{\boldsymbol{p}}_N} \quad (37)$$

$$\widetilde{\boldsymbol{P}}_N = \widetilde{\boldsymbol{P}}_{N-1} - \frac{\widetilde{\boldsymbol{P}}_{N-1}\widetilde{\boldsymbol{p}}_N\widetilde{\boldsymbol{p}}_N^T\widetilde{\boldsymbol{P}}_{N-1}}{1+\widetilde{\boldsymbol{p}}_N^T\widetilde{\boldsymbol{P}}_{N-1}\widetilde{\boldsymbol{p}}_N} \quad (38)$$

where

$$\widetilde{\boldsymbol{P}}_N = \left(\sum_{t=1}^{N} \widetilde{\boldsymbol{p}}_t\widetilde{\boldsymbol{p}}_t^T\right)^{-1} . \quad (39)$$

It follows from (14) that

$$\widetilde{z}_t = \widetilde{\boldsymbol{p}}_t^T\boldsymbol{\theta} + \widetilde{v}_t \quad (40)$$

where $\widetilde{v}_t$ is the filtered signal for v_t as

$$\widetilde{v}_t = \alpha(q^{-1})v_t = \sum_{i=0}^{l} \alpha_i v_{t-i} . \quad (41)$$

Define the the sum of squared residual $\widehat{f}_N$ as

$$\widehat{f}_N = \sum_{t=1}^{N} \widehat{\widetilde{\xi}}_t^2 = \sum_{t=1}^{N}(\widetilde{z}_t - \widetilde{\boldsymbol{p}}_t^T\widehat{\boldsymbol{\varphi}}_N)^2 = \sum_{t=1}^{N}\widetilde{z}_t\widetilde{v}_t - \widehat{\boldsymbol{\varphi}}_N^T\sum_{t=1}^{N}\widetilde{\boldsymbol{p}}_t\widetilde{v}_t \quad (42)$$

where $\widehat{\widetilde{\xi}}_t$ is the residual of the estimator $\widehat{\boldsymbol{\varphi}}_N$ defined by

$$\widehat{\widetilde{\xi}}_t = \widetilde{z}_t - \widetilde{\boldsymbol{p}}_t^T\widehat{\boldsymbol{\varphi}}_N . \quad (43)$$

Taking probability limit of (42) yields

$$\underset{N\to\infty}{p\lim} \frac{1}{N}\sum_{t=1}^{N}\widetilde{z}_t\widetilde{v}_t = E[\widetilde{z}_t\widetilde{v}_t] = \sigma_e^2\left(\sum_{i=0}^{l}\alpha_i^2 + \boldsymbol{\rho}^T\boldsymbol{a}\right) \quad (44)$$

and

$$\underset{N\to\infty}{p\lim} \frac{1}{N}\sum_{t=1}^{N}\widetilde{\boldsymbol{p}}_t\widetilde{v}_t = E[\widetilde{\boldsymbol{p}}_t\widetilde{v}_t] = -\sigma_e^2\begin{bmatrix}\boldsymbol{\rho}\\ \boldsymbol{0}_n\end{bmatrix} - \widetilde{\boldsymbol{D}}\boldsymbol{\theta} \quad (45)$$

where

$$\boldsymbol{\rho}^T = \left[\sum_{i=0}^{l-1}\alpha_i\alpha_{i+1},\ \sum_{i=0}^{l-2}\alpha_i\alpha_{i+2},\ \cdots,\ \sum_{i=0}^{l-n}\alpha_i\alpha_{i+n}\right] \quad (46)$$

$$\widetilde{\boldsymbol{D}} = \mathrm{diag}\{\sigma_e^2\boldsymbol{H}_n;\ \sigma_d^2\boldsymbol{H}_n\} \quad (47)$$

$$\boldsymbol{H}_n = \sum_{i=0}^{l}\alpha_i^2\boldsymbol{I}_n + \sum_{j=1}^{n-1}\sum_{i=0}^{l-j}\alpha_i\alpha_{i+j}\left[(\boldsymbol{S}_n)^{(j)} + (\boldsymbol{S}_n^T)^{(j)}\right] \quad (48)$$

$$\boldsymbol{S}_n = \begin{bmatrix}\boldsymbol{0}_{n-1}^T & 0\\ \boldsymbol{I}_{n-1} & \boldsymbol{0}_{n-1}\end{bmatrix} \quad (49)$$

and $\boldsymbol{0}_n$ is an $n\times 1$ zero vector.

Finally,

$$\underset{N\to\infty}{p\lim}\frac{1}{N}\widehat{f}_N = \sigma_e^2\left\{\sum_{i=0}^{l}\alpha_i^2 + \boldsymbol{\rho}^T\left(\boldsymbol{a} + \underset{N\to\infty}{p\lim}\widehat{\boldsymbol{\varphi}}_{a,N}\right)\right\} + \sigma_e^2\underset{N\to\infty}{p\lim}\widehat{\boldsymbol{\varphi}}_{a,N}^T\boldsymbol{H}_n\boldsymbol{a} + \sigma_d^2\underset{N\to\infty}{p\lim}\widehat{\boldsymbol{\varphi}}_{b,N}^T\boldsymbol{H}_n\boldsymbol{b} \quad (50)$$

where $\widehat{\boldsymbol{\varphi}}_N^T = [\widehat{\boldsymbol{\varphi}}_{a,N}^T,\ \widehat{\boldsymbol{\varphi}}_{b,N}^T]$.

Now, let

$$\alpha_i = \begin{cases}1\ ,\ i=0\\ 0\ ,\ 1\le i\le l\end{cases} \quad (51)$$

then the estimator $\widehat{\boldsymbol{\varphi}}_N$ in (34) becomes the least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$, and $\widehat{f}_N$ becomes the sum of squared residual defined by

$$\widehat{g}_N = \sum_{t=1}^{N}\widehat{\xi}_t^2 \quad (52)$$

where $\widehat{\xi}_t$ is the residual of the least-squares estimate $\widehat{\boldsymbol{\theta}}_{LS,N}$ defined by

$$\widehat{\xi}_t = z_t - \boldsymbol{p}_t^T\widehat{\boldsymbol{\theta}}_{LS,N} . \quad (53)$$

It follows from (50) that

$$\underset{N\to\infty}{p\lim}\frac{1}{N}\widehat{g}_N = \sigma_e^2 + \sigma_e^2\underset{N\to\infty}{p\lim}\widehat{\boldsymbol{a}}_{LS,N}^T\boldsymbol{a} + \sigma_d^2\underset{N\to\infty}{p\lim}\widehat{\boldsymbol{b}}_{LS,N}^T\boldsymbol{b} . \quad (54)$$

From (50) and (54), the estimates of input and output noise variances σ_e^2 and σ_d^2 can be obtained by solutions of system of equations

$$\begin{bmatrix}1+\widehat{\boldsymbol{a}}_{LS,N}^T\widehat{\boldsymbol{a}}_{BC,N-1} & \widehat{\boldsymbol{b}}_{LS,N}^T\widehat{\boldsymbol{b}}_{BC,N-1}\\ \widehat{\alpha}_N+\widehat{\boldsymbol{\varphi}}_{a,N}^T\boldsymbol{H}_n\widehat{\boldsymbol{a}}_{BC,N-1} & \widehat{\boldsymbol{\varphi}}_{b,N}^T\boldsymbol{H}_n\widehat{\boldsymbol{b}}_{BC,N-1}\end{bmatrix}\begin{bmatrix}\widehat{\sigma_e^2}\\ \widehat{\sigma_d^2}\end{bmatrix} = \frac{1}{N}\begin{bmatrix}\widehat{g}_N\\ \widehat{f}_N\end{bmatrix} \quad (55)$$

where

$$\widehat{\alpha}_N = \sum_{i=0}^{l}\alpha_i^2 + \boldsymbol{\rho}^T\left(\widehat{\boldsymbol{a}}_{BC,N-1} + \widehat{\boldsymbol{\varphi}}_{a,N}\right) \quad (56)$$

$$\widehat{\boldsymbol{\theta}}_{BC,N}^T = [\widehat{\boldsymbol{a}}_{BC,N}^T,\ \widehat{\boldsymbol{b}}_{BC,N}^T] . \quad (57)$$

6. THE PREVIOUSLY PROPOSED BIAS-COMPENSATION PRINCIPLE BASED METHODS

In this section, Jia et al.'s BCLS method (Jia *et al.*, 2001) and Zheng's BELS method (Zheng, 2001) are briefly described, which are different estimation methods of noise variances.

6.1 Jia et al.'s BCLS method

Consider an auxiliary estimator $\widehat{\boldsymbol{\phi}}_N$ defined by

$$\widehat{\boldsymbol{\phi}}_N = \left(\sum_{t=1}^{N} \boldsymbol{p}_{t-1}\boldsymbol{p}_{t-1}^T\right)^{-1} \sum_{t=1}^{N} \boldsymbol{p}_{t-1} z_t \ . \tag{58}$$

Define $\widehat{j}_N$ by

$$\widehat{j}_N = \sum_{t=1}^{N} (z_t - \boldsymbol{p}_{t-1}^T \widehat{\boldsymbol{\phi}}_N) z_{t-1} \tag{59}$$

then the following expression can be obtained

$$\begin{aligned} &\underset{N\to\infty}{p\lim} \frac{1}{N}\widehat{j}_N \\ &\quad = \sigma_e^2 \underset{N\to\infty}{p\lim} \widehat{\boldsymbol{\phi}}_{a,N}^T \boldsymbol{a} + \sigma_d^2 \underset{N\to\infty}{p\lim} \widehat{\boldsymbol{\phi}}_{b,N}^T \boldsymbol{b} \end{aligned} \tag{60}$$

where $\widehat{\boldsymbol{\phi}}_N^T = [\widehat{\boldsymbol{\phi}}_{a,N}^T,\ \widehat{\boldsymbol{\phi}}_{b,N}^T]$.

From (54) and (60), the estimates of input and output noise variances σ_e^2 and σ_d^2 can be obtained by solutions of system of equations

$$\begin{bmatrix} 1+\widehat{\boldsymbol{a}}_{LS,N}^T \widehat{\boldsymbol{a}}_{BC,N-1} & \widehat{\boldsymbol{b}}_{LS,N}^T \widehat{\boldsymbol{b}}_{BC,N-1} \\ \widehat{\boldsymbol{\phi}}_{a,N}^T \widehat{\boldsymbol{a}}_{BC,N-1} & \widehat{\boldsymbol{\phi}}_{b,N}^T \widehat{\boldsymbol{b}}_{BC,N-1} \end{bmatrix} \begin{bmatrix} \widehat{\sigma_e^2} \\ \widehat{\sigma_d^2} \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \widehat{g}_N \\ \widehat{j}_N \end{bmatrix} \tag{61}$$

6.2 Zheng's BELS method

Define an argumented parameter vector as

$$\bar{\boldsymbol{\theta}}^T = [\boldsymbol{\theta}^T,\ \boldsymbol{c}^T],\ \boldsymbol{c}^T = [b_{n+1},\ b_{n+2}] = \boldsymbol{0}_2^T \ . \tag{62}$$

The corresponding auxiliary regression vector is given by

$$\bar{\boldsymbol{p}}_t^T = [\boldsymbol{p}_t^T,\ \boldsymbol{x}_t^T],\ \boldsymbol{x}_t^T = [w_{t-n-1},\ w_{t-n-2}] \ . \tag{63}$$

The least-squares estimate of $\bar{\boldsymbol{\theta}}$ is given by

$$\underset{N\to\infty}{p\lim} \widehat{\bar{\boldsymbol{\theta}}}_{LS,N} = \boldsymbol{R}_{\bar{p}\bar{p}}^{-1} \boldsymbol{r}_{\bar{p}z} \tag{64}$$

$$\underset{N\to\infty}{p\lim} \widehat{\bar{\boldsymbol{\theta}}}_{LS,N} = \bar{\boldsymbol{\theta}} + \boldsymbol{R}_{\bar{p}\bar{p}}^{-1} \bar{\boldsymbol{D}} \bar{\boldsymbol{\theta}} \tag{65}$$

where $\boldsymbol{R}_{\bar{p}\bar{p}} = E[\bar{\boldsymbol{p}}_t \bar{\boldsymbol{p}}_t^T]$, $\boldsymbol{r}_{\bar{p}z} = E[\bar{\boldsymbol{p}}_t z_t]$ and $\bar{\boldsymbol{D}} = \mathrm{diag}\{\boldsymbol{D};\ \sigma_d^2 \boldsymbol{I}_2\}$. Applying the matrix inversion formula to $\boldsymbol{R}_{\bar{p}\bar{p}}$, the last two components of $\widehat{\bar{\boldsymbol{\theta}}}_{LS,N}$ may have two expressions. Then the following expression can be established from (64) and (65)

$$\begin{aligned} &\sigma_e^2 \boldsymbol{R}_{px}^T \boldsymbol{R}_{pp,1}^{-1} \boldsymbol{a} + \sigma_d^2 \boldsymbol{R}_{px}^T \boldsymbol{R}_{pp,2}^{-1} \boldsymbol{b} \\ &\quad = \boldsymbol{r}_{xz} - \boldsymbol{R}_{px}^T \underset{N\to\infty}{p\lim} \widehat{\boldsymbol{\theta}}_{LS,N} \end{aligned} \tag{66}$$

where $\boldsymbol{R}_{px} = E[\boldsymbol{p}_t \boldsymbol{x}_t^T]$, $\boldsymbol{r}_{xz} = E[\boldsymbol{x}_t z_t]$. $\boldsymbol{R}_{pp,1}^{-1}$ and $\boldsymbol{R}_{pp,2}^{-1}$ are composed of the first n and the last n columns of $\boldsymbol{R}_{pp}^{-1}$, respectively.

From (54) and (66), the estimates of input and output noise variances σ_e^2 and σ_d^2 can be obtained by solutions of system of equations

$$\begin{bmatrix} 1+\widehat{\boldsymbol{a}}_{LS,N}^T \widehat{\boldsymbol{a}}_{BC,N-1} & \widehat{\boldsymbol{b}}_{LS,N}^T \widehat{\boldsymbol{b}}_{BC,N-1} \\ \widehat{\boldsymbol{R}}_{px}^T \widehat{\boldsymbol{R}}_{pp,1}^{-1} \widehat{\boldsymbol{a}}_{BC,N-1} & \widehat{\boldsymbol{R}}_{px}^T \widehat{\boldsymbol{R}}_{pp,2}^{-1} \widehat{\boldsymbol{b}}_{BC,N-1} \end{bmatrix} \begin{bmatrix} \widehat{\sigma_e^2} \\ \widehat{\sigma_d^2} \end{bmatrix} = \begin{bmatrix} \widehat{g}_N / N \\ \widehat{\boldsymbol{r}}_{xz} - \widehat{\boldsymbol{R}}_{px}^T \widehat{\boldsymbol{\theta}}_{LS,N} \end{bmatrix} \tag{67}$$

Zheng's BELS method can be extended for colored output noise (Zheng, 2002).

7. SIMULATION RESULTS

By computer simulation, the proposed BCLS algorithm is compared with Jia *et al.*'s BCLS algorithm, Zheng's BELS algorithm and LS algorithm. Consider the following second-order system:

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.169901q^{-1} + 0.143831q^{-2}}{1 - 1.575157q^{-1} + 0.606531q^{-2}} \ . \tag{68}$$

The noise free input u_t is white signal with variance $\sigma_u^2 = 1$. The noise variance on input side is set as $\sigma_d^2 = 0.1$ which yields $\mathrm{SNR} = 10\log_{10}(\sigma_u^2/\sigma_d^2) = 10$ [dB]. The noise variance on output side is set as $\sigma_e^2 = 0.3987$ which yields $\mathrm{SNR} = 10\log_{10}(\sigma_y^2/\sigma_e^2) = 10$ [dB]. The filter $\alpha(q^{-1})$ is designed as $l = 3$, $\alpha_0 = 1$, $\alpha_1 = 2$, $\alpha_2 = 2$, $\alpha_3 = 1$.

Computer simulation for comparison are carried out through $M = 100$ independent runs with a data length of 4000. Fig. 1 gives a plot of the RMSE which is defined by

$$\mathrm{RMSE} = 20\log_{10} \sqrt{\frac{1}{M} \sum_{k=1}^{M} \frac{\|\widehat{\boldsymbol{\theta}}_{k,t} - \boldsymbol{\theta}\|^2}{\|\boldsymbol{\theta}\|^2}} \ \text{[dB]} \tag{69}$$

where $\widehat{\boldsymbol{\theta}}_{k,t}$ denotes the estimate of $\boldsymbol{\theta}$ at time step t in the kth independent run. The mean values of the estimates of σ_e^2 and σ_d^2 are shown in Fig. 2 and Fig. 3, respectively.

Simulation results indicate that LS method gives biased results. On the contrary, the proposed BCLS method, Jia *et al.*'s method and Zheng's

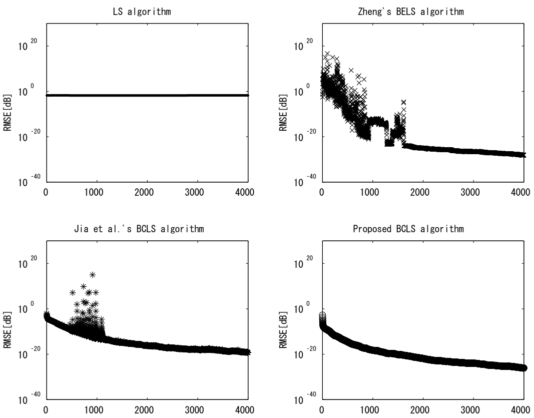

Fig. 1. RMSE of parameter estimates.

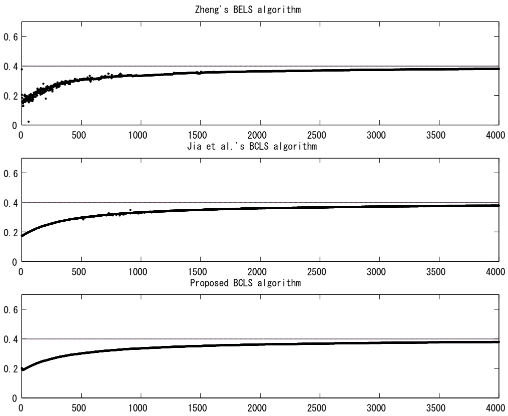

Fig. 2. The mean values of the estimates of σ_e^2.

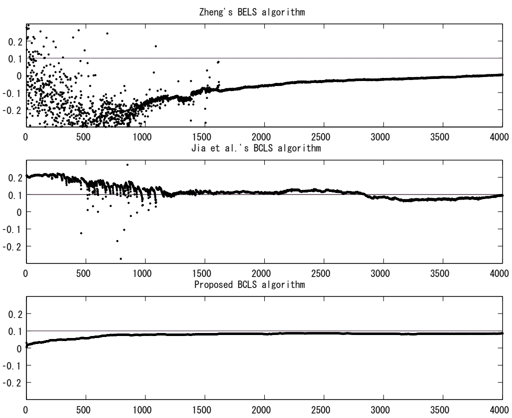

Fig. 3. The mean values of the estimates of σ_d^2.

method can give consistent estimates. Especially, since the proposed algorithm provides the good estimates of noise variances σ_e^2 and σ_d^2 compared with Jia et al.'s algorithm and Zheng's algorithm, the resulting BCLS estimates are more accurate than those obtained with Jia *et al.*'s algorithm and Zheng's algorithm.

8. CONCLUSION

In this paper, the method of consistent estimation of noisy input-output system has been studied. A new BCLS based algorithm has been proposed by introducing a generalized least-squares type estimator. Since the proposed approach can give the good estimates of the noise variances, the estimation accuracy of the resulting BCLS estimates can be improved. It is demonstrated that the proposed method can give consistent parameter estimates via simulation results.

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