

**EXPERIMENTAL RESULTS OF A CASCADE  
OBSERVER FOR SENSORLESS INDUCTION  
MOTOR ON LOW FREQUENCIES  
BENCHMARK**

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Abstract: The purpose of this paper is to design a cascade observer to reconstruct the angular speed of induction motor as well as an estimator of fluxes and load torque. Due to the complexity of induction motor observation at low frequencies, the observer and the estimator are tested and validated on reference trajectories of a new sensorless induction motor benchmark particularly in the case when the motor state is unobservable. This benchmark is applied on an experimental set-up located at IRCCyN (Nantes, France). *Copyright*© 2005 IFA C.

Keywords: Induction motor, cascade observer, low frequencies sensorless benchmark, experimental results.

## 1. INTRODUCTION

The most efficient control strategies of induction motor, such as field oriented control and nonlinear control, require velocity measurement. To limit the cost, there is a growing interest in sensorless control (involving an estimation of speed and position). Recently, many schemes for sensorless control of induction motors have been proposed in the literature. Generally, using the induction motor state equations, the flux and speed can be calculated from the stator voltage and current values (Kubota, 93). A model reference adaptive system (MRAS) (Campbell, 02), is also an alternative method to sensorless induction motor control. In another proposed scheme (Kubota, 93), the flux is obtained by a full order Luenberger observer. However, these methods perform well except at very low speed, near zero stator frequency

(Neves, 99). Indeed, observability problems at low frequency have not often been taken into account in motor control design. A possibility to circumvent the difficulty is to inject high frequency signals in the stator voltage (Holtz, 00). Nevertheless, few works have addressed the observability problem. In (Canudas, 00) and (Ibarra, 04) conditions for lost of observability are analyzed. For usual operating conditions, the hardest situations for lost of observability is that the excitation voltage frequency is zero and the motor is operating at constant speed. From this point of view, the purpose of this paper is to propose a cascade observer to estimate the induction motor angular speed and an estimator for fluxes and load torque. The observer and the estimator are tested on a benchmark (see (web, 03)), in which the reference trajectories are defined to drive the motor from high to low frequencies. Robustness tests are defined in the setting of this benchmark with defined inductance and resistance

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variations. This paper is organized as follows. Section 2 introduces the model of induction motor. Section 3 presents our benchmark designed to test observers. The design of a cascade observer and an estimator in order to estimate the unmeasurable variables is given in section 4. The experimental results obtained from the proposed observer and estimator are presented in section 5. Finally, some conclusions are drawn.

## 2. INDUCTION MOTOR MODEL

The equation of the induction motor can be simplified by the Concordia transformation. The transformed dynamic equation are given in a  $(\alpha, \beta)$  fixed reference frame (Chiasson, 95) and the motor can be described by (1)

$$\begin{pmatrix} \dot{\phi}_{r\alpha} \\ \dot{\phi}_{r\beta} \\ \dot{i}_{s\alpha} \\ \dot{i}_{s\beta} \\ \dot{\Omega} \end{pmatrix} = \begin{pmatrix} -a\phi_{r\alpha} - p\Omega\phi_{r\beta} + aM_{sr}i_{s\alpha} \\ -a\phi_{r\beta} + p\Omega\phi_{r\alpha} + aM_{sr}i_{s\beta} \\ b(a\phi_{r\alpha} + p\Omega\phi_{r\beta}) - \gamma i_{s\alpha} \\ b(a\phi_{r\beta} - p\Omega\phi_{r\alpha}) - \gamma i_{s\beta} \\ m(\phi_{r\alpha}i_{s\beta} - \phi_{r\beta}i_{s\alpha}) - c\Omega - \frac{1}{J}T_l \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ m_1 & 0 \\ 0 & m_1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_{s\alpha} \\ u_{s\beta} \end{pmatrix} \quad (1)$$

where  $i_{s\alpha}, i_{s\beta}, \phi_{r\alpha}, \phi_{r\beta}, u_{s\alpha}, u_{s\beta}, \Omega, T_l$  denote respectively the stator currents, the rotor fluxes, the stator voltage inputs, the angular speed and the load torque. The subscripts  $s$  and  $r$  refer to the stator and rotor. The parameters  $a, b, c, \gamma, \sigma, m$  and  $m_1$  are defined by:

$$a = (R_r/L_r), \quad b = (M_{sr}/\sigma L_s L_r), \quad c = (f_v/J), \quad \gamma = \left( \frac{L_r^2 R_s + M_{sr}^2 R_r}{\sigma L_s L_r^2} \right), \quad \sigma = (1 - (M_{sr}/L_s L_r)),$$

$$m = (pM_{sr}/JL_r), \quad m_1 = (1/\sigma L_s).$$

$R_s$  and  $R_r$  are the resistances.  $L_s$  and  $L_r$  are the self-inductances,  $M_{sr}$  is the mutual inductance between the stator and rotor windings.  $p$  is the number of pole-pair.  $J$  is the inertia of the system (motor and load) and  $f_v$  is the viscous damping coefficient.

The control inputs are the stator voltages. The load torque is viewed as a disturbance. Only stator currents and stator voltages are measured.

## 3. OBSERVER BENCHMARK

The observability problem of induction motor has been underlined by many authors ((Canudas de Wit, 00), (Ibarra, 04)). The authors of (Canudas de Wit, 00) have characterized sufficient conditions leading to observable and unobservable situations. Sufficient conditions of unobservability are: the excitation voltages frequency is zero and the rotor speed is constant.

To define a benchmark to test observers on and near the unobservability conditions, we have defined trajectories (Fig. 1) for which the initial conditions of speed and stator pulsation are such that the motor is observable. Afterwards the pulsation of the stator

voltages is forced to zero corresponding to constant fluxes (Fig. 2) while the rotor velocity remains constant, making the state unobservable between 4 and 5 seconds and between 6 and 7 seconds. Between 5 and 6 seconds, the rotor moves with a constant acceleration, allowing to check the observer convergence when the state is slightly observable. Finally, the induction motor is driven outside the unobservability conditions (Fig. 1). Practically, to apply this benchmark, the main difficulty lies in the simultaneous control of speed and stator pulsation so that the slip pulsation  $\omega_g = \omega_s - p\omega$  does not exceed a limiting value  $\omega_g = R_r M_{sr} i_q / L_r \phi_d$ , which corresponds to the highest admissible stator current. The reference slip pulsation is given in Figure 3.c. In order to respect the above condition, it is necessary to drive the speed of the motor by a connected synchronous motor which is controlled to follow the speed trajectory. Simultaneously, the frequency of the voltages applied to the induction motor stator follows the stator pulsation reference shown in Figure 3. Moreover robustness tests are defined by realistic variation of stator resistance and stator inductance. This benchmark is applied on the experimental set-up located at the "Institut de Recherche en Communications et Cybernétique de Nantes" (IRCCyN, see (web, 03)).

The frequency of the voltages applied to the stator of induction motor is controlled by classical U/f control which is independent of motor measurements and estimated state. At the same time, the speed of the induction motor is controlled by the connected synchronous motor using speed measurement. Our cascade observer and the estimator use only the measurement of stator voltages and stator currents.

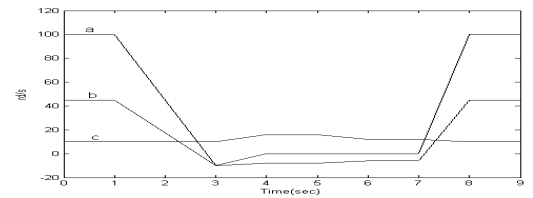


Fig. 1. Observer Benchmark trajectories : a) reference stator voltage pulsation (rd/s), b) reference speed (rd/s), c) reference slip pulsation versus time (s).

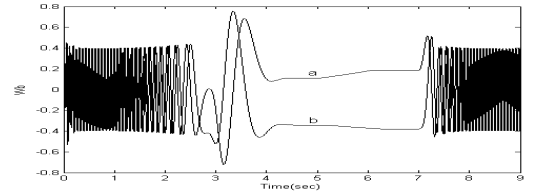


Fig. 2. Components of rotor flux : a)  $\phi_{r\alpha}$ , b)  $\phi_{r\beta}$  (Wb) versus time (s).

## 4. CASCADE OBSERVER

### 4.1 Design of the observer

In this section the design of a sensorless cascade observer for induction motor is introduced. It is well

known that there is no systematic method to design an observer for a given nonlinear control system. The induction motor model may be seen as an interconnection between several subsystems, where each of these subsystems satisfies some required properties for design an observer.

With this aim of view, the model of induction motor (1) can be rewritten in the following form :

$$\begin{pmatrix} \dot{i}_{s\alpha} \\ \dot{\Omega} \end{pmatrix} = \begin{pmatrix} 0 & bp\phi_{r\beta} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} i_{s\alpha} \\ \Omega \end{pmatrix} + \begin{pmatrix} -\gamma i_{s\alpha} + ab\phi_{r\alpha} + m_1 u_{s\alpha} \\ m(\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - c\Omega - T_l/J \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} \dot{i}_{s\beta} \\ \dot{\phi}_{r\alpha} \\ \dot{\phi}_{r\beta} \end{pmatrix} = \begin{pmatrix} -\gamma i_{s\beta} - bp\phi_{r\alpha}\Omega + ab\phi_{r\beta} + m_1 u_{s\beta} \\ aM_{sr}i_{s\alpha} - a\phi_{r\alpha} - p\Omega\phi_{r\beta} \\ aM_{sr}i_{s\beta} - a\phi_{r\beta} + p\Omega\phi_{r\alpha} \end{pmatrix}. \quad (3)$$

The above system (2) and (3) can be represented in an interconnected compact form as:

$$\begin{aligned} \dot{X}_1 &= A_1(u, y, X_2)X_1 + g_1(u, y, X_2, X_1) \\ y_1 &= C_1X_1 \end{aligned} \quad (4)$$

$$\begin{aligned} \dot{X}_2 &= A_2X_2 + g_2(u, y, X_2, X_1) \\ y_2 &= C_2X_2 \end{aligned} \quad (5)$$

where  $A_1 = \begin{pmatrix} 0 & bp\phi_{r\beta} \\ 0 & 0 \end{pmatrix}$ ,

$$g_1(u, y, X_2, X_1) = \begin{pmatrix} -\gamma i_{s\alpha} + ab\phi_{r\alpha} + m_1 u_{s\alpha} \\ m(\phi_{r\alpha} i_{s\beta} - \phi_{r\beta} i_{s\alpha}) - c\Omega - T_l/J \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -\gamma & 0 & ab \\ 0 & -a & 0 \\ -ab & 0 & -a \end{pmatrix},$$

$$g_2(u, y, X_2, X_1) = \begin{pmatrix} -bp\Omega\phi_{r\alpha} + m_1 u_{s\beta} \\ -p\Omega\phi_{r\beta} + aM_{sr}i_{s\alpha} \\ p\Omega\phi_{r\alpha} + a(M_{sr} + b)i_{s\beta} \end{pmatrix}, \text{ and}$$

$X_1 = \text{col}(x_{11}, x_{12})$  with  $x_{11} = i_{s\alpha}$ ,  $x_{12} = \Omega$ ,  $C_1 = (1 \ 0)$ ;  $X_2 = \text{col}(x_{21}, x_{22}, x_{23})$  with  $x_{21} = i_{s\beta}$ ,  $x_{22} = \phi_{r\alpha}$ ,  $x_{23} = \phi_{r\beta}$ ,  $C_2 = (1 \ 0 \ 0)$ .  $u = [u_{s\alpha}, u_{s\beta}]^T$ ,  $y = [i_{s\alpha}, i_{s\beta}]^T$ .

Our main purpose is to design an observer for subsystem (4) which is based on the interconnected approach (see (Besançon, 98)) and an estimator for subsystem (5).

In order to design cascade observer for (4), we make the following assumption:

**Assumption A1.** The variables  $u$ ,  $y$  and  $X_2$  are considered as known signals of subsystem (4).

Define  $v := [u, y, X_2]^T$ .

Suppose that the assumption above is satisfied, subsystem (4) can be written in the following form (see (Besançon, 98)):

$$\begin{aligned} \dot{X}_1 &= A_1(v)X_1 + g_1(v, X_1) \\ y_1 &= C_1X_1 \end{aligned} \quad (6)$$

To design observer for subsystem (6), we introduce the following assumptions

**Assumption A2.**

1.  $v$  is bounded and assumed to be regularly persistent (Hammouri, 90) in order to guarantee the observability property of subsystem (6).

2.  $X_1 \in D_1$  of  $\mathcal{R}^{n_1}$  and  $X_2 \in D_2$  of  $\mathcal{R}^{n_2}$ , where  $n_1$  and  $n_2$  are the dimensions of subsystems (4) and (5) respectively.

3.  $A_1(u, y, X_2)$  is globally Lipschitz with respect to  $X_2$  and uniformly with respect to  $(u, y)$ .

4.  $g_1(u, y, X_2, X_1)$  is globally Lipschitz with respect to  $X_1$  and uniformly with respect to  $(u, y, X_2)$ .

5.  $g_2(u, y, X_2, X_1)$  is globally Lipschitz with respect to  $X_2$  and uniformly with respect to  $(u, y, X_1)$ .

From A2 an observer for the above form of system (6) is given by (see (Schreier, 01)):

$$\begin{aligned} \dot{Z}_1 &= A_1(v)Z_1 + g_1(v, Z_1) + M(v)C_1(X_1 - Z_1) \\ \hat{y}_1 &= C_1Z_1 \end{aligned} \quad (7)$$

where  $Z_1 = \text{col}(z_{11}, z_{12})$  with  $z_{11} = \hat{i}_{s\alpha}$ ,  $z_{12} = \hat{\Omega}$  and the gains of the observer are given by :

$$M(v)C_1 = \Gamma^{-1}(v)\Delta_\theta^{-1}KC_1 \quad (8)$$

where  $\Gamma = \text{diag}(1, \zeta(v))$   $\Delta_\theta = \text{diag}(\frac{1}{\theta}, \frac{1}{\theta^2})$  with  $\zeta(v) = bp\hat{\phi}_{r\beta}$ ,  $\theta > 0$  and  $K = (K_1, K_2)^T$  is such that the matrix  $(\bar{A} - KC_1)$  is stable where  $\bar{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ .

From A2.1, the observability property of (6) is satisfied and thus  $\phi_{r\beta}$  is not equal to zero except for very short time (The induction motor requires to be fluxed for electromechanical energy conversion). Practically, to avoid large gain of  $\Gamma^{-1}$  a discontinuity offset at zero for  $\phi_{r\beta}$  is used in the simulation scheme.

**Remark.** The fluxes  $\phi_{r\alpha}$  and  $\phi_{r\beta}$  are not measurable, then we estimate them by an estimator of subsystem (5). The actual variables of fluxes are replaced by their estimated.

The estimator for subsystem (5) is given by the following equations:

$$\begin{aligned} \dot{Z}_2 &= A_2Z_2 + g_2(u, y, Z_2, Z_1) \\ \hat{y}_2 &= C_2Z_2 \end{aligned} \quad (9)$$

Moreover, the estimator of load torque  $T_l$  is given by the following equation

$$\begin{aligned} \hat{T}_l &= Jm(-\hat{\phi}_{r\beta}z_{11} + \hat{\phi}_{r\alpha}\hat{i}_{s\beta}) - Jcz_{12} - J\frac{d}{dt}z_{12} \\ &+ J\frac{\theta^2 K_2}{bp\hat{\phi}_{r\beta}}(x_{11} - z_{11}). \end{aligned} \quad (10)$$

with  $\frac{d}{dt}z_{12}$  is given by numerical differentiation.

**Assumption A3.** The initial conditions are known such that estimator (9) of (5) is well initialized. The induction motor parameters are known with a sufficient precision such that the errors between  $\hat{\phi}_{r\alpha}$ ,  $\hat{\phi}_{r\beta}$ , given by (9) and  $\hat{T}_l$  by (10) and respectively their real values are supposed sufficiently bounded. This assumption is totally realistic from a practical point of view: at initial conditions, all states values are null and the parameters uncertainties are known and limited.

#### 4.2 Analysis of observer stability

In this section we present the stability analysis of the proposed observer. For that, we denote the estimation errors as

$$e_1 = X_1 - Z_1; \quad e_2 = X_2 - Z_2$$

whose the errors dynamics are given by

$$\begin{aligned} \dot{e}_1 &= [A_1(u, y, Z_2) - M(u, y, Z_2)C_1]e_1 \\ &\quad + g_1(u, y, X_2, X_1) - g_1(u, y, Z_2, Z_1) \\ &\quad + [A_1(u, y, X_2) - A_1(u, y, Z_2)]X_1 \\ \dot{e}_2 &= A_2e_2 + g_2(u, y, X_2, X_1) - g_2(u, y, Z_2, Z_1). \end{aligned}$$

Taking the following change of coordinates

$$\varepsilon_1 = \Gamma(u, y, Z_2)\Delta_\theta e_1, \quad \varepsilon_2 = \frac{1}{\theta^{n_1}}e_2$$

it follows that

$$\begin{aligned} \dot{\varepsilon}_1 &= (\theta(\bar{A}_1 - KC_1))\varepsilon_1 + \Lambda_1 + \Lambda_2 + \Lambda_3 \\ \dot{\varepsilon}_2 &= A_2\varepsilon_2 + \frac{1}{\theta^{n_1}} \{g_2(u, y, X_2, X_1) - g_2(u, y, Z_2, Z_1)\} \end{aligned}$$

where

$$\begin{aligned} \Lambda_1 &= \Gamma(u, y, Z_2)\Delta_\theta(g_1(u, y, X_2, X_1) - g_1(u, y, Z_2, Z_1)), \\ \Lambda_2 &= \Gamma(u, y, Z_2)\Delta_\theta([A_1(u, y, X_2) - A_1(u, y, Z_2)]X_1), \\ \Lambda_3 &= \dot{\Gamma}(u, y, Z_2)\Gamma^{-1}(u, y, Z_2)\varepsilon_1. \end{aligned}$$

**Assumption A4.** We assume that:

$$\|\dot{\Gamma}(u, y, Z_2)\Gamma^{-1}(u, y, Z_2)\| \leq \rho,$$

where  $\dot{\Gamma}(u, y, Z_2) = \text{diag}(0, bp(aM_{sr}i_{s\beta} - a\hat{\phi}_{r\beta} + p\hat{\Omega}\hat{\phi}_{r\alpha}))$  and  $\Gamma^{-1}$  defined in (8). This assumption is justified because of the bounds of the considered variables (A2.2) and the persistent assumption A2.1.

Now, we establish the following result.

**Lemma.** If Assumptions A1, A2, A3 and A4 hold, then the system (7)-(9) is an asymptotic observer for system (6)-(5). Moreover, the convergence velocity of the estimation error  $e = (e_1, e_2)^T$  can be chosen to be as fast as the one imposed by the estimator (9) of subsystem (5).

**Proof.**

In order to prove the convergence of the observer, consider the following Lyapunov function

$$V = V_1 + V_2$$

where  $V_1 = \varepsilon_1^T P \varepsilon_1$  and  $V_2 = \varepsilon_2^T \varepsilon_2$ , and  $P = P^T > 0$  is such that  $(\bar{A}_1 - KC_1)^T P + P(\bar{A}_1 - KC_1) = -Q$ .  $Q = Q^T > 0$ .

Taking the time derivative of  $V$  along of  $\varepsilon_1$  and  $\varepsilon_2$ , we obtain

$$\begin{aligned} \dot{V} &= \theta \varepsilon_1^T (\bar{A}_1 - KC_1)^T P \varepsilon_1 + \theta \varepsilon_1^T P (\bar{A}_1 - KC_1) \varepsilon_1 \\ &\quad + 2 \varepsilon_1^T P (\Lambda_1 + \Lambda_2 + \Lambda_3) + \varepsilon_2^T \{A_2 + A_2^T\} \varepsilon_2 \\ &\quad + \frac{2}{\theta^{n_1}} \varepsilon_2^T \{g_2(u, y, X_2, X_1) - g_2(u, y, Z_2, Z_1)\} \\ &= -\theta \varepsilon_1^T Q \varepsilon_1 \\ &\quad + 2 \varepsilon_1^T P (\Lambda_1 + \Lambda_2 + \Lambda_3) + \varepsilon_2^T \{A_2 + A_2^T\} \varepsilon_2 \\ &\quad + \frac{2}{\theta^{n_1}} \varepsilon_2^T \{g_2(u, y, X_2, X_1) - g_2(u, y, Z_2, Z_1)\}. \end{aligned}$$

From assumption A2 and A4 we have

$$\begin{aligned} \dot{V} &\leq -\theta \varepsilon_1^T Q \varepsilon_1 \\ &\quad + 2 \|\varepsilon_1\| P \left\{ \frac{k_1}{\theta^{n_1}} \|\varepsilon_1\| + \frac{k_2 k_x}{\theta^{n_1}} \|\varepsilon_2\| + \rho \|\varepsilon_1\| \right\} \\ &\quad + \varepsilon_2^T \{A_2 + A_2^T\} \varepsilon_2 + \frac{2}{\theta^{n_1}} \|\varepsilon_2\| \{l_2 \|\varepsilon_2\|\} \\ &\leq -\theta \varepsilon_1^T Q \varepsilon_1 \\ &\quad + 2 \|\varepsilon_1\| P \{k_1 l_1 \|\varepsilon_1\| + k_2 k_x \|\varepsilon_2\| + \rho \|\varepsilon_1\|\} \\ &\quad + \varepsilon_2^T \{A_2 + A_2^T\} \varepsilon_2 + \frac{2}{\theta^{n_1}} \|\varepsilon_2\| \{\theta^{n_1} l_2 \|\varepsilon_2\|\}. \end{aligned}$$

Regrouping the terms of  $\|\varepsilon_1\|$  and  $\|\varepsilon_2\|$ , we get

$$\begin{aligned} \dot{V} &\leq -\theta \mu_1 \|\varepsilon_1\|_P^2 \\ &\quad + 2 \|\varepsilon_1\| P \{ (k_1 l_1 + \rho) \|\varepsilon_1\| + (k_2 k_x) \|\varepsilon_2\| \} \\ &\quad + \varepsilon_2^T \{A_2 + A_2^T\} \varepsilon_2 + 2 \|\varepsilon_2\| \{l_2 \|\varepsilon_2\|\} \end{aligned}$$

then

$$\begin{aligned} \dot{V} &\leq -(\theta \mu_1 - 2(k_1 l_1 + \rho) \|P\|) \|\varepsilon_1\|_P^2 \\ &\quad + (k_2 k_x \|P\|) \|\varepsilon_1\| \|\varepsilon_2\| \\ &\quad - (\mu_2 - 2l_2) \|\varepsilon_2\|^2 \end{aligned} \quad (11)$$

where  $\mu, \mu_1, l_1, l_2, k_1, k_2, k_x$  are positive constants chosen to satisfy Lipschitz conditions and inequalities (11) and  $\mu_2 = \min(2\gamma, 2a) > 0$ .

Now, by writing in terms of the Lyapunov function  $V_1$  and  $V_2$

$$\begin{aligned} \dot{V} &\leq -(\theta \mu_1 - 2(k_1 l_1 + \rho) \|P\|) V_1 \\ &\quad + (k_2 k_x \|P\|) k_p \sqrt{V_1} \sqrt{V_2} - (\mu_2 - 2l_2) V_2 \end{aligned}$$

or in compact form

$$\dot{V} \leq -\pi_1 V_1 + \pi_2 \sqrt{V_1} \sqrt{V_2} - \pi_3 V_2$$

where

$$\begin{aligned} \pi_1 &= (\theta \mu_1 - 2(k_1 l_1 + \rho) \|P\|) > 0, \\ \pi_2 &= (k_2 k_x \|P\|) k_p > 0, \\ \pi_3 &= (\mu_2 - 2l_2) > 0. \end{aligned}$$

Finally, with  $\alpha > 0$

$$\begin{aligned} \dot{V} &\leq -\alpha V_2 - \pi_1 V_1 + \pi_2 \sqrt{V_1} \sqrt{V_2} - (\pi_3 - \alpha) V_2 \\ &\leq -\alpha V_2 - (\pi_1 - \frac{\pi_2^2}{4(\pi_3 - \alpha)^2}) V_1 \\ &\quad - \left\{ \frac{\pi_2}{2(\pi_3 - \alpha)} \sqrt{V_1} - \sqrt{(\pi_3 - \alpha)} \sqrt{V_2} \right\}^2. \end{aligned}$$

where  $\alpha$  is such that  $(\pi_3 - \alpha) > 0$ .

Taking  $\beta = (\pi_1 - \frac{\pi_2^2}{4(\pi_3 - \alpha)^2}) > 0$ , and  $\kappa = \min(\alpha, \beta)$ , it follows that

$$\begin{aligned} \dot{V} &\leq -\kappa (V_1 + V_2) \\ &\leq -\kappa V. \end{aligned}$$

This ends the proof.

### 4.3 Compensation of bad observer behavior at low frequencies (under unobservable conditions)

In the next section, it is shown that obviously the proposed observer diverges at low frequencies when the induction motor is under unobservable conditions. When stator frequency of the induction motor is near zero, some components of cascade observer gains become very large, then the part of feedback due to the measure is important and insignificant (unobservable conditions). To avoid this bad behavior, the idea is to switch to a structure without information feedback (without error estimation gains) i.e: to an estimator when the induction motor is near and under unobservable conditions.

## 5. EXPERIMENTAL RESULTS

This section presents experimental results of proposed cascade observer and estimator which are tested on our observer benchmark. The parameters  $K_1$ ,  $K_2$ ,  $\theta$  are chosen as follows:  $K_1 = 0.1$ ,  $K_2 = 0.01$  and  $\theta = 300$  in order to satisfy convergence conditions. Experiments were carried out with the following induction motor values:  $1.5kW$  normal rate power;  $1430rpm$  nominal angular speed;  $220V$  nominal voltage;  $7.5A$  nominal current;  $n_p = 2$  number of pole pairs. The values of the nominal parameters of the motor are:  $R_s = 1.633\Omega$  stator resistance;  $R_r = 0.93\Omega$  rotor resistance;  $L_s = 0.142H$  stator self-inductance;  $L_r = 0.076H$  rotor self-inductance;  $M_{sr} = 0.099H$  mutual inductance;  $J = 0.0111/rad/s^2$  inertia (motor and load);  $f_v = 0.0018Nm/rad/s$  viscous damping coefficient. The experimental sampling time  $T$  is equal to  $200\mu s$ . In conformity with Observer Benchmark described in section 3, the experimental results have been evaluated in the following steps: - Step 1: Nominal conditions; - Step 2: Nominal conditions with observer/estimator switching; - Step 3: Stator resistance variation  $+50\%$ ; - Step 4: Stator inductance variation  $+20\%$ .

Experimental results for Step 1 are reported in Fig. 3, 4 and 5. The estimated and measured speed are shown in Fig. 3. As foreseeable estimated speed tracks good its measure under observable conditions, but the estimated speed diverges when the motor is near and under unobservable conditions. The conclusion is the same for estimated torque (see Fig. 5). The norm of flux cannot be measured on the set-up. However only estimated flux is given in Fig. 4 which diverges like estimated speed and torque near and under unobservable conditions.

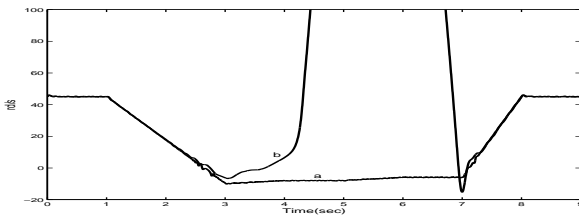


Fig. 3. Measured speed (a) and its estimation (b), (rd/s) versus time (s).

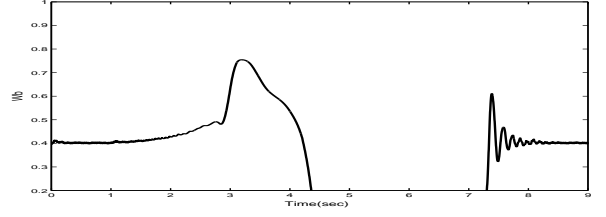


Fig. 4. Flux estimation, (Wb) versus time (s).

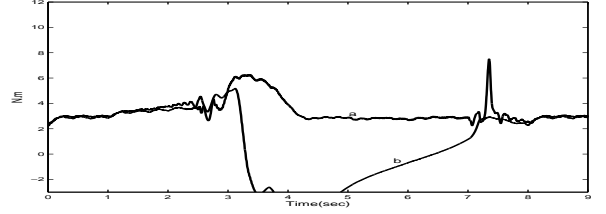


Fig. 5. Measured torque (a) and its estimation (b), (N.m) versus time (s).

### Step 1: Nominal conditions.

In Step 2 with the switch of the observer in estimator mode near and under unobservable conditions, results are depicted in Fig. 6, 7 and 8. As may be observed, the estimated speed shown in Fig. 6 is stable near and under unobservable conditions where it appears only a small static error. When the motor is under observable conditions, the speed tracking is the same than in the former case (see Fig. 3). Conclusion is identical for the estimated flux and torque (see Fig. 7 and Fig. 8).

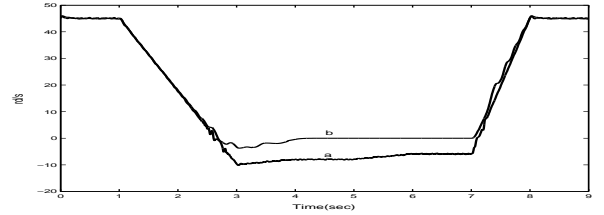


Fig. 6. Measured speed (a) and its estimation (b), (rd/s) versus time (s).

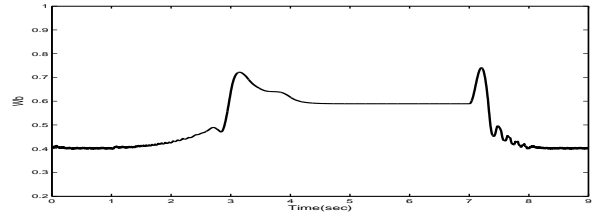


Fig. 7. Flux estimation, (Wb) versus time (s).

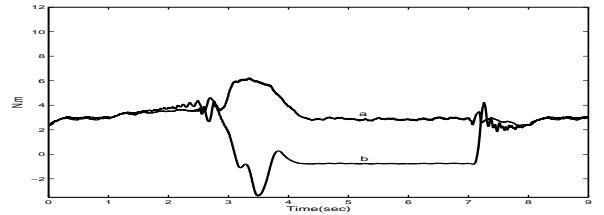


Fig. 8. Measured torque (a) and its estimation (b), (N.m) versus time (s).

### Step 2: Nominal conditions with observer/estimator switching.

Now, the interest is to verify the robustness of the observer with respect to motor parameters variations (Step 3). The comparison is made with Step 2. Results in Step 3 with resistance variation +50% are shown in Fig. 9, 10 and 11. It is noted that the estimated speed (Fig. 9) is analogous with the one in Step 2 (Fig. 6) even near and under unobservable conditions. Under observable conditions, it appears a small static error for estimated torque (Fig. 11) but estimated flux (Fig. 10) is nearly similar compared with Step 2. The results are different near and under unobservable conditions: it appears a small static error for estimated flux (Fig. 10) but estimated torque (Fig. 11) is nearly similar compared with Step 2.

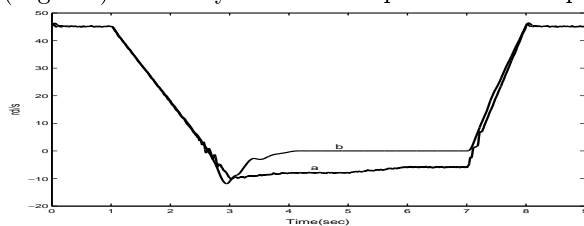


Fig. 9. Measured speed (a) and its estimation (b), (rd/s) versus time (s).

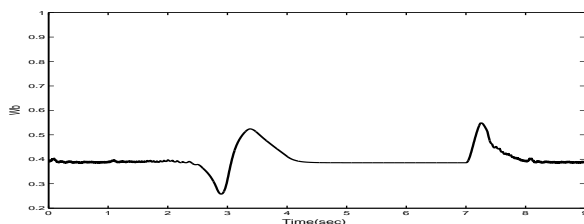


Fig. 10. Flux estimation, (Wb) versus time (s).

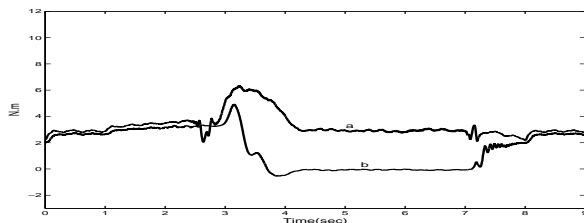


Fig. 11. Measured torque (a) and its estimation (b), (N.m) versus time (s).

Step 3: Resistance variation +50%.

The tests carried out in the step of stator inductance variation (+20%) gave also good results compared with Step 2.

## 6. CONCLUSION

In this paper, a cascade observer and an estimator to reconstruct the angular speed of sensorless induction motor and to estimate the fluxes and load torque even at low frequency conditions were proposed.

Cascade observer and estimator are tested and validated on an experimental set-up using a significant benchmark which evaluates the performances of the observer at low frequencies. Experimental results show, as it was foreseeable, that the observer diverges

near and under unobservable conditions. In order to escape this behavior, we have forced the observer to switch in estimator mode (without error estimation gains) for low frequencies conditions. The robustness of the cascade observer and the estimator is verified with respect to significant tests by stator resistance variation (+50%) and inductance variation (+20%). For the two cases, the angular speed is well estimated even at low frequencies (near and under unobservable conditions).

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