PASSIVITY BASED CONTROL OF A SEISMICALLY EXCITED BUILDING [1]

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Abstract: The design of a passivity based control law with interconnection and damping assignment to attenuate motion of a building seismically excited is presented. The actuator is a magneto-rheological damper placed between ground and first story. Significant motion reduction is attained when compared with the case of building free response.

Keywords: Passive compensation, passivity based control, interconnection and damping assignment.

1. INTRODUCTION

Passivity based control (PBC) tries to exploit intrinsic characteristics of a system that contribute to its stability. The control law is derived based upon an energy storage function that has a minimum at a desired point. A recent extension to PBC is the technique of interconnection and damping assignment (IDA), that aims to increase system damping and redefine, if necessary, system interconnection in order to induce system trajectories to a desired stable equilibrium (Ortega *et al.*, 1998).

To apply PBC-IDA it is convenient to pose system dynamics as port controlled Hamiltonian (PCH) equations, because they allow a clean identification of interconnection and damping elements of the system and of its passive nature, if present. The design of the PBC-IDA control law for a PCH system follows then a systematic procedure.

In this paper PBC-IDA design methodology is applied to control the motion of a seismically excited building. The actuator in this case is a magneto-rheological damper placed between ground and first story. There are several advantages on choosing this semi-active actuator, as it is robust under environment changes and because its energy consumption is small (Symans, M. R. and Constantinou, A. C., 1997).

The paper recovers basic equations from PBC-IDA and states the conditions that the control law must satisfy in order for the feedback system to have a prescribed behavior. Conditions are then proven for the case of the control law here proposed. Simulation results are provided where the performance of the controller is compared with the free response case. Concluding remarks are also included.

2. PASSIVITY BASED CONTROL OF PORT CONTROLLED HAMILTONIAN SYSTEMS

Using Euler-Lagrange equations and Legendre transformation $H = p\dot{q} - L$, where q are the generalized coordinates, p the generalized momenta, L the Lagrangian and H the Hamiltonian, it is possible to write the port controlled Hamiltonian equations (PCH) as (Blankenstein, G. and van der Schaft, A. J., 2001)

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$$\dot{s} = [J(s) - R(s)] \frac{\partial H}{\partial s}(s) + g(s)u \,, \tag{1}$$
$$y = g^T(s) \frac{\partial H}{\partial s}(s) \,,$$

where $s = [q^T \quad p^T]^T \, \epsilon \, R^{2n}$ represents energy variables, $J(s) = -J^T(s)$ the interconnection matrix, $R(s) = R^T(s) \geq 0$ the damping matrix, $H(s) : R^n \rightarrow R, H(s) \geq 0$ the energy storage function and $u, y \, \epsilon R^m$ the port power variables. The energy balance of the PCH system guarantees that PCH in Eq. (1) is passive.

The first step to design a PBC control law is to propose the closed loop desired dynamics with PCH structure, that is

$$\dot{s} = [J_d(s) - R_d(s)] \frac{\partial H_d}{\partial s}(s) \,. \tag{2}$$

Terms in Eq. (2) must satisfy $J_d(s) = -J_d^T(s)$, $R_d(s) = R_d^T(s) \geq 0$ and $H_d \geq 0$ with minimum at s_*, the desired stable equilibrium for the closed loop system. It is always possible to write $J_d(s) = J(s) + J_a(s)$, $R_d(s) = R(s) + R_a(s)$ and $H_d(s) = H(s) + H_a(s)$, where terms with subindex a are referred to as *assigned* behavior.

There are several ways to propose the assigned terms $J_a(s)$, $R_a(s)$ and $H_a(s)$. In this paper the fact that building dynamics is well known[3] is exploited. To guarantee that the assigned terms are appropriate (Ortega *et al.*, 2002) propose the left annihilator test, that consists on verifying

$$g^\perp(s)[J_d(s) - R_d(s)] \frac{\partial H_a(s)}{\partial s} =$$
$$- g^\perp(s)[J_a(s) - R_a(s)] \frac{\partial H(s)}{\partial s} \,, \tag{3}$$

where $g^\perp(s)$ is the left annihilator of $g(s)$, i.e., $g^\perp(s)g(s) = 0$. A solution to Eq. (3) must be obtained in terms of $H_a(s)$. Once a solution is found for Eq. (3), it is necessary to verify the following proposition, taken directly from (Ortega *et al.*, 2002), to assure that the proposed $J_a(s)$ and $R_a(s)$ will take the system to the desired equilibrium.

Proposition 1: Given $J(s)$, $R(s)$, $H(s)$, $g(s)$ and the desired equilibrium $s_* \epsilon R^n$, assume that $\beta(s)$ $J_a(s)$ $R_a(s)$ and vector $\kappa(s)$ can be found such that

$$[J(s)+J_a(s)-(R(s)+R_a(s))]\kappa(s) =$$
$$- [J_a(s)-R_a(s)] \frac{\partial H_d}{\partial s}(s) \,, \tag{4}$$

and that the following properties hold

- Structure preservation.
- Integrability.
- Equilibrium assignment.
- Lyapunov stability.

Under these conditions, the control law $u = \beta(s)$ yields the closed loop dynamics in Eq. (2), with desired energy storage function given by

$$H_d(s) = H(s) + H_a(s) \,, \tag{5}$$

with $H_a(s)$ satisfying

$$\frac{\partial H_a}{\partial s}(s) = \kappa(s). \tag{6}$$

The desired equilibrium will be locally stable. It will be asymptotically stable if the largest invariant set under the closed loop dynamics is equal to s_*, i.e.,

$$s_* = \left\{ s \epsilon R^n | \left[\frac{\partial H_d}{\partial s} \right]^T R_d \frac{\partial H_d}{\partial s}(s) = 0 \right\}. \tag{7}$$

An estimate of the domain of attraction is given by $\{s \epsilon R^n | H_d(s) \leq c\}$. □

Once the left annihilator test and *Proposition 1* are verified, the PBC-IDA control law can be stated as

$$u = \beta(s) = [g^T(s)g(s)]^{-1} g^T(s) \tag{8}$$
$$\left\{ [J_d(s) - R_d(s)] \frac{\partial H_d}{\partial s}(s) - [J(s) - R(s)] \frac{\partial H}{\partial s}(s) \right\} .$$

3. PBC-IDA CONTROL LAW FOR A BUILDING

The PBC-IDA control law will be developed for a prototype building subject to seismic excitation in one direction and assuming that the control signal will be provided by a magneto-rheological damper. Fig. 1 illustrates the arrangement of building, damper and ground motion.

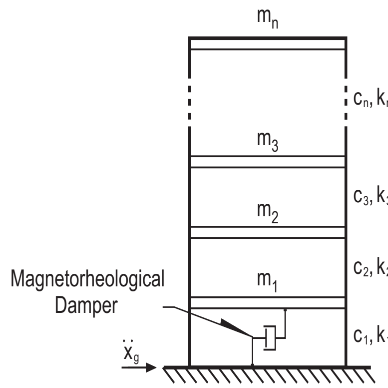

Fig. 1. Building with n stories.

In Fig. 1, m_i is the concentrated mass of story i, c_i and k_i are the damping and stiffness coefficients of column between stories i and $i-1$, respectively and $\ddot{x}_g$ is the ground acceleration induced by the earthquake. In matrix form, the dynamics of the building are given by (Chopra, 1995)

$$M\ddot{x} + C\dot{x} + Kx = -M\ddot{x}_g + F \,,$$

[3] See, for example, (Chopra, 1995; Paz, 1997).

where $x,\ \dot{x},\ \ddot{x} \in \mathcal{R}^n$ are the vectors of stories displacements, velocities and accelerations, $F = l\ f = [1\ 0 \cdots 0]^T f$, with f the force applied by the magneto-rheological damper [4], $\ddot{x}_g \in \mathcal{R}^n$ is the vector of ground accelerations applied to all the stories [5], $M = M^T > 0$ is the diagonal inertia matrix, $C = C^T \geq 0$ is the damping matrix and $K = K^T > 0$ is the stiffness matrix. C and K are tridiagonal. In this paper it is assumed that measurements of $\ddot{x}$ and $\ddot{x}_g$ are available, and that from them x, $\dot{x}$ can be derived by an observer.

The energy variables are

$$\begin{aligned} q &= x \\ p &= M\dot{x}\,, \end{aligned}$$

where p is the linear momentum.

The energy storage function is

$$H(s) = \frac{1}{2}p^T M^{-1} p + \frac{1}{2} q^T K q\,. \tag{9}$$

Rearranging previous equations yields the PCH equations in the form of Eq. (1)

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix} \right) \begin{bmatrix} Kq \\ M^{-1}p \end{bmatrix} \tag{10}$$

$$+ \begin{bmatrix} 0 \\ l \end{bmatrix} u + \begin{bmatrix} 0 \\ -I \end{bmatrix} M\ddot{x}_g$$

$$y = [\,0\ \ I\,] \begin{bmatrix} Kq \\ M^{-1}p \end{bmatrix},$$

where $u = f$ is the system control signal. From Eq. (10), it follows that the interconnection and damping matrices are given by

$$J(s) = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}, \quad R(s) = \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix}. \tag{11}$$

To design the PBC-IDA control law for the structure, it is proposed to keep the interconnection term and to assign additional damping, therefore

$$J_a = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad R_a = \begin{bmatrix} 0 & 0 \\ 0 & C_d - C \end{bmatrix}, \tag{12}$$

where C_d is the desired damping matrix.
The desired energy storage function is proposed as

$$H_d(q,p) = \frac{1}{2}p^T M^{-1} p + \frac{1}{2} q^T K_d q\,, \tag{13}$$

where K_d is the desired stiffness matrix, that normally adds stiffness to the system. From analyzing Eq. (13), it is clear that the desired equilibrium point is set to $s = [q^T \quad p^T]^T = [0 \quad 0]^T$, i.e., to the structure with no motion.

[4] In this case, the force is applied directly to the first story.
[5] All elements of $\ddot{x}_g$ are equal.

Once all the terms for the IDA have been proposed, the PCH closed loop equations are given by

$$\begin{bmatrix} \dot{q}_d \\ \dot{p}_d \end{bmatrix} = \left(\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & C_d \end{bmatrix} \right) \begin{bmatrix} K_d q \\ M^{-1}p \end{bmatrix}. \tag{14}$$

Using Eq. (8) the control law is

$$f = -l^T \left\{ (K_d - K)q - (C_d - C)M^{-1}p + I\ddot{x}_g \right\}. \tag{15}$$

To assure that this control law will stabilize the closed loop system in the desired equilibrium, $s_*^T = [0 \quad 0]^T$, it is necessary to verify the left annihilator, Eq. (3), and the conditions in *Proposition 1*. Using Eq. (10), it follows that the annihilator equation is of form

$$[\,g_1(s)^\perp\ \ g_2(s)^\perp\,] \begin{bmatrix} 0 \\ g_2(s) \end{bmatrix} = 0\,. \tag{16}$$

From Eq. (16) it follows that $g_1(s)^\perp = \bigstar$ [6] and $g_2(s)^\perp = 0$.

Substituting the values in Eqs. (10) and (14) into Eq. (3) yields

$$\begin{bmatrix} g_1(s)^\perp M^{-1}p \\ g_2(s)^\perp[-K_d q - C_d M^{-1}p] \end{bmatrix} = \begin{bmatrix} g_1(s)^\perp M^{-1}p \\ g_1(s)^\perp[-Kq - CM^{-1}p] \end{bmatrix}. \tag{17}$$

From the first row of Eq. (17) it follows that $g_1(s)^\perp = \bigstar$ and from the second that there is not a value of $g_2(s)^\perp$ different from zero that satisfies the equality. Therefore, from Eqs. (16) and (17) the proper values are $g_1(s)^\perp = \bigstar$ and $g_2(s)^\perp = 0$. Solving Eq. (5), it follows that

$$H_a = \frac{1}{2} q^T (K_d - K) q\,. \tag{18}$$

To obtain the value of $\kappa(s)$, Eq. (6) is solved

$$\kappa(s) = \begin{bmatrix} (K_d - K)q \\ 0 \end{bmatrix}. \tag{19}$$

With the values of $\kappa(s)$ and $H_a(s)$, Eq. (4) is verified using the building model to obtain

$$\begin{bmatrix} \kappa_2(s) + \dfrac{\partial H}{\partial p}(s) \\ \kappa_1(s) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial H}{\partial p}(s) \\ (K_d - K)q \end{bmatrix}. \tag{20}$$

For the equality in the first row to hold it is necessary that $\kappa_2(s) = 0$, and from the second row that $\kappa_1(s) = (K_d - K)q$, result that is consistent with Eq. (19).

The expression to verify structure preservation is

$$J_d(s) = J(s, \beta(s)) + J_a(s) = -[J(s, \beta(s)) + J_a(s)]^T \tag{21}$$

$$R_d(s) = R(s) + R_a(s) = [R(s) + R_a(s)] \geq 0\,. \tag{22}$$

[6] $\bigstar$ arbitrary value.

Substituting Eqs. (11) and (12) into Eq. (21) for the desired interconnection yields

$$J_d = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} = -\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}^T, \qquad (23)$$

and for the desired damping

$$R_d = \begin{bmatrix} 0 & 0 \\ 0 & C_d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & C_d \end{bmatrix}^T. \qquad (24)$$

To satisfy integrability is necessary to verify that $\kappa(s)$ is the gradient of an scalar function, therefore

$$\frac{\partial \kappa}{\partial s}(s) = \left[\frac{\partial \kappa}{\partial s}(s)\right]^T. \qquad (25)$$

Evaluating Eq. (25) with the values of the building model it follows that

$$\begin{bmatrix} \frac{\partial \kappa_1(q,p)}{\partial q} & \frac{\partial \kappa_1(q,p)}{\partial p} \\ \frac{\partial \kappa_2(q,p)}{\partial q} & \frac{\partial \kappa_2(q,p)}{\partial p} \end{bmatrix} = \begin{bmatrix} \frac{\partial \kappa_1(q,p)}{\partial q} & \frac{\partial \kappa_2(q,p)}{\partial q} \\ \frac{\partial \kappa_1(q,p)}{\partial p} & \frac{\partial \kappa_2(q,p)}{\partial p} \end{bmatrix}. \qquad (26)$$

From Eq. (26), it follows that elements $(1,1)$ and $(2,2)$ of matrices in both sides are identical. The equality of the other two elements requires that

$$\frac{\partial \kappa_1(q,p)}{\partial p} = \frac{\partial \kappa_2(q,p)}{\partial q}. \qquad (27)$$

One possibility for Eq. (27) to hold is that $\kappa_1(s)$ is only a function of q and $\kappa_2(s)$ a function of p, i.e., $\kappa_1(q)$ and $\kappa_2(p)$. In this case, Eq. (27) holds trivially. This requirement is fulfilled by the results in Eq. (19). If s_* is the equilibrium of the desired energy storage function, $H_d(s)$, $\kappa(s)$ must satisfy

$$\kappa(s_*) = -\frac{\partial H}{\partial s}(s_*). \qquad (28)$$

Substituting $\kappa(s)$ from Eq. (19) and the partial derivative of the energy function in Eq. (9) into Eq. (28) it follows that

$$\begin{bmatrix} (K_d - K)q \\ 0 \end{bmatrix} = \begin{bmatrix} -Kq \\ -M^{-1}p \end{bmatrix}. \qquad (29)$$

Evaluating Eq. (29) in the equilibrium given by $s_* = [q \quad p]_*^T = [0 \quad 0]$ yields

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (30)$$

Finally, stability in the sense of Lyapunov is verified using

$$\frac{\partial K}{\partial s}(s) > -\frac{\partial^2 H}{\partial s^2}(s_*). \qquad (31)$$

Applying Eq. (31) to the model of the building

$$\begin{bmatrix} (K_d - K) & 0 \\ 0 & 0 \end{bmatrix} > -\begin{bmatrix} K & 0 \\ 0 & M^{-1} \end{bmatrix}. \qquad (32)$$

Taking determinants in Eq. (32) yields

$$0 > -KM^{-1}. \qquad (33)$$

Condition (33) holds because $K > 0$ y $M^{-1} > 0$.

All the conditions stated in the left annihilator and in *Proposition 1* have been verified. With this it is assured that the PBC-IDA control law in Eq. (15) makes the desired equilibrium point s_* assymptotically stable.

The control law f in Eq. (15) is used to determine the voltage v that a magneto-rheological damper (MRD) has to apply to the building. For this purpose the model of the MRD proposed in (Jiménez-Fabián, R. E. and Álvarez Icaza-Longoria, L. A., 2003) is employed to obtain an explicit equation of the voltage value required in the MRD that best matches the required control force f in by the PCB-IDA control law. The equation for the voltage is

$$v = \frac{(f - \sigma_2 \dot{x}_1) a_0 |\dot{x}_1|}{\dot{x}_1 - a_0 a_1 f |\dot{x}_1| \dot{x}_1 + a_0 a_1 \sigma_2 \dot{x}_1 |\dot{x}_1|}, \qquad (34)$$

where $\dot{x}_1$ the relative velocity at the damper ends, in this case the first story velocity, and a_0, a_1, σ_0, σ_1, σ_2 model constants. Velocity $\dot{x}_1$ is assumed to be available, as stated before. In the prototype under consideration $v \in [0,\, 2.5][V]$, the value in Eq. (34) is truncated accordingly. It is possible to incorporate the model of the MRD in the PCH equations. However, in this paper the fact that the dynamics of the MRD is much faster than that of the building is exploited, to justify the approximation in Eq. (34).

4. SIMULATION RESULTS

Once the system was model in PCH equations and the PBC-IDA control law in Eq. (8) was designed, numerical simulations were executed for different selections of the desired energy function, H_d, and the assigned damping, R_d. The parameters for the model, a three stories building, are in the appendix, together with the relevant constants of the MRD.

The excitation for the building came from a record of the north-south (NS) component of the earthquake on September 19, 1985 taken at the "Secretara de Comunicaciones y Transportes" (SCT) in Mexico City. The corresponding ground acceleration is shown in Fig. 2.

Figs. 3, 4 y 5 show the displacement of the building stories under the effect of the earthquake in Fig. (2) for one typical simulation. Two curves are included in each figure. One represents the free response of the building and the other the displacement when the PBC-IDA control law is used. It can be noticed that the curves corresponding to the controlled case show significant

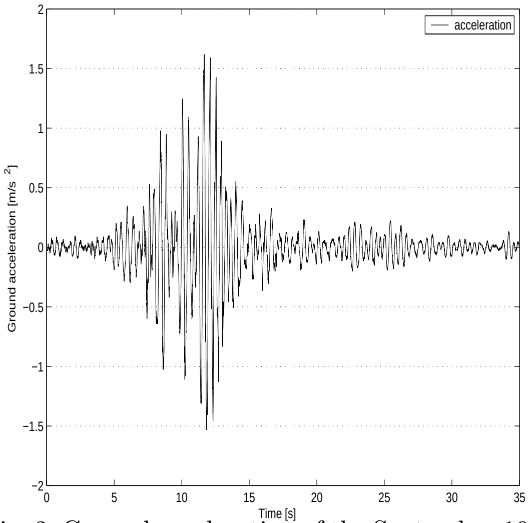

Fig. 2. Ground acceleration of the September 1985 earthquake at SCT-NS, Mexico City.

reductions of the displacements, reduction that is sharper in the case of the first story. It is expected that if the number of stories increase, the value of the displacements will be bigger for taller stories. It is important to note, however, that the most important quantities to reduce are inter-story displacement. The control algorithm proposed in this paper achieves a significant reduction of these displacements, independently of the number of stories.

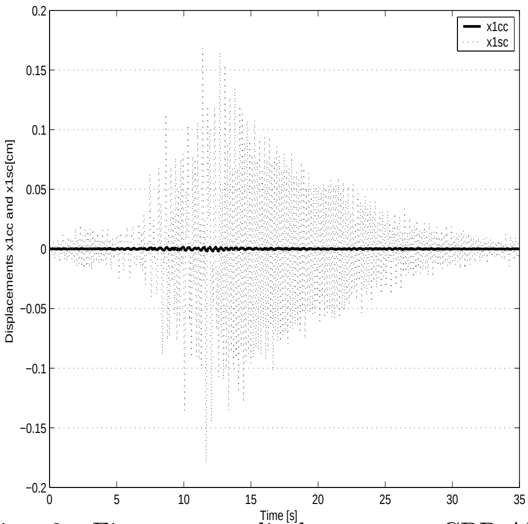

Fig. 3. First story displacements; CBP-AIA (x1cc): solid, free response (x1sc): dotted.

Fig. 6 shows the voltage at the MRD that corresponds to the results in Figs. 3, 4 y 5. It can be noticed that the maximum voltage is $1.49[V]$, well below the maximum range of $2.5[V]$ of the MRD.

Finally, Table **??** has a summary of four significant simulation results under the effect of the PBC-IDA control law.

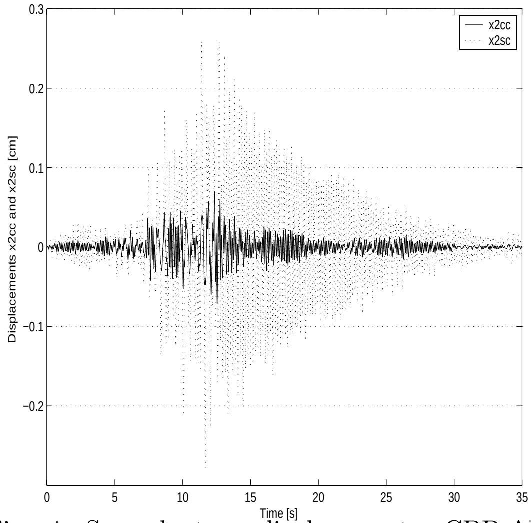

Fig. 4. Second story displacements; CBP-AIA (x2cc): solid, free response (x2sc): dotted.

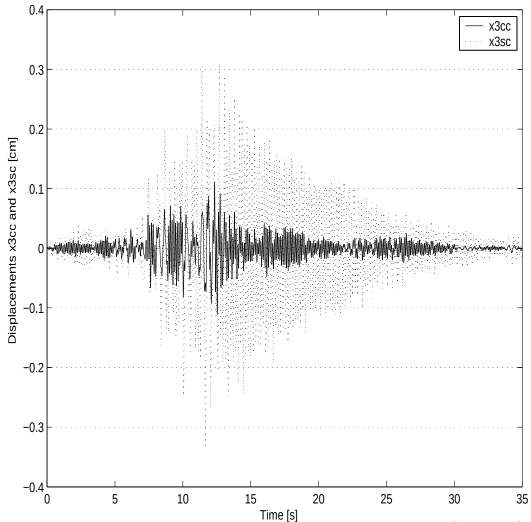

Fig. 5. Third story displacements; CBP-AIA (x3cc): solid, free response (x3sc): dotted.

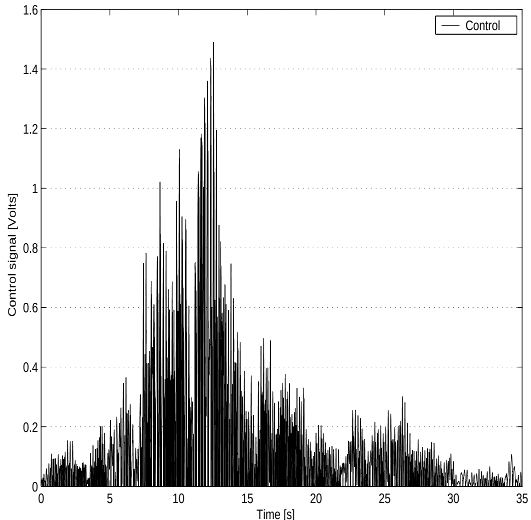

Fig. 6. Voltage at the magneto-rheological damper.

5. CONCLUSION

The methodology of passivity based control with interconnection and damping assignment (PBC-

Table 1 Summary of results

Force $[N]$	Volt $[V]$	Displacement $[cm]$ No cont.	PBC-IDA	Red. of x_i [%]	Characteristic values
		0.17	0.0016	99.06	-40.73 ± 136.78
620	1.49	0.26	0.0700	73.08	-40.67 ± 197.52
		0.31	0.1100	64.52	-40.68 ± 299.77
		0.17	0.0135	92.06	-0.08 ± 78.13
590	0.7	0.26	0.0800	69.23	-0.16 ± 110.49
		0.31	0.1220	60.65	-0.12 ± 95.69
		0.17	0.0130	92.35	-1.58 ± 442.78
2000	2.5	0.26	0.0800	69.23	-0.82 ± 313.64
		0.31	0.1200	61.29	-1.16 ± 380.84
		0.17	0.0110	93.53	-14.31 ± 139.56
600	1.17	0.26	0.0810	68.85	-12.01 ± 128.98
		0.31	0.1230	60.32	-9.29 ± -115.16

IDA) was applied to control the motion of a building under seismic excitation and assuming that the actuator is a magneto-rheological damper. Building model was posed as port controlled Hamiltonian equations and desired damping and energy storage functions were proposed that allowed to derived a PBC-IDA control law. All the conditions required in (Ortega *et al.*, 2002) to guarantee stability of a desired equilibrium were verified. Simulation results using this control law in a three story building subject to the ground acceleration of an earthquake recorded at Mexico City were presented. Results indicated a significant reduction on the story displacements, when compared with the free response case. Reduction was between 60 % and 99 % for the simulations presented. Voltage requirements at the MRD never exceeded the maximum allowable value.

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Appendix A. PARAMETERS FOR THE SIMULATION

Parameters for the MRD are taken from (Jiménez-Fabián, R. E. and Álvarez Icaza-Longoria, L. A., 2003) and for the building from (Dyke, S.J. *et al.*, 1996)

Table A.1. MRD constants

Ct.	Value
a_0	$0.003\ [(V \cdot s^2)/(Kg \cdot m)]$
a_1	$-0.1444\ [V^{-1}]$
σ_0	$1059300\ [Kg/(V \cdot s^2)]$
σ_1	$5800\ [Kg/s]$
σ_2	$2300\ [Kg/s]$

Inertia, damping and stiffness matrices for the building

$$M=\begin{bmatrix} 98.3 & 0 & 0 \\ 0 & 98.3 & 0 \\ 0 & 0 & 98.3 \end{bmatrix}[Kg],$$

$$C=\begin{bmatrix} 175 & -50 & 0 \\ -50 & 100 & -50 \\ 0 & -50 & 50 \end{bmatrix}\left[\frac{N \cdot s}{m}\right]$$

$$K = 10^5 \begin{bmatrix} 12 & -6.84 & 0 \\ -6.84 & 13.7 & -6.84 \\ 0 & -6.84 & 6.84 \end{bmatrix}\left[\frac{N}{m}\right]$$

Natural frequencies of building are $\omega_1 = 34[rad/s]$, $\omega_2 = 99[rad/s]$, $\omega_3 = 149[rad/s]$.