STOCHASTIC POWER CONTROL FOR TIME-VARYING FLAT FADING WIRELESS CHANNELS

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Abstract: The performance of stochastic optimal power control of time-varying wireless short-term flat fading channels, in which the evolution of the dynamical channel is described by a stochastic state space representation, is determined. The solution of the stochastic optimal control is obtained through path-wise optimization, which is solved by linear programming using a predictable power control strategy. The algorithm can be implemented using an iterative power control algorithm. The performance measure of the algorithm is interference or outage probability. The algorithm can be used as long as the time duration for successive adjustments of transmitter powers is less than the coherence time of the channel. *Copyright © 2005 IFAC*

Keywords: Power control, Stochastic control, State-space models, Power spectral density (PSD), Random processes, Random variables.

1. INTRODUCTION

Power control is important to improve performance of wireless communication systems. Most of the research that has been done in this field deals with static wireless channel models. But in reality, wireless channels are dynamic due to the relative motion between the transmitter and the receiver and the temporal variations of the propagation environment (Charalambous and Menemenlis, 2001). Therefore dynamical channel models are more realistic than static ones. The random variables characterizing the instantaneous power of each multipath component in short-term fading (STF) model are generalized to dynamical models comprising random processes with time varying statistics.

Since wireless channels have random and time varying properties, this paper suggests using dynamical (time varying) channel models. The dynamics of the channel is captured by stochastic differential equations (SDE's). A stochastic power control algorithm (PCA) is applied to determine the optimal transmitted powers. The proposed PCA is based on predictable power control strategy (PPCS) that was first developed by Charalambous, *et al*. (2001). The PPCS algorithm is proven to be effectively applicable to such dynamical models for an optimal power control (PC). The outage probability is used as a performance measure for the proposed algorithm. Simulation results illustrate the efficiency and the advantages of the algorithm developed. In this paper, centralized power control (CPC) and closed loop PC schemes are used.

Moreover, iterative algorithms can be used to find the optimal powers for the proposed PCA.

The power allocation problem has been studied extensively as an eigenvalue problem for nonnegative matrices (Zander, 1992), as iterative PCA's that converge each user's power to the minimum power (Foschini and Miljanic, 1993), and as optimization-based approaches (Kandukuri and Boyd, 2002). Much of this previous work deals with static channel models. The scheme introduced by Kandukuri and Boyd, (2002) whereby the statistics of the received signal to interference ratio (SIR) are used to allocate power, rather than an instantaneous SIR. The allocation decisions can then be made on a much slower time scale.

In this paper, the performance of the PCA is measured by outage probability and calculations of the outage probability have been simulated. Simulation results are provided comparing the performance of the proposed method (i.e. PPCS) with the performance of no power control (NPC) for both Rayleigh and Ricean fading channels.

2. DYNAMICAL SHORT TERM FLAT FADING CHANNEL MODEL

The traditional STF model is based on Ossanna, (1964) and later expanded by Clarke, (1968) and Aulin, (1979). Aulin's model is shown in Fig.1. This model assumes that at each point between a transmitter and a receiver, the total received wave consists of the superposition of *N* plane waves each having travelled via a different path. The *n*th wave is characterized by its field vector $E_n(t)$ given by:

$$
E_n(t) = I_n(t) \cos \omega_c t - Q_n(t) \sin \omega_c t
$$

= Re{ $r_n e^{j\Phi_n(t)} e^{j\omega_c t}$ } (1)

where $\{I_n(t), Q_n(t)\}\$ are the corresponding inphase and quadrature components, $r_n(t) = \sqrt{I_n^2(t) + Q_n^2(t)}$ is the signal envelope, $\Phi_n(t) = \tan^{-1}(Q_n(t) / I_n(t))$ is the phase and ω_c is the carrier frequency. The total field $E(t)$ is given by:

Fig. 1: Aulin's 3D multipath scattering model.

$$
E(t) = I(t)\cos\omega_c t - Q(t)\sin\omega_c t \tag{2}
$$

where $\{I(t), Q(t)\}$ are inphase and quadrature components of the total wave with $I(t) = \sum_{n=1}^{N} I_n(t)$ and $Q(t) = \sum_{n=1}^{N} Q_n(t)$. By the central limit theorem, for large *N* (typically $N \geq 6$), the inphase and quadrature components have Gaussian distributions $N(\bar{x}; \sigma^2)$, with same values for mean and variance (Clarke, 1968). The mean is \bar{x} := $E[I(t)] = E[Q(t)]$ and the variance is $\sigma^2 := Var(I(t)) = Var(Q(t))$. In the case where there is no specular or line of sight (LOS) component between the transmitter and the intended receiver, then the mean $\bar{x} = 0$ and the received signal amplitude has Rayleigh distribution. In the presence of the LOS component, $\bar{x} \neq 0$ and the received signal envelope is Ricean distributed. Also, it is assumed that *I*(*t*) and *Q*(*t*) are uncorrelated and thus independent since they are Gaussian distributed (Aulin, 1979).

The main idea in constructing the dynamical model for short term flat fading channels is to factorize the Doppler power spectral density (DPSD) into an approximate $4th$ order even transfer function, and then any stochastic realization can be used to obtain a state space representation for inphase and quadrature components.

Consider the expression for the DPSD given by (Aulin, 1979):

$$
S_{D}(f) = \begin{cases}\n0, & |f| > f_{m} \ (3) \\
\frac{E_{0}}{4f_{m} \sin \beta_{m}}, & f_{m} \cos \beta_{m} \le |f| \le f_{m} \\
\frac{E_{0}}{4\pi f_{m} \sin \beta_{m}}\left[\frac{\pi}{2} - \sin^{-1}\left(\frac{2\cos^{2}\beta_{m} - 1 - (f/f_{m})^{2}}{1 - (f/f_{m})^{2}}\right)\right], & |f| < f_{m} \cos \beta_{m}\n\end{cases}
$$

where

$$
p_{\alpha}(\alpha) = \frac{1}{2\pi}, \quad 0 \le |\alpha| \le 2\pi, \ 0 \ \text{elsewhere},
$$

$$
p_{\beta}(\beta) = \frac{\cos \beta}{2\sin \beta_m}, \quad |\beta| \le |\beta_m| \le \frac{\pi}{2}, \ 0 \ \text{elsewhere},
$$

and $\{\alpha, \beta\}$ define the direction of the incident wave onto the receiver as illustrated in Fig. 1, $p_X(x)$ denotes a probability density function of the random variable *X*. f_m is the maximum Doppler frequency, and $E_0/2 = Var(I(t)) = Var(Q(t))$. In order to approximate the DPSD in (3) , a 4th order even function in the form $\tilde{S}_D(s) = H(s)H(-s)$ with factorization shown below is used (Charalambous and Menemenlis, 2000).

$$
\tilde{S}_D(s) = \frac{K^2}{s^4 + 2\omega_n^2 (1 - 2\zeta^2)s^2 + \omega_n^4}
$$
\n
$$
H(s) = \frac{K}{s^2 + 2\zeta\omega_n + \omega_n^2}
$$
\n(4)

where $\tilde{S}_D(s)$ is the approximation of $S_D(s)$. Equation (4) has three arbitrary parameters $\{\zeta, \omega_n, K\}$, which can be adjusted such that the approximate curve coincides with the actual curve at different points. In fact, if these parameters are chosen such that:

$$
\zeta = \sqrt{\frac{1}{2} \left(1 - \sqrt{1 - \frac{S_D(0)}{S_D(j\omega_{\text{max}})}} \right)},
$$

\n
$$
\omega_n = \frac{\omega_{\text{max}}}{\sqrt{1 - 2\zeta^2}},
$$

\n
$$
K = \omega_n^2 \sqrt{S_D(0)}
$$
\n(5)

where

$$
S_D(0) = \frac{E_0}{4\pi f_m \sin \beta_m} \left[\frac{\pi}{2} - \sin^{-1} \left(2 \cos^2 \beta_m - 1 \right) \right],
$$

$$
\omega_{\text{max}} = \left(\frac{1 + \cos \beta_m}{2} \right) 2\pi f_m, \ S_D(j\omega_{\text{max}}) = \frac{E_0}{4 f_m \sin \beta_m},
$$

then the approximate density $\tilde{S}_D(s)$ coincides with the exact density $S_p(s)$ at $\omega = 0$ and $\omega = \omega_{\text{max}}$.

The SDE which corresponds to $H(s)$ in (4) with the initial conditions is:

$$
\ddot{x}(t) + 2\zeta \omega_n \dot{x} + \omega_n^2 x(t) = K \dot{w}(t)
$$
 (6)

where $\dot{x}(0), x(0)$ are given and $\{\dot{w}(t)\}_{t>0}$ is a whitenoise process. (6) can be re-written in terms of inphase and quadrature components as:

$$
\ddot{x}_I(t) + 2\zeta \omega_n \dot{x}_I + \omega_n^2 x_I(t) = K \dot{w}_I(t) \tag{7}
$$

$$
\ddot{x}_Q(t) + 2\zeta \omega_n \dot{x}_Q + \omega_n^2 x_Q(t) = K \dot{w}_Q(t)
$$
 (8)

where $\dot{x}_1(0), x_1(0), \dot{x}_0(0)$, and $x_0(0)$ are given, ${\hat{w}_1(t)}_{t\ge0}$, and ${\hat{w}_2(t)}_{t\ge0}$ are two independent and identically distributed (i.i.d) white Gaussian noises with distribution $N(0; \sigma_{\varphi}^2)$. Equations (7) and (8) can be realized in state-space controllable canonical form as:

$$
\dot{X}_I(t) = A_I X_I(t) + B_I \dot{w}_I(t), \qquad X_I(0) \in \mathfrak{R}^2; \n\dot{X}_Q(t) = A_Q X_Q(t) + B_Q \dot{w}_Q(t), \quad X_Q(0) \in \mathfrak{R}^2,
$$
\n(9)

where $A_I = A_Q = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, B_I = B_Q = \begin{bmatrix} 0 \\ K \end{bmatrix}$ $A_{I} = A_{Q} = \begin{bmatrix} 0 & 1 \\ -\omega_{n}^{2} & -2\zeta\omega_{n} \end{bmatrix}, B_{I} = B_{Q} = \begin{bmatrix} 0 \\ K \end{bmatrix},$

and $X(t, \tau)$ is the power path loss. Define,

$$
A := \begin{bmatrix} A_I & 0 \\ 0 & A_Q \end{bmatrix}, B := \begin{bmatrix} B_I & 0 \\ 0 & B_Q \end{bmatrix},
$$

\n
$$
X := \begin{bmatrix} X_I \\ X_Q \end{bmatrix}, \dot{w} := \begin{bmatrix} \dot{w}_I \\ \dot{w}_Q \end{bmatrix}, v := \begin{bmatrix} v_I \\ v_Q \end{bmatrix},
$$

\n
$$
C(t) := [\cos \omega_c t \quad 0 \quad -\sin \omega_c t \quad 0],
$$

\n
$$
D(t) := [\cos \omega_c t \quad -\sin \omega_c t]
$$

then the stochastic state space realizations for Rayleigh and Ricean flat fading channels can be described as:

$$
\dot{X}(t) = AX(t) + f_s(t) + B\dot{w}(t), \quad X(0) \in \mathbb{R}^4; \n y(t) = s(t)C(t)X(t) + D(t)v(t)
$$
\n(10)

where $\{y(t)\}_{t\geq0}$ is the output signal, $\{v_I(t)\}_{t\geq0}$ and ${v_Q(t)}_{\sim 0}$ are two i.i.d white Gaussian noises with distribution $N(0; \sigma_n^2)$, and $f_s(t)$ describes the specular or LOS component for Ricean fading given by:

$$
f_s(t) = \begin{bmatrix} -r_0 \omega_0 \sin(\omega_0 t + \phi_0) \\ r_0 \omega_n^2 \cos(\omega_0 t + \phi_0) \\ r_0 \omega_0 \cos(\omega_0 t + \phi_0) \\ r_0 \omega_n^2 \sin(\omega_0 t + \phi_0) \end{bmatrix}
$$
(11)

where $f_s(t) = 0$ for Rayleigh fading. Equation (10) captures the variations in the path loss $X(t, \tau)$.

Now consider a cellular network with *M* transmitters and *M* receivers. The received signal at the *n*th base station can be expressed in terms of inphase and quadrature components as:

$$
y_n(t) = \sum_{j=1}^{M} u_j(t) s_j(t) \begin{pmatrix} I_{nj}(t) \cos \omega_c t \\ -Q_{nj}(t) \sin \omega_c t \end{pmatrix} + d_n(t) \quad (12)
$$

where u_i is the control input of transmitter *j* which acts as scaling on the information signal s_j , I_{nj} and Q_{nj} are the inphase and quadrature components respectively, and d_n is the channel disturbance or noise at receiver *n*. It can be shown that ${I_{nj}(t), Q_{nj}(t)}$ are realizable through the multidimensional linear SDE:

$$
d\left[\begin{array}{c} X_{I_{nk}} \\ X_{Q_{nk}} \end{array}\right] = A_{nk} \left[\begin{array}{c} X_{I_{nk}} \\ X_{Q_{nk}} \end{array}\right] dt + f_{nk} dt + B_{nk} dw_{nk},
$$

\n
$$
I_{nk} = C_{nk} X_{I_{nk}},
$$

\n
$$
Q_{nk} = C_{nk} X_{Q_{nk}}
$$
\n(13)

for $1 \leq n, k \leq M$. Here $\{w_{nk}(t)\}_{t>0}$ is a vector of standard Brownian motion, $\left\{X_{I_{nk}}(t), X_{Q_{nk}}(t)\right\}_{t>0}$ are state vectors representing power path loss associated with the inphase and the quadrature components of the channel, and f_{nk} is the LOS component for Ricean STF channel model. Let $H_{nk} := [s_k C_{nk} \sin \omega_c t \quad -s_k C_{nk} \cos \omega_c t]$ and $X_{nk} := \begin{bmatrix} X_{I_{nk}} & X_{Q_{nk}} \end{bmatrix}$, then the state space representation of a wireless network can be written:

$$
dX_{nk} = A_{nk} X_{nk} dt + f_{nk} dt + B_{nk} dw_{nk}
$$
 (14)

$$
y_n(t) = \sum_{j=1}^{M} u_j(t) H_{nj} X_{nj}(t) + d_n(t), \ \ 1 \le n, k \le M \ \ (15)
$$

The next section describes the PCA that achieves the minimal transmitted power in stochastic short-term flat fading channels.

3. POWER CONTROL MODEL

3.1 Stochastic Power Control Scheme

Consider a wireless network of *M* transmitters and *M* receivers. The optimal PCA can then be posed in terms of the following optimization (Kandukuri and Boyd, 2002):

$$
\min_{(p_1 \ge 0, \dots, p_M \ge 0)} \sum_{i=1}^M p_i, \tag{16}
$$

subject to

$$
\frac{p_n g_{nn}}{\sum_{j\neq i}^M p_j g_{nj} + \eta_n} \ge \overline{\gamma}_n, \qquad (17)
$$

which is equivalent to

$$
\min_{(p_i \ge 0, \dots, p_M \ge 0)} \sum_{i=1}^M p_i, \tag{18}
$$

subject to

$$
\frac{p_n g_{nn}}{\sum_{j=1}^M p_j g_{nj} + \eta_n} \ge \gamma_n \tag{19}
$$

where $\gamma_n := \frac{r_n}{\overline{\gamma}_n + 1}, 0 < \gamma_n < 1.$ $\gamma_n := \frac{\gamma_n}{\overline{\gamma}_n + 1}, 0 < \gamma$ $=\frac{r_n}{\overline{\gamma}_n+1}$, $0 < \gamma_n < 1$. Here p_n denotes the

power of transmitter $n, g_{nj} > 0$ denotes the channel gain of transmitter *j* to the receiver assigned to transmitter *n*, $\overline{\gamma}_n > 0$ is the required SIR and $\eta_n > 0$ is the noise power level at the *n*th receiver, $1 \le n, j \le$ *M*. Equations (18) and (19) in dynamic case using the path-wise QoS of each user with respect to the power signals over a time interval [0,*T*] are given as (Charalambous, *et al*., 2001):

$$
\min_{(p_1 \ge 0, \dots, p_M \ge 0)} \left\{ \sum_{i=1}^M \int_0^T p_i(t) \right\}, \quad \text{subject to} \\
\frac{\int_0^T p_n(t) \| H_{nn} X_{nn}(t) \|^2 dt}{\sum_{j=1}^M \int_0^T p_j(t) \| H_{nj}(t) X_{nj}(t) \|^2 dt + \int_0^T \| d_n(t) \|^2 dt} \ge \gamma_n
$$
\n(20)

where $(1 \le n \le M)$ and H_{nj} , $X_{nj}(t)$, $p_j(t)$, $d_n(t)$ are the same as defined in the dynamical STF channel model, and $\|\cdot\|$ is the Euclidean norm.

In wireless cellular networks, it is practical to observe and estimate channels at base stations and then communicate the information to the transmitters to adjust their control input signals $\{u_k(t)\}_{k=1}^M$. Since channel experiences delays, and the control are not feasible continuously in time but only at discrete time instants, the concept of predictable strategies is introduced (Charalambous, *et al*., 2001).

 Let the control input signal for a transmitter at discrete time be $\{u(t); t = t_1, t_2, \dots, T\}$ and let the channel information at any time *t* be denoted by $\{I(t), Q(t), s(t)\}\$. At time t_{i-1} , the base station observes the channel information $\left\{ I_k(t_{j-1}), Q_k(t_{j-1}), s_k(t_{j-1}) \right\}_{k=1}^M$. Using the concept of predictable strategy, the base station determines the control strategy $\left\{ u_k(t_j) \right\}_{k=1}^M$ for the next time instant *tj*. The latter is communicated back to the transmitters, which hold these values during the time interval $\left(t_{j-1}, t_j\right)$. At time t_j , a new set of channel information $\left\{ I_k(t_j), Q_k(t_j), s_k(t_j) \right\}_{k=1}^M$ is observed at the base station and the time t_{i+1} control strategies $\left\{ u_k(t_{j+1}) \right\}_{k=1}^M$ are computed and then communicated to the transmitters and held constant during the time interval $[t_i, t_{i+1})$. Such decision strategies are called predictable strategies. Using the concept of PPCS over any time interval defined as $[t_k, t_{k+1}]$, the equivalence of (18) and (19) is:

$$
\min_{p_i(t_{k+1})} \sum_{i=1}^M p_i(t_{k+1}), \text{ subject to} \tag{21}
$$
\n
$$
p(t_{k+1}) \ge \Gamma G_1^{-1}(t_k, t_{k+1}) \times (G(t_k, t_{k+1}) p(t_{k+1}) + \eta(t_{k+1}))
$$

where

$$
g_{ni}(t_k, t_{k+1}) := \int_{t_k}^{t_{k+1}} \left\| H_{ni}(t) X_{ni}(t) \right\|^2 dt, \quad 1 \le n, i \le M,
$$

\n
$$
\eta_{ni}(t_k, t_{k+1}) := \int_{t_k}^{t_{k+1}} \left\| d_n(t) \right\|^2 dt, \quad 1 \le n, i \le M,
$$

\n
$$
G_l(t_k, t_{k+1}) = diag(g_{11}(t_k, t_{k+1}), \cdots, g_{MM}(t_k, t_{k+1})),
$$

\n
$$
G(t_k, t_{k+1}) = \left\{ (g_{ni}(t_k, t_{k+1}) \right\}_{M \times M}, \quad 1 \le n, i \le M,
$$

\n
$$
\eta(t_k, t_{k+1}) = (\eta_1(t_k, t_{k+1}), \cdots, \eta_M(t_k, t_{k+1}))^T,
$$

\n
$$
p(t_{k+1}) = (p_1(t_{k+1}), \cdots, p_M(t_{k+1}))^T,
$$

\n
$$
\Gamma - diag(y, \cdots, y_n) = diag(j) \quad \text{denotes, } 2 \quad \text{diam}
$$

 $\Gamma = diag(\gamma_1, \cdots, \gamma_M)$, *diag* (.) denotes a diagonal matrix with its argument as diagonal entries, and "^T" stands for matrix or vector transpose. The optimization in (21) is a linear programming problem in $M \times 1$ vector of unknowns $p(t_{k+1})$. Here $[t_k, t_{k+1}]$ denotes time interval of the signal such that the channel model does not change significantly, i.e., it is shorter than the coherence time of the channel.

The performance measure is interference or outage probability. It is defined as the probability that a randomly chosen link will fail due to excessive interference (Zander, 1992). Therefore, smaller outage probability implies larger capacity of the wireless network. A link with received SIR $\bar{\gamma}_{\text{rvd}}$ less than or equal to threshold SIR $\bar{\gamma}_h$ is considered a communication failure. The outage probability $F(\overline{\gamma}_{\text{rod}})$ is expressed as $F(\overline{\gamma}_{th}) = \Pr{\overline{\gamma}_{\text{rod}} \leq \overline{\gamma}_{th}}$, where $F(\overline{\gamma}_{\text{rcvd}})$ is the distribution of $\overline{\gamma}_{\text{rcvd}}$.

3.2 Iterative Power Control Scheme

Since PC only occurs at discrete time instants using
PPCS, the iterative algorithm described by Foschini
and Miljanic (1993) can be used to determine the
optimal transmitted powers. Define

$$
F(t_k, t_{k+1}) \triangleq \Gamma^* G_I^{-1}(t_k, t_{k+1})^* G(t_k, t_{k+1})
$$
 and

$$
u(t_k, t_{k+1}) = \Gamma^* G_l^{-1}(t_k, t_{k+1})^* \eta(t_{k+1}),
$$
 where

$$
u(t_k, t_{k+1}) = \left(\frac{\gamma_1 \eta_1(t_{k+1})}{G_{11}(t_k, t_{k+1})}, \frac{\gamma_2 \eta_2(t_{k+1})}{G_{22}(t_k, t_{k+1})}, \dots, \frac{\gamma_M \eta_M(t_{k+1})}{G_{MM}(t_k, t_{k+1})}\right)^T
$$

and $F_t(t, t_{k+1}) = \gamma_t G_{ij}(t_k, t_{k+1})$

and
$$
F_{ij}(t_k, t_{k+1}) = \frac{\gamma_i G_{ij}(t_k, t_{k+1})}{G_{ii}(t_k, t_{k+1})}, 1 \le i, j \le M.
$$

Then the expectation (31) can be amplitude as

Then the constraint in (21) can be rewritten as:

$$
(I - F(t_k, t_{k+1}))P(t_{k+1}) \ge u(t_k, t_{k+1})
$$
\n(22)

The matrix $F(t_k, t_{k+1})$ has nonnegative elements and is irreducible. The existence of a feasible power vector $P(t_{k+1}) > 0$ satisfying (22) is equivalent to $\rho_{F(t_k, t_{k+1})} < 1$, where $\rho_{F(t_k, t_{k+1})}$ is the maximum modulus eigenvalue of $F(t_k, t_{k+1})$. The power vector $P^{*}(t_{k+1}) = (I - F(t_{k}, t_{k+1}))^{-1} u(t_{k}, t_{k+1})$ is the optimal power vector satisfying (21), and the iteration

$$
P^{n+1}(t_{k+1}) = F(t_k, t_{k+1})P^n(t_{k+1}) + u(t_k, t_{k+1}) \quad (23)
$$

converges to $P^*(t_{k+1})$ when $\rho_{F(t_k, t_{k+1})} < 1$, where *n* is the number of iterations. Equation (23) can be written as follows:

$$
P_{i}^{n+1}(t_{k+1}) = \left(\frac{\gamma_{i}}{G_{ii}(t_{k}, t_{k+1})}\right)^{*}
$$

$$
\left(\sum_{j=1}^{M} G_{ij}(t_{k}, t_{k+1}) P_{j}^{n}(t_{k+1}) + \eta_{i}(t_{k}, t_{k+1})\right)
$$
(24)

and also can be written as:

$$
P_i^{n+1}(t_{k+1}) = \frac{\gamma_i}{\gamma_{i_{\text{row}}}^n(t_k, t_{k+1})} P_i^n(t_{k+1})
$$
 (25)

where
$$
\gamma_i := \frac{\overline{\gamma_i}}{\overline{\gamma_i} + 1}
$$
, $\gamma_{i_{\text{red}}} := \frac{\overline{\gamma_{i_{\text{red}}}}}{\overline{\gamma_{i_{\text{red}}}} + 1}$, $i = 1, ..., M$, and

n is the number of iterations. It is shown that the iterative PC in (24) and (25) converges to the optimal (minimal) power vector (Foschini and Miljanic, 1993; Bambos and Kandukuri, 2002). The numerical implementation of the iterative scheme can be carried out during processing in the interval $[t_k, t_{k+1}]$.

4. NUMERICAL RESULTS

In this section, we give numerical examples to determine the outage probability of the PPCS algorithm for both Rayleigh and Ricean fading channel models. The wireless cellular model has the following features:

- Number of transmitters/receivers is $M = 24$.
- Carrier frequency $= 910$ MHz.
- The average velocities of mobiles are generated as independent random variables uniformly distributed in $[20 - 100]$ km/hr.
- E_{0ii} 's are independent random variables uniformly distributed in the range [400-600].
- E_{0ij} 's ($i \neq j$) are independent random variables uniformly distributed in the range [25-150].
- Angles of arrival β_{m} 's for each link are generated as independent random variables uniformly distributed in $[0 - 36]$ degrees, where β_{m} is the direction of the incident wave between transmitter *j* and receiver *i*.
- η_n 's are independent Gaussian random variables with zero mean and variance 4*10-8.

The parameters $\{\zeta, \omega_n, K\}$ are extracted from the DPSD as described in (5). Both Rayleigh and Ricean cases are simulated.

4.1 Rayleigh flat fading channel

This scenario represents flat Rayleigh fading where the signal envelop at the receivers exhibit Rayleigh distributed density. The outage probabilities as a function of SIR and time for both PC and NPC cases are shown in Fig. 2. It shows how the outage probability changes with respect to SIR threshold $(\bar{\gamma}_{th})$ and time. As the $\bar{\gamma}_{th}$ increases, the outage probability also increases. This is obvious since we expect more users to fail. Changes with respect to time are due to the dynamicity of the channel model.

The average outage probabilities over all time intervals are shown in Fig. 3 (a). The performance of PPCS is compared with the one for fixed transmitter power (i.e. NPC). Results show that the PPCS algorithm outperforms the reference algorithm. It is noticed that the outage probability for PPCS is less than the one for NPC by about 20%. For example, at 15 dB SIR threshold, the outage probability of Rayleigh flat channel is reduced from 0.6 for NPC case to 0.3 for PPCS case.

4.2 Ricean flat fading channel

This scenario represents Ricean fading where the STF channel model has LOS path. The same procedure is followed as Rayleigh fading channels. The average outage probability of this case for both PPCS and NPC cases are shown in Fig. 3 (b). From Fig. 3 (b), the outage probability for PPCS is less than the one for NPC. And the performance of flat Ricean fading is better than the one for flat Rayleigh fading channels. This is because the existence of LOS component in Ricean channels.

Fig. 2: Outage probability for dynamical flat Rayleigh short term fading model. (a) Using PPCS algorithm. (b) Using NPC.

Fig. 3: Average outage probability for (a) Rayleigh channel (b) Ricean channel.

5. CONCLUSIONS

PPCS power control scheme as developed by Charalambous, *et al*. (2001) is applied to timevarying flat STF channel model. The dynamics of the channel is captured by stochastic state space representation. The optimization problem is solved by linear programming. Iterative algorithms can be used to solve for the optimization problem. The performance measure is interference or outage probability. Numerical results indicate that the performance of PPCS outperforms the performance of NPC. PPCS algorithm can be used as long as the channel model does not change significantly, that is $[t_k, t_{k+1}]$ is a subset of the coherence time of the channel.

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