IMPLEMENTATION AND TESTING OF SAMPLED-DATA $\mathcal{H}_\infty$ CONTROL OF JET ENGINES.

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Abstract: This paper shows how $\mathcal{H}_\infty$-design methods successfully can be used for control of jet engines in a multivariable framework. The work concludes experiences from both analysis and design studies concerning control of the Volvo Aero jet engine RM12, which is currently placed in the Swedish air fighter JAS39 Gripen. The emphasis is on useful methods for design and implementation of $\mathcal{H}_\infty$-controllers. The work considers selection of operating points and choice of weight functions for different specific tasks, design of sampled-data $\mathcal{H}_\infty$-controllers of low order, and finally analysis of the closed loop system.

Keywords: H-infinity control, Sampled-data control, Multivariable feedback control, Engine control

1. INTRODUCTION

This work considers several aspects on a control system for a jet engine, with special emphasis on implementation of an $\mathcal{H}_\infty$-controller for the Volvo Aero jet engine RM12. It is shown how the use of modern model based MIMO-design and analysis tools can improve the functionality of the whole system. Härefors showed in (Härefors 1995) that an $\mathcal{H}_\infty$-controller is most suitable for this purpose, and scheduling was performed between linear continuous controllers to handle the nonlinear engine. Christiansson has developed methods for design of sampled-data controllers of low order so as to be more easily implemented in software, see (Christiansson 2003). The paper presents further useful ideas for design and implementation, and ends with simulations and analysis of the closed loop system showing that the concept is most promising.

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2. SAMPLED-DATA $\mathcal{H}_\infty$-CONTROL

This section presents the proposed method for designing a sample-data $\mathcal{H}_\infty$-controller for the jet engine, which includes a number of considerations to make. The main idea was presented in (Christiansson and Lennartson 2003), however more practical issues that are important for the implementation are given here. This includes selection of operating points, reduction of the plant order, the often omitted discussion on how to achieve desired performance for the closed loop, handling of the nonlinear system, and reduction of the controller order. These subjects are covered in the following subsections and verified to some extent through simulations in Section 4.

2.1 The plant model and its reduction

The system, modeled in $\mathrm{MATRIX_X}$, is validated since long for different purposes at Volvo Aero.

The jet engine is modeled as a non linear thermo dynamical system of order 34 with additional dynamic sensor and actuator models, see more in (Härefors 1995). The first step in the design procedure is to choose operating points covering the flight envelope. In this study 93 operating points were chosen as a combination of flight conditions and power settings covering normal flights.

For linearisation purposes an initial study based on physical insight and Hankel singular values showed that the number of plant states in the linear models could be reduced considerably, down to 9th order, see Fig 3; very slow states were removed, and very fast states were reduced using balanced reduction. For one operating point, a normalised linear plant is given in (1).

$$A = \tag{1}$$
$$\begin{bmatrix} -134 & -134 & 5.14 & 4.97 & -0.07 & 50.7 & 50.7 & -86.5 & 135 \\ -21.1 & -144 & 0.36 & 0.3 & -0.03 & 18.4 & 18.4 & -34.1 & 78.6 \\ 166 & 166 & -209 & -122 & 5.61 & 2.55 & 2.55 & 247 & -43.5 \\ -67.6 & -67.6 & -1.19 & -179 & 99.7 & -0.93 & -0.93 & -77.9 & 28.9 \\ 0 & 0 & 0 & 0 & -100 & 0 & 0 & 0 & 0 \\ 80.4 & 80.5 & 65.2 & 36 & 0.22 & -156 & -110 & 4.66 & -18.5 \\ -31.2 & -31.2 & 13.2 & 95.4 & 3.11 & 21.2 & -91.9 & 0.97 & 8.73 \\ -1.21 & -1.21 & 1.5 & 2.37 & 0.01 & -0.07 & -0.07 & -3.83 & 0.17 \\ -0.89 & -0.89 & 3.03 & 3.56 & 0.04 & -1.63 & -1.63 & 0.17 & -2.61 \end{bmatrix}$$
$$B' = \begin{bmatrix} 0 & 0 & 0 & 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -94.3 & -19 & 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} -0.04 & -0.04 & -0.06 & 0.82 & 0.02 & 0.18 & 0.18 & 0 & 0 \\ 0.03 & 0.03 & 0.03 & 0.03 & 0 & 0.94 & 0.94 & 0 & -0.01 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The reduced plant order has been kept the same over the whole flight envelope in order to obtain the same order for all controllers. All these reduced plant models are linear and thus useful for $\mathcal{H}_\infty$-design. The controller is in this paper considered to have access to two measured engine outputs (two among rotational speeds, pressure and temperature) as inputs, and to have two control outputs (main fuel flow and output nozzle area), however the method is useful for any system setup.

When sampling is applied, as in this application, also anti-alias filters for all measured outputs will be included in the linear plant model for design.

2.2 Weight functions and performance

Before starting the actual sampled-data controller design it is important to choose appropriate weight functions for each operating point. These choices are most vital for the final performance, which is why this part of the work is very important and surprisingly neglected in many papers. Weight functions can be placed either *outside* the loop, or *in* the loop, so called “loop shaping” proposed in (McFarlane and Glover 1992), which is used in this work.

In the current work emphasis has been laid on integral action on all controlled outputs so as to avoid static control errors, and on keeping temperature changes smooth so as to avoid temperature cycles and associated increased wear of the engines. Stability is guaranteed in all cases and the control signals are kept within reasonable size. The $\mathcal{H}_\infty$-design also includes a minimisation of the so called γ-value to obtain a (sub)optimal controller. In fact - the ordinary loop-shaping approach needs no γ-iteration in pure continuous or discrete-time, but it is needed for sampled-data design. Performance demands are preferably considered in continuous time in the design phase for choosing weight functions, since the constraints are more easily interpreted in continuous time than in discrete time. The selection of weight function parameters can then be carried out using any optimisation routine that can handle constraints; here Matlab optimisation toolbox was used.

For the jet engine it has shown to be simple and good enough to use diagonal weights of PI-type on the plant input side, however they could, in general, have any structure and also be placed at plant output. For all operating points such weights have been optimised bearing the performance demands for the closed loop in mind. Each PI-weight has two parameters to optimise (gain and integral factor) at each operating point. In order to make simpler controllers the lowest integral gain was chosen for all, followed by a new optimisation of the respective gains. This weight selection is made in continuous time and treated further in discrete-time, see below.

2.3 Change of control mode

In a joint project, see (Ring *et al.* 2005), a change of control mode has been developed, such that the controllers could use different sets of input signals (measurements) according to Table 1. For each mode, mode-1 to mode-4, a sampled-data controller was designed, again bearing the respective performance needs in Section 2.2 in mind. Thus, four controllers are designed for each operating point, and one is active at a time. An anti-windup scheme is applied so that the inactive integral states do not windup. The controllers are able to handle mode switches and actuator restrictions. While inactivated each mode is positioned in its zero (optimal) setting and the controller states are used for matching the active engine controller.

The reason for the change-of-mode facility is to accommodate for possible sensor faults or other control objectives, since different control modes can consider different design objectives. When all weight functions have been chosen, the sampled-data design proceeds.

Table 1. Control modes depending on measurement combinations.

	Tot pressure at turbine outlet	Tot temperature at turbine outlet
Low-pressure spool speed	mode-1	mode-2
High-pressure spool speed	mode-3	mode-4

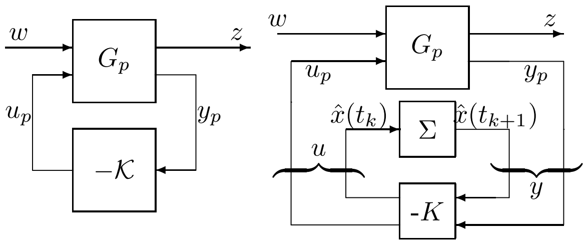

Fig. 1. The system to consider. Left: Normal configuration, $\mathcal{K}$ is dynamic. Right: The parametric approach, K is static. The controller dynamics is indicated by the Σ-device.

2.4 Sampled-data design

We recall the sampled-data $\mathcal{H}_\infty$-design criterion associated with the left part of Fig. 1: *Given the continuous-time plant $G_p(s)$ and a constant $\gamma > 0$, find a stabilising discrete-time controller $-\mathcal{K}(z)$ that achieves the induced $\mathcal{L}_2/\ell_2$ norm $\|G_{zw}\|_\infty < \gamma$.* G_{zw} is interpreted as the closed system with disturbance inputs w and performance output z, both possibly mixed continuous/discrete-time, which is why the induced norm is a mixed continuous/discrete-time norm. To obtain a (sub) optimal controller the γ-value shall also be minimised.

In sampled-data control a discrete-time controller is implemented in a computer to control a continuous-time plant at a certain sampling frequency; the current work has chosen 100 [Hz] due to the plant dynamics. The sampling frequency also depends on the hardware used in the controller, however not discussed in this paper.

Once appropriate weight functions are chosen as in Section 2.2, the sampled-data controllers are designed for each of the different operating points and control modes using a “lifting” method that takes into consideration the system behaviour also between the sampling instants. This design method is described in detail in (Christiansson 2003) and is mainly based upon Riccati equations as in the classical reference (Green and Limebeer 1995) combined with lifting as in (Sågfors and Toivonen 1996, Toivonen and Sågfors 1997) which was further developed in (Christiansson and Lennartson 2003). One observation is that the lifting procedure depends on the γ-value in question and iterations are thus often

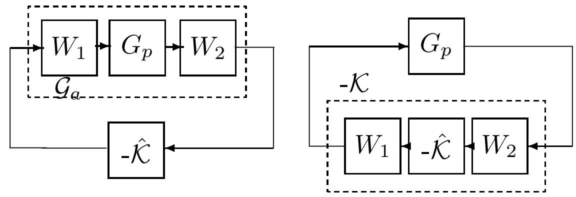

Fig. 2. Figure that shows how the weight functions W_i are moved to be within the controller $\mathcal{K}$ instead of in the augmented plant $\mathcal{G}_a$.

needed. A consequence of this ´´lifting” is that the sampling interval can be relatively long compared to using continuous-time design with discretisation afterwards, since the behaviour is kept during the whole sampling interval. The longer the sampling interval is, the more time is left for the computer to fulfil other important tasks. The design results in a discrete-time controller

$$\mathcal{K}: \quad \begin{cases} \hat{x}(t_{k+1}) = A_K \hat{x}(t_k) + B_K y(t_k) \\ u(t_k) = C_K x(t_k) + D_K y(t_k) \end{cases} \tag{2}$$

2.5 Full order controller

The discrete-time controller is designed for the continuous augmented plant $\mathcal{G}_a$, which includes the plant itself with sensor and actuator dynamics and anti-alias filters represented by G_p and PI-weight functions W_i, see Fig. 2. This design results in a discrete-time preliminary controller $-\hat{\mathcal{K}}$ of the same order as $\mathcal{G}_a$. Finally the PI-weights - discretised with an explicit Euler representation with same parameters as the continuous weights W_i - are moved to be part of the controller $-\mathcal{K}$, resulting in even higher controller order - typical in loop-shaping. This final controller order is here denoted “full order”.

2.6 Parametric approach to lower controller order

Classical $\mathcal{H}_\infty$-design methods (continuous-time, discrete-time or sampled-data) yield a controller order that equals that of the plant augmented with actuators, sensors, anti-alias filters and weight functions. For loop-shaping the order of the weight functions are in fact included twice as observed above. The so obtained controller order is often too high to be easily implemented, especially when some kind of switching is applied, since then a number of controllers need to be active all the time. This is often so even when the plant model itself is reduced before the actual controller design begins. The “full order” controller design problem is a convex problem, which implies that when a solution exists, it is unique. However, the lower order problem is normally not convex, and care must be taken so as not to arrive in a local optimum. The controller order can be lowered using different techniques, and here a parametric approach using Linear Matrix Inequalities (LMIs) is presented briefly, see more in (Christiansson 2003).

The right part of Fig. 1 shows a parametric representation of the controller such that all its dynamics are augmented with the plant, while the rest of the controller K can be considered as the static feedback matrix gain K. Later this will be combined with the dynamics to form $\mathcal{K}$. There exist useful routines for obtaining static feedback controllers in both continuous and discrete time.

The closed loop system from w to z in Fig. 1 can be written in discrete time as

$$\mathcal{G}_{zw_d} : \begin{bmatrix} x(t_{k+1}) \\ z(t_k) \end{bmatrix} = \begin{bmatrix} A_{\mathrm{cl}_d} & B_{\mathrm{cl}_d} \\ C_{\mathrm{cl}_d} & D_{\mathrm{cl}_d} \end{bmatrix} \begin{bmatrix} x(t_k) \\ w(t_k) \end{bmatrix} \quad (3)$$

The plant model is here considered as a lifted discrete-time system. The matrices indexed $_{\mathrm{cl}_d}$ include the controller K linearly. LMIs will now be used as the key to solve the static feedback design problem, see more on LMIs in e.g. (Gahinet and Apkarian 1994, Boyd *et al.* 1994, Scherer 2000). The Bounded Real LMI-lemma is in discrete-time, see e.g. (Gahinet and Apkarian 1994), formulated as:

Lemma 1. Consider the discrete-time system (3). Then the following are equivalent

- The transfer $\|\mathcal{G}_{zw_d}\|_\infty < \gamma$ and A_{cl_d} is stable
- There exists an $S = S' > 0$ such that

$$\begin{bmatrix} A'_{\mathrm{cl}_d} S A_{\mathrm{cl}_d} - S & A'_{\mathrm{cl}_d} S B_{\mathrm{cl}_d} & C'_{\mathrm{cl}_d} \\ B'_{\mathrm{cl}_d} S A_{\mathrm{cl}_d} & B'_{\mathrm{cl}_d} S B_{\mathrm{cl}_d} - \gamma I & D'_{\mathrm{cl}_d} \\ C_{\mathrm{cl}_d} & D_{\mathrm{cl}_d} & -\gamma I \end{bmatrix} < 0 \quad (4)$$

The key to obtain a stable closed loop fulfilling the performance measure $\|\mathcal{G}_{zw_d}\|_\infty < \gamma$ can thus be solved by finding variables that fulfil (4). The unknown variables are now S (the solution to the static feedback Riccati equation) and K, which is linearly included in A_{cl_d}, ... The problem is thus not a true LMI-problem but rather a BMI (bilinear MI), such that one variable at a time is searched for. Furthermore, to obtain a (sub)optimal controller, the γ-value shall be minimised.

The procedure is now the following:

(1) "Lift" the augmented plant $\mathcal{G}_a$ into a discrete-time model at some (large) γ-value.
(2) Design a full-order sampled-data controller $\mathcal{K}_{\mathrm{full}}$, (2), for the augmented plant. This involves γ-iteration, and in fact repeated lifting (step1) for each γ-value. This problem is convex.
(3) Reduce the order of the discrete-time controller from step 2 using a suitable model reduction scheme; balance the controller, then reduce the order such that the least important state(s) is (are) removed keeping the norm mainly constant, giving the reduced controller K_{red}.
(4) Given K_{red} from step 3 (implying that $A_{\mathrm{cl}_d}, B_{\mathrm{cl}_d}, C_{\mathrm{cl}_d}$, D_{cl_d} defined in (3) are known), solve the discrete-time LMI (4) for $S = S' > 0$ (linear in S) and minimise γ.
(5) Given S from step 4, solve the LMI (4) for K (now linear in K since all $A_{\mathrm{cl}_d}, B_{\mathrm{cl}_d}, C_{\mathrm{cl}_d}$, D_{cl_d} are linear in K).
(6) Iterate steps 4 and 5 until convergence.
(7) Go back to step 3 and repeat the controller reduction until the closed loop degrades too much (too large γ-value).

If the procedure gives approved solutions it assures stability for the closed loop, since the S- and K-calculations rely on the Bounded Real Lemma. If the full-order controller only is reduced as in step 3 without the BMI-calculations in steps 4 and 5, neither stability nor performance can be assured. The discretisation or lifting should be redone when the γ-value changes too much since the lifting depends on this value. The procedure is in fact applicable for mere continuous- or discrete-time systems as well, with due modifications.

2.7 Nonlinear controller

The nonlinear behaviour is taken into account in the design through controller switching together with appropriate anti windup and bumpless transfer considerations, see (Ring *et al.* 2005). The plant shows similar dynamic behavior for many operating points which has motivated some further simplifications. One such investigation has been simulations for mode-1 when the controller matrices were the same for a number of operating points while the inputs and outputs were scaled according to the plant gains at the respective operating point. To some extent this showed to be very promising, see Fig. 5. This approach reduces the computational effort in the hardware significantly.

3. DESIGN CONSIDERATIONS AND TESTING

The discrete-time controllers are finally implemented in $\mathrm{MATRIX_X}$ together with the engine model, actuators and sensors described in Section 2.1. The procedure presented above results in stable closed loop systems at each operating point fulfilling given design criteria. It is hopefully clear from above how to put emphasis if new tuning is needed, and such a redesign should iterate from the choice of weight functions in Section 2.2.

Results from the proposed control system are compared with results from today's FADEC-control (Full Authority Digital Engine Control), and although the current design has been used for long with experienced tuning the new results can compete very well. In this new concept there are obvious clear points on where to put more emphasis if needed, since all the design is made in the same framework. Some motivating and explaining simulation results are shown in next section.

4. SIMULATION RESULTS

Fig. 3 presents Hankel singular values for the plant at one operating point seen from a controller point of view, as was discussed in Section 2.1. The engine model has in total 34 states, and the

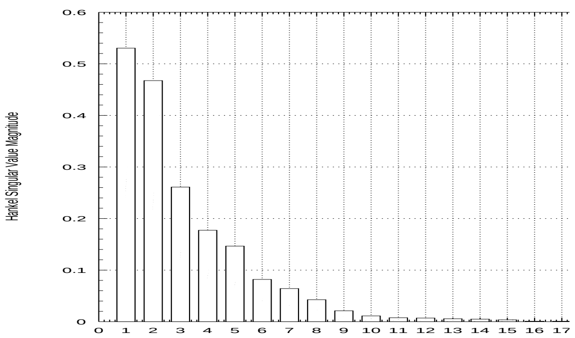

Fig. 3. The largest Hankel singular values for normalised system considered for controller design.

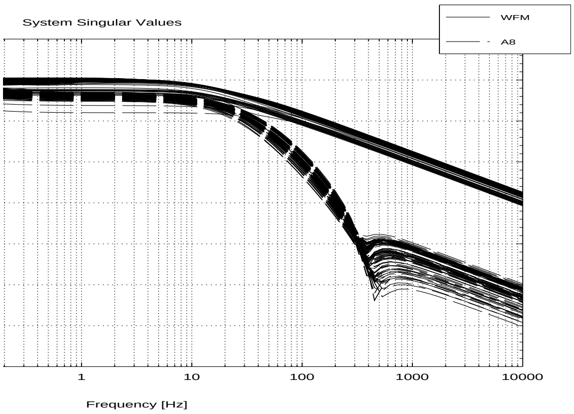

Fig. 4. Normalised frequency plots for the engine models with control signals WFM (the upper curves) and A8 (the lower). The y-axis is in steps of 20 [dB] from -140 to +20 [dB] .

singular values suggest that a considerable model reduction can be made.

Fig. 4 shows normalised frequency plots for the engine model at all operating points with main fuel flow WFM and output nozzle area A8 as control variables. It can be seen that when normalised, there is not much difference between the operating points. This motivates simplifications in the design.

Another fact that motivates that the design might be simplified can be seen in Fig. 5. The plots show step responses at different operating points covering all power lever angles (PLA). The solid plots refer to when the controller is the same for all operating points but with different input/output scalings, and the dotted plots refer to when the controller is optimised for the operating point in question. The differences are minor.

Fig. 6 motivates the BMI-controller reduction by showing results with controllers from "full order" $n_K = 15$ to order $n_K = 2$ at one operating point. The increase in γ-value when the controller order n_K is decreased is a measure of the performance degradation, see Table 2. The conclusion is that

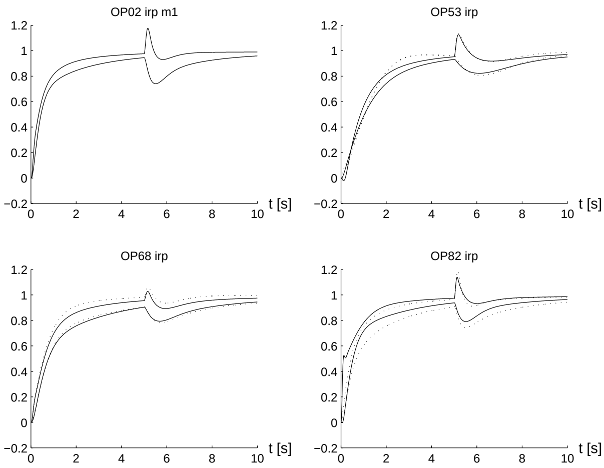

Fig. 5. Time responses after step in reference followed by step in disturbance for different operating points at four different PLA. Solid plots use the same controllers with input/output scalings while dotted are optimised at the operating point.

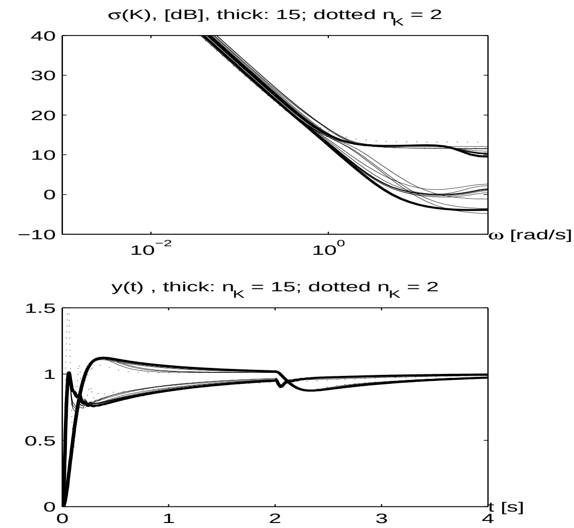

Fig. 6. Normalised results from linear sampled-data controllers with order $n_K = 15$ down to $n_K = 2$ at one operating point. Top: frequency plots for the different controllers. Bottom: system output due to a step in reference signal followed by a step disturbance at plant input. Both outputs are shown in the same plot.

the performance degradation is minor until the order is as low as 6. Recall from Section 2.6 that using the BMI-procedure stability and performance are assured. In fact, controllers of order $n_K = 4$ have successfully been used in the nonlinear $\mathrm{MATRIX_X}$ model at one operating point, see Fig. 7. There is hardly any difference in results from full controller order down to fourth order.

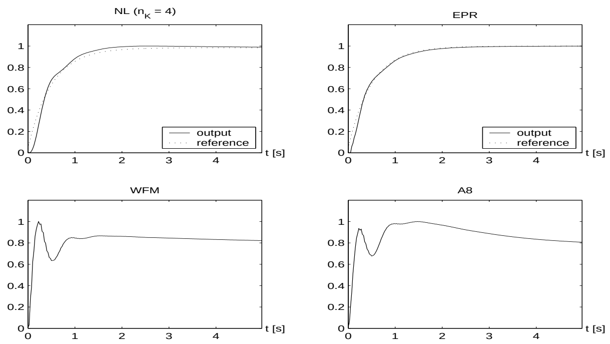

Fig. 7. Simulation results when the controller order is 4 and the desired power reference value is changed in a step. Upper plots show the controlled signals *low pressure rotational speed* and *engine pressure ratio* together with their reference values. Lower plots show the control signals *main fuel flow* and *output nozzle area*. All signals are normalised.

Table 2. Table showing corresponding controller order (n_K) and obtained γ-values for the MIMO jet-engine control

γ	1.74	...	1.74	1.75	1.78	1.85	1.87	1.97
n_k	15	...	8	7	6	5	4	3

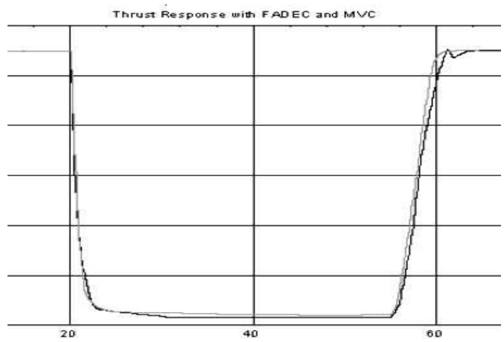

Fig. 8. The engine thrust as a result of a step in PLA-signal from max value down to min value and after a while back again. The response is shown for both the FADEC-system (black) and the proposed concept (grey).

Finally, Fig. 8 compares the engine thrust when the reference switches from max to min and back to max again for both the FADEC system of today and the proposed system. The latter gives a smoother and faster behaviour. The FADEC version switches between three different controllers, while the new concept uses one nonlinear controller with switching as described in Section 2.7.

5. SUMMARY

This joint work has given more understanding on how to take advantage of modern design and analysis tools in order to improve the control of advanced jet engines. Hereby $\mathcal{H}_\infty$-design techniques using PI-weights, sampled-data “lifting” and controller switching have shown to be easily implementable.

The main contribution of this work is to show how different parts in the design procedure can be adapted to different demands and thereby be taken into consideration. Both design and analysis are carried out in a common MIMO framework, and is therefore easily adaptable to a number of different MIMO systems.

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