ADVANCED MODEL-BASED DIAGNOSIS OF SENSOR FAULTS IN VEHICLE DYNAMICS CONTROL SYSTEMS

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Abstract: In this paper, a model-based scheme is presented for the detection and isolation of sensor faults in the vehicle lateral dynamics control systems. Core of the scheme is the handling of model uncertainties. This scheme has been successfully tested using real driving data.

Keywords: Sensor fault diagnosis; Observer based FDI; Lateral dynamics control systems; Automotive systems

1. INTRODUCTION

The rapid development of electronic control systems like ABS (Anti-lock Breaking System), ESP (Electronic Stability Program), TCS (Traction Control System) and their wide integration in cars mark an important technological progress in the automotive industry in the past years. A central functionality of these control systems is to improve the active safety by stabilising the vehicle in extreme driving situations (Kiencke and Nielsen, 2000). As far as a critical driving situation is identified, controllers will be activated until the vehicle returns to the steady driving. As information provider for the controllers, performance of sensors embedded in ABS, ESP and TCS plays a key role in the vehicle stabilisation. To meet the demand for high dependability of the embedded sensors, fault detection and isolation (FDI) systems are integrated in the electronic control systems. They ensure an automatic early detection and isolation of all possible faults in the sensors.

Recently, it was reported that a new generation of fault diagnosis system based on model-based FDI technology has been successfully developed and integrated into ESP as a series component (Ding *et al.*, 2004). Driven by the strong demands from the practice, research on the development of advanced fault diagnosis strategies for vehicle dynamics control systems is receiving more and more attention. Following the reported results, application of advanced model-based FDI including robust/adaptive observer, parity space methods, of computational intelligent technology marks the state of the art in the research fields (Isermann, 2001).

The study reported in this paper is a part of European project Intelligent Fault Tolerant Control in Integrated Systems (IFATIS). One of the objectives of IFATIS is to establish a design framework of model-based monitoring systems, on the basis

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of advanced model-based FDI technology, for vehicle lateral dynamics control systems (Ding *et al.*, 2003). The major requirements on the design schemes are: (a) the monitoring system should deliver reliable and fault tolerant estimates of the vehicle lateral dynamics (b) it should include a modularly structured fault diagnosis system for all embedded lateral dynamics control systems.

In this paper, an FDI scheme and its application to the design of an FDI module used for detecting and isolating faults in the lateral acceleration, yaw rate and steering wheel angle sensors will be presented. These three sensors are integrated in the vehicle lateral dynamics control systems. An early and reliable detection and isolation of faults in these sensors has the highest priority and correspondingly strict technical requirements should be satisfied. Indeed, this FDI module builds the central unit of the overall fault diagnosis system and delivers also the needed information for the fault tolerant estimation modules.

2. PROBLEM FORMULATION AND PRELIMINARY WORK

2.1 System models

There are a number of mathematical models for the description of vehicle lateral dynamics (Mitschke, 1990; Kiencke and Nielsen, 2000). In this study, the well-known bicycle model is used, which is given in the state space form as follows:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta K_{\phi R}}{mv} & \frac{Y_r K_{\phi R}}{mv} - 1 \\ \frac{N_\beta}{I_z} & \frac{N_r}{I_z} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{c'_{\alpha V} K_{\phi R}}{mv} \\ \frac{l_V c'_{\alpha V}}{I_z} \end{bmatrix} \delta_L^* + \begin{bmatrix} -\frac{g}{v} \\ 0 \end{bmatrix} \sin \alpha_x \qquad (1)$$

$$Y_\beta = -\left(c'_{\alpha V} + c_{\alpha H}\right), Y_r = \left(l_H c_{\alpha H} - l_V c'_{\alpha V}\right)/v$$
$$N_\beta = l_H c_{\alpha H} - l_V c'_{\alpha V}, N_r = \left(l_V^2 c'_{\alpha V} - l_H^2 c_{\alpha H}\right)/v$$

where β denotes the side slip angle, r the yaw rate and δ_L^* the relative steering wheel angle. α_x is the unknown bank angle. The parameters used in (1) are explained in Appendix.

The decision for the use of bicycle model has been made based on a compromise between the needed on-line computation and sufficient description of system dynamics. As reported in (Ding *et al.*, 2004), the FDI systems integrated in the existing lateral dynamics control systems have often been designed on the basis of static models. Most of these models are derived from the bicycle model on the assumption that $\dot{\beta} = \dot{r} = 0$. To improve the FDI performance, it is reasonable to relax the above restriction. On the other hand, implementation of high order models will enhance the demands for on-line computation considerably. As shown in (1), the bicycle model is a system of the second order and thus the associated on-line computation for the model-based FDI system is acceptable. However, the validity conditions of model (1), for instance (lateral acceleration) $a_y \leq 4m/s^2$ or $\dot{v} \approx 0$, may lead to significant model uncertainties in some driving situations. Moreover, tyre cornering stiffness $c'_{\alpha V}, c_{\alpha H}$ may considerably vary during driving on road with low or varying friction values (Mitschke, 1990; Boerner, 2004).

Following the relationship (Mitschke, 1990)

$$K_{\phi R} a_y = v\left(\dot{\beta} + r\right) + g \sin \alpha_x$$

we have the (first) sensor model

$$\begin{bmatrix} a_y \\ r \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{m} & \frac{Y_r}{m} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{c'_{\alpha V}}{m} \\ 0 \end{bmatrix} \delta_L^* \qquad (2)$$

Note that model (1) can also be re-written into

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ \frac{c_{\alpha H}\left(l_H + l_V\right)}{I_z} & -\frac{l_H c_{\alpha H}\left(l_H + l_V\right)}{v I_z} \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{K_{\phi R}}{v} \\ \frac{l_V m}{I_z} \end{bmatrix} a_y + \begin{bmatrix} -\frac{g}{v} \\ 0 \end{bmatrix} \sin \alpha_x \qquad (3)$$

with a_y as input. Then, the associated sensor model is described by

$$\begin{bmatrix} \delta_L^* \\ r \end{bmatrix} = \begin{bmatrix} \frac{-Y_\beta}{c'_{\alpha V}} & \frac{-Y_r}{c'_{\alpha V}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \frac{m}{c'_{\alpha V}} \\ 0 \end{bmatrix} a_y \qquad (4)$$

The faults in the lateral acceleration, yaw rate and steering wheel angle sensors will be denoted by $f_{a_y}, f_r, f_{\delta_L^*}$ respectively. They are modelled, following the practical requirements, as an additive term in the associated sensor models. Both of the system models, (1) and (3), with their associated sensor models (2) and (4) will be used below.

2.2 Requirements on the FDI system

The major requirements on the FDI system are (a) low false alarm rate: *for a couple of hundreds of driving maneuvers defined in a test catalog the number* of the *false alarms should be very limited* (b) early detection: the detecting time should not exceed $0.2sec$. In addition, it is desired that the FDI system is modularly structured so that a system extension by adding additional sensors will not lead to a total re-design.

The central problem related to the design of the FDI system is to find a compromise between high fault detectability and low false alarm rate. The major difficulty for the solution is that a large number of driving maneuvers listed in the test catalog will lead the vehicle dynamics leaving the valid range of the bicycle model temporarily. Thus, handling of model uncertainties is the major focus of this study.

2.3 A brief overview of preliminary work

As mentioned above, the FDI units mounted in the existing lateral dynamics control systems were often developed on the basis of the static models with a strongly limited valid range. To achieve a low false alarm rate, an unit of identifying model validity is additionally integrated in the FDI units. In case that model invalidity is identified, the thresholds and evaluation times will be significantly increased (Ding *et al.*, 2004). The major disadvantages of this kind of FDI systems are (a) the non-transparent construction of the model validity identification unit that consists of a network of logic relationships (b) the strong dependency of threshold determination on the tests. The thresholds would be repeatedly modified by driving tests so that no false alarm exists for all driving maneuvers listed in the test catalog.

In the last years, efforts of applying advanced FDI methods to improve the performance of FDI systems have been remarkably enhanced. In (Ding *et al.*, 2003; Boerner, 2004), different adaptive FDI schemes have been developed. Their major focus is on the identification of the cornering stiffness. The resulted improvement of the FDI performance is limited. The robust control theory based handling of model uncertainties marks another research effort in this field. Norm-based robust FDI schemes with adaptive thresholds have been reported in (AKS, 2003). Since the model uncertainties are only temporarily dominant, the norm-based FDI schemes that are in some sense a worst-case handling of model uncertainties seem to be too conservative to enhance the fault detectability. Motivated by this observation, efforts have also been made to solve the FDI problems in the probabilistic framework (Schwall and Gerdes, 2002).

3. FDI SYSTEM DESIGN

3.1 Basic ideas and system structure

Fig.1 sketches the structure of the FDI system to be designed. Three observers are used for the generation of residual signals $R_{1,a_y}, R_{2,r}, R_{3,r}$. They also deliver estimates $\hat{r}, \hat{a}_y, \hat{\delta}_L^*$ respectively. In addition, three indicators of model uncertainties, $I_1\left(R_{1,a_y}\right), I_2\left(R_{2,r}\right),\ I_3\left(R_{3,r}, R_{1,r}, R_{1,a_y}\right),$

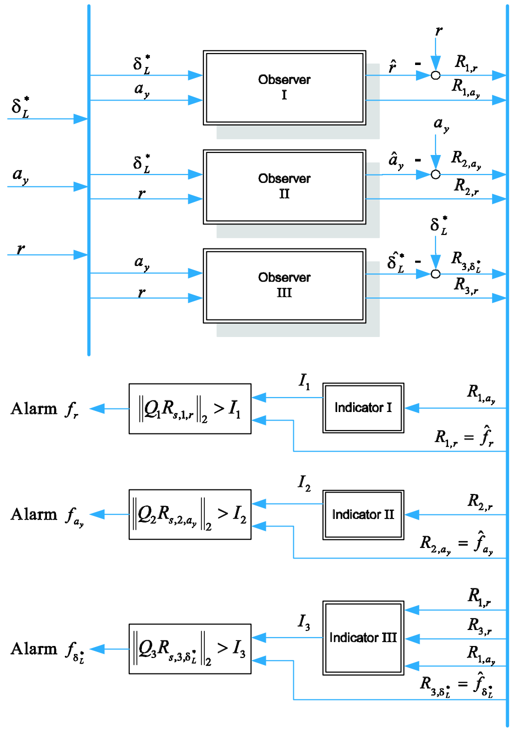

Fig. 1. Structure of the FDI system

are built, which would be, as will be shown in the sequent sub-sections, small in case of moderate model uncertainties and become considerably larger with increasing model uncertainties. The basic idea behind the FDI system in Fig.1 is to establish adaptive thresholds based on the estimates of the model uncertainties delivered by the indicators. A fault would be detected and isolated if the model uncertainties are moderate or the size of the fault is very large.

It is worth mentioning that the residual generation scheme shown in Fig.1 is similar to the so-called Generalised Observer Scheme (GOS) (Chen and Patton, 1999). The major reasons for the utilisation of this structure are (a) the system is modularly structured (b) $\hat{r}, \hat{a}_y, \hat{\delta}_L^*$ are delivered independent of r, a_y, δ_L^* respectively. Thus, this FDI system also provides a fault tolerant measurement (Ding *et al.*, 2003).

3.2 Re-modelling

The first step to the FDI system design is a re-modelling which includes a state space transformation and system discretisation. Note that in models (1) and (3), the system matrix is a function of v. In order to design an observer whose dynamics is independent of v, model (1) will first be transformed into observer canonical form. Recall that each observer integrated in the FDI system will be driven by one output, two different state transformations corresponding to different output signals are carried out, as described below:

State transformation I with a_y as output

$$z_I = T_1 \begin{bmatrix} \beta \\ r \end{bmatrix}, T_1 = \begin{bmatrix} \frac{Q_{\beta,r}}{mI_z} & \frac{-Y_\beta}{mI_z} \\ \frac{Y_\beta}{m} & \frac{Y_r}{m} \end{bmatrix}$$
$$Q_{\beta,r} = N_\beta Y_r - Y_\beta N_r$$

State transformation II with r as output

$$z_{II} = T_2 \begin{bmatrix} \beta \\ r \end{bmatrix}, T_2 = \begin{bmatrix} \frac{N_\beta}{I_z} & \frac{-Y_\beta K_{\phi R}}{mv} \\ 0 & 1 \end{bmatrix}$$

They lead, after a straightforward calculation, to

$$\dot{z}_I = \tilde{A}_1 z_I + \tilde{B}_1 \begin{bmatrix} a_y \\ \delta_L^* \end{bmatrix} + \tilde{E}_{1,w} w_I + \tilde{E}_{1,\alpha_x} \sin\alpha_x$$
$$a_y = c_1 z_I + v_I + d_1 \delta_L^*, r = c_2 z_I$$
$$\tilde{A}_1 = \begin{bmatrix} 0 & -\frac{N_\beta}{I_z} \\ 1 & 0 \end{bmatrix}, \tilde{E}_{1,\alpha_x} = T_1 \begin{bmatrix} -\frac{g}{v} \\ 0 \end{bmatrix}, d_1 = \frac{c'_{\alpha V}}{m}$$
$$\tilde{B}_1 = \begin{bmatrix} \frac{K_{\phi R} Q_{\beta,r}}{I_z mv} & -\frac{Y_\beta l_V c'_{\alpha V}}{mI_z} \\ \frac{Y_\beta K_{\phi R}}{mv} + \frac{N_r}{I_z} & \frac{c'_{\alpha V}(l_V Y_r - N_r)}{mI_z} \end{bmatrix}$$
$$\tilde{E}_{1,w} = T_1, c_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, c_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} T_1^{-1}$$

as well as

$$\dot{z}_{II} = \tilde{A}_2 z_{II} + \tilde{B}_2 \begin{bmatrix} r \\ \delta_L^* \end{bmatrix} + \tilde{E}_{2,w} w_I + \tilde{E}_{2,\alpha_x} \sin\alpha_x$$
$$r = c_3 z_{II}, a_y = c_4 z_{II} + v_I + d_1 \delta_L^*$$
$$\tilde{A}_2 = \begin{bmatrix} 0 & -\frac{N_\beta}{I_z} \\ 1 & 0 \end{bmatrix}, \tilde{E}_{2,\alpha_x} = T_2 \begin{bmatrix} -\frac{g}{v} \\ 0 \end{bmatrix}$$
$$\tilde{B}_2 = \begin{bmatrix} \frac{K_{\phi R} Q_{\beta,r}}{I_z mv} & \frac{K_{\phi R} c'_{\alpha V}(N_\beta - Y_\beta l_V)}{I_z mv} \\ \frac{Y_\beta K_{\phi R}}{mv} + \frac{N_r}{I_z} & \frac{l_V c'_{\alpha V}}{I_z} \end{bmatrix}$$
$$\tilde{E}_{2,w} = T_2, c_3 = \begin{bmatrix} 0 & 1 \end{bmatrix}, c_4 = \begin{bmatrix} \frac{Y_\beta}{m} & \frac{Yr}{m} \end{bmatrix} T_2^{-1}$$

Moreover, model (3) is re-written into

$$\dot{x} = \tilde{A}_3 x + \tilde{B}_3 \begin{bmatrix} r \\ a_y \end{bmatrix} + E_{3,w} w_{II} + E_{3,\alpha_x} \sin\alpha_x$$
$$r = c_5 x, \delta_L^* = c_6 x + v_{II} + d_2 a_y$$
$$x = \begin{bmatrix} \beta \\ r \end{bmatrix}, \tilde{A}_3 = \begin{bmatrix} 0 & -1 \\ \frac{c_{\alpha H}(l_H + l_V)}{I_z} & 0 \end{bmatrix}, E_{3,w} = I$$
$$\tilde{B}_3 = \begin{bmatrix} 0 & \frac{K_{\phi R}}{v} \\ -\frac{l_H c_{\alpha H}(l_H + l_V)}{vI_z} & \frac{l_V m}{I_z} \end{bmatrix}, E_{3,\alpha_x} = \begin{bmatrix} -\frac{g}{v} \\ 0 \end{bmatrix}$$
$$c_5 = \begin{bmatrix} 0 & 1 \end{bmatrix}, c_6 = \begin{bmatrix} \frac{-Y_\beta}{c'_{\alpha V}} & \frac{-Y_r}{c'_{\alpha V}} \end{bmatrix}, d_2 = \frac{m}{c'_{\alpha V}}$$

In the above models, $w_I, w_{II} \in \mathbf{R}^2, v_I, v_{II} \in \mathbf{R}$ are unknown and represent the model uncertainties caused by the changes in the original system parameters in the state space and output equations, respectively. These changes can be, for instance, caused by varying cornering stiffness $c'_{\alpha V}, c_{\alpha H}$, varying m, v as well as some dynamic effects which have been neglected by the bicycle model (Mitschke, 1990).

For the on-line implementation, the above three models are now discretised by means of the zero-order-hold. The sampling time is $10ms$. As a result, we have:

discrete-time model I

$$z_I(k+1) = A_1 z_I(k) + B_1 \begin{bmatrix} a_y(k) \\ \delta_L^*(k) \end{bmatrix} + E_{1,w} w_I(k)$$
$$+ E_{1,\alpha_x} \sin\alpha_x(k), r(k) = c_2 z_I(k) \quad (5)$$
$$a_y(k) = c_1 z_I(k) + v_I(k) + d_1 \delta_L^*(k) \quad (6)$$

discrete-time model II

$$z_{II}(k+1) = A_2 z_{II}(k) + B_2 \begin{bmatrix} r(k) \\ \delta_L^*(k) \end{bmatrix} + E_{2,w} w_I(k)$$
$$+ E_{2,\alpha_x} \sin\alpha_x, r(k) = c_3 z_{II}(k) \quad (7)$$
$$a_y(k) = c_4 z_{II}(k) + v_I(k) + d_1 \delta_L^*(k) \quad (8)$$

discrete-time model III

$$x(k+1) = A_3 x(k) + B_3 \begin{bmatrix} r(k) \\ a_y(k) \end{bmatrix} + E_{3,w} w_{II}(k)$$
$$+ E_{3,\alpha_x} \sin\alpha_x(k), r(k) = c_5 x(k) \quad (9)$$
$$\delta_L^*(k) = c_6 x(k) + v_{II}(k) + d_2 a_y(k) \quad (10)$$

3.3 Observer design

As shown in Fig.1, three observers are used for the purpose of residual generation. Based on models (5)-(10), these observers are designed by means of the standard pole assignment method.

Observer I: It is constructed as follows

$$\hat{z}_I(k+1) = A_1 \hat{z}_I(k) + B_1 \begin{bmatrix} a_y(k) \\ \delta_L^*(k) \end{bmatrix} + L_1 R_{1,a_y}(k)$$
$$\hat{a}_y(k) = c_1 \hat{z}_I(k) + d_1 \delta_L^*(k), \hat{r}(k) = c_2 \hat{z}_I(k)$$
$$R_{1,a_y}(k) = a_y(k) - \hat{a}_y(k), R_{1,r}(k) = r(k) - \hat{r}(k)$$

Define $e_I(k) = z_I(k) - \hat{z}_I(k)$. Then, the dynamics of the residual generator is governed by

$$e_I(k+1) = \bar{A}_1 e_I(k) + \bar{w}_I(k) - L_1 v_I(k) \quad (11)$$
$$\begin{bmatrix} R_{1,a_y}(k) \\ R_{1,r}(k) \end{bmatrix} = \begin{bmatrix} c_1 e_I(k) + v_I(k) \\ c_2 e_I(k) + f_r \end{bmatrix} \quad (12)$$

with $\bar{A}_1 = A_1 - L_1 c_1, \bar{w}_I(k) = E_{1,w} w_I(k) + E_{1,\alpha_x} \sin\alpha_x(k)$.

Observer II: Similar to Observer I**,** it is constructed as follows

$$\hat{z}_{II}(k+1) = A_2 \hat{z}_{II}(k) + B_2 \begin{bmatrix} r(k) \\ \delta_L^*(k) \end{bmatrix} + L_2 R_{2,r}(k)$$

$$\hat{r}(k) = c_3\hat{z}_{II}(k), \hat{a}_y(k) = c_4\hat{z}_{II}(k) + v_I(k) + d_1\delta_L^*$$
$$R_{2,r}(k) = r(k) - \hat{r}(k), R_{2,a_y}(k) = a_y(k) - \hat{a}_y(k)$$

Again, by defining $e_{II}(k) = z_{II}(k) - \hat{z}_{II}(k)$ the dynamics of the residual generator is described by

$$e_{II}(k+1) = \bar{A}_2 e_{II}(k) + \bar{w}_{II}(k) - L_2 f_r \quad (13)$$
$$\begin{bmatrix} R_{2,r}(k) \\ R_{2,a_y}(k) \end{bmatrix} = \begin{bmatrix} c_3 e_{II}(k) + f_r \\ c_4 e_{II}(k) + \bar{v}_I(k) + f_{a_y} \end{bmatrix} (14)$$

with $\bar{A}_2 = A_1 - L_2 c_2, \bar{w}_{II}(k) = E_{2,w} w_I(k) + E_{2,\alpha_x} \sin\alpha_x(k)$, $\bar{v}_I(k) = v_I(k) - f_{a_y}$.

Observer III: It is constructed as follows

$$\hat{x}(k+1) = A_3\hat{x}(k) + B_3 \begin{bmatrix} r(k) \\ a_y(k) \end{bmatrix} + L_3 R_{3,r}(k)$$
$$\hat{\delta}_L^*(k) = c_6\hat{x}(k) + d_2 a_y(k), \hat{r}(k) = c_5\hat{x}(k)$$
$$R_{3,r}(k) = r(k) - \hat{r}(k), R_{3,\delta_L^*}(k) = \delta_L^*(k) - \hat{\delta}_L^*(k)$$

and, by defining $e_{III}(k) = x(k) - \hat{x}(k)$ and $\bar{A}_3 = A_3 - L_3 c_5$, the dynamics of the residual generator is governed by

$$e_{III}(k+1) = \bar{A}_3 e_{III}(k) + \bar{w}_{II}(k) - L_3 f_r (15)$$
$$\begin{bmatrix} R_{3,r} \\ R_{3,\delta_L^*} \end{bmatrix} = \begin{bmatrix} c_5 e_{III}(k) + f_r \\ c_6 e_{III}(k) + \bar{v}_{II}(k) + f_{\delta_L^*} \end{bmatrix} \quad (16)$$

with $\bar{w}_{II}(k) = E_{3,w} w_{II}(k) + E_{3,\alpha_x} \sin\alpha_x(k)$, $\bar{v}_{II}(k) = v_{II} - f_{\delta_L^*}$.

As shown above, each observer leads to the generation of two residual signals. These will be used for different purposes. Residual signals $R_{1,a_y}(k), R_{2,r}(k), R_{3,r}(k)$ are called *Control Signals* and used to estimate the influences of the model uncertainties including the bank angle and the failed sensors on $R_{1,r}(k), R_{2,a_y}(k), R_{3,\delta_L^*}(k)$, respectively. The latter three residual signals are called *Detection Signals* which will be used to indicate the sensor faults.

3.4 Uncertainties indicators and detection logic

In this sub-section, an approach to the design of model uncertainties indicators and its application will be presented. Consider system model

$$e(k+1) = (A - Lc_1)\, e(k) + w(k) - Lv(k) \in \mathbf{R}^2$$
$$r_1(k) = c_1 e(k) + v(k), r_2(k) = c_2 e(k) + f(k) \in \mathbf{R}$$

where $w(k), v(k)$ are unknown and represent the model uncertainties, $r_1(k), r_2(k)$ represent control and detection signals, respectively. After a straightforward calculation, it turns out

$$r_{s,1}(k) = \Gamma_1 d(k), r_{s,2}(k) = \Gamma_2 d(k) + f_s(k) \quad (17)$$
$$r_{s,i}(k) = \begin{bmatrix} r_i(k-s) \\ \vdots \\ r_i(k) \end{bmatrix}, f_s(k) = \begin{bmatrix} f(k-s) \\ \vdots \\ f(k) \end{bmatrix}$$
$$d(k) = \begin{bmatrix} e^T(k-s) & \phi^T(k-s) & \cdots & \phi^T(k) \end{bmatrix}^T$$
$$\phi^T(k) = \begin{bmatrix} w^T(k) & v(k) \end{bmatrix}, i = 1, 2$$

with s being the length of the evaluation window,

$$\Gamma_1 = \begin{bmatrix} c_1 & 0\ 1 & & & \\ c_1\bar{A} & c_1\ -c_1L & 0\ 1 & & \\ \vdots & \vdots & \ddots & \ddots & \\ c_1\bar{A}^s & c_1\bar{A}^{s-1} & -c_1\bar{A}^{s-1}L & \cdots & 0\ 1 \end{bmatrix}$$
$$\Gamma_2 = \begin{bmatrix} c_2 & 0 & & & \\ c_2\bar{A} & c_2\ -c_2L & 0 & & \\ \vdots & \vdots & \ddots & \ddots & \\ c_2\bar{A}^s & c_2\bar{A}^{s-1} & -c_2\bar{A}^{s-1}L & \cdots & 0 \end{bmatrix} \quad (18)$$

and $\bar{A} = A - Lc_1$. Suppose that Γ_1 has the full row rank and denote $\Gamma_1^+ = \Gamma_1^T \left(\Gamma_1\Gamma_1^T\right)^{-1}$. Then, $d(k)$ can be written into

$$d(k) = \Gamma_1^+ r_{s,1}(k) + \left(I - \Gamma_1^+\Gamma_1\right) z$$

with some (unknown) vector z. Substituting it into $r_{s,2}(k) = \Gamma_2 d(k) + f_s(k)$ leads to

$$r_{s,2}(k) = \Gamma_2\Gamma_1^+ r_{s,1}(k) + \Gamma_2\left(I - \Gamma_1^+\Gamma_1\right) z + f_s(k) \quad (19)$$

Since z is unknown, it is reasonable to reduce the influence of the second term in (19) on $r_{s,2}(k)$ and at the same time to enhance the influence of $f_s(k)$. To this end, introduce matrix Q and solve the following optimisation approach

$$\min_Q \frac{\sigma_{\max}\left(\frac{\partial(Qr_{s,2}(k))}{\partial z}\right)}{\sigma_i\left(\frac{\partial(Qr_{s,2}(k))}{\partial f_s}\right)}, i = 1, \cdots, s+1 \quad (20)$$

where $\sigma(\cdot)$ denotes the singular value of some matrix. As shown in (Ding *et al.*, 1998), optimisation problem (20) can be solved in two steps: (a) do an SVD on $\Gamma_2\left(I - \Gamma_1^+\Gamma_1\right)$:

$$\Gamma_2\left(I - \Gamma_1^+\Gamma_1\right) = U\Sigma V, \Sigma = \begin{bmatrix} \Sigma_1 & 0 \end{bmatrix}$$
$$\Sigma_1 = diag(\sigma_1, \cdots, \sigma_{s+1}), \sigma_1 \le \cdots \le \sigma_{s+1}$$

(b) compute $Q = \left(U\Sigma_1^{1/2}\right)^{-1}$, which gives the optimum solution. Let

$$\|Qr_{s,2}(k)\|_2 = \sqrt{r_{s,2}(k)^T Q^T Q r_{s,2}(k)}$$

be the residual evaluation function, then the model uncertainties indicator is defined as

$$I = \left\|Q\Gamma_2\Gamma_1^+ r_{s,1}(k)\right\|_2 + \sigma_1^{-1/2}\,|f_{\min}| \quad (21)$$

where, taking into account the possibly remaining influence of z, an additional term $f_{\min}$ that denotes the (given) minimum detectable fault defined according to the technical requirements is added to the indicator. In the following, the above approach will be applied for the design of different model uncertainties indicators. Extension will be made as needed.

Design of indicator I: Indicator I is used to estimate the influence of the model uncertainties on the detection signal $R_{1,r}$. To this end, using $R_{1,r}$ as detection and R_{1,a_y} as control signals, the indicator is constructed as follows:

$$I_1 = \left\| Q_1 \Gamma_{r,1} \Gamma_{a_y,1}^{+} R_{s,1,a_y}(k) \right\| + \sigma_{1,1}^{-1/2} \left| f_{r,\min} \right|$$

$$Q_1 = \left(U_1 \Sigma_1^{1/2} \right)^{-1}, R_{s,1,a_y}(k) = \begin{bmatrix} R_{1,a_y}(k-3) \\ \vdots \\ R_{1,a_y}(k) \end{bmatrix}$$

$$\Gamma_{a_y,1}^{+} = \Gamma_{a_y,1}^{T} \left(\Gamma_{a_y,1} \Gamma_{a_y,1}^{T} \right)^{-1}$$

$$\Gamma_{a_y,1} = \begin{bmatrix} c_1 & 0 \; 1 & & \\ c_1\bar{A}_1 & c_1 \; -c_1 L_1 & 0 \; 1 & \\ \vdots & \vdots & \ddots & \ddots \\ c_1\bar{A}_1^3 & c_1\bar{A}_1^2 \; -c_1\bar{A}_1^2 L_1 & \cdots & 0 \; 1 \end{bmatrix}$$

$$\Gamma_{r,1} = \begin{bmatrix} c_2 & 0 & & \\ c_2\bar{A}_1 & c_2 \; -c_2 L_1 & 0 & \\ \vdots & \vdots & \ddots & \ddots \\ c_2\bar{A}_1^3 & c_2\bar{A}_1^2 \; -c_2\bar{A}_1^2 L_1 & \cdots & 0 \end{bmatrix}$$

$$\Gamma_{r,1} \left(I - \Gamma_{a_y,1}^{+} \Gamma_{a_y,1}^{T} \right) = U_1 \left[\Sigma_1 \; 0 \right] V_1,$$
$$\Sigma_1 = diag(\sigma_{1,1}, \cdots, \sigma_{1,s+1}), \sigma_{1,1} \leq \cdots \leq \sigma_{1,s+1}$$

As a result, the detection logic is set to be

$$\left\| Q_1 R_{s,1,r} \right\|_2 > I_1 \Rightarrow f_r \neq 0$$

Design of indicator II: Indicator II is used to estimate the influence of the model uncertainties on the detection signal R_{2,a_y}. Note that in the output equation of R_{2,a_y}, (14), an additional term $\bar{v}_I(k)$ exists. On the other side, it follows from (1) and (2) that $\bar{v}_I(k)$ will not be affected by α_x and for $\alpha_x = 0$

$$\bar{v}_I(k) = \begin{bmatrix} \frac{v}{K_{\phi R}} & 0 \end{bmatrix} w_I(k)$$
$$= \begin{bmatrix} \frac{v}{K_{\phi R}} & 0 \end{bmatrix} T_2^{-1} \bar{w}_{II}(k) := E_{II} \bar{w}_{II}(k)$$

as well as for $\alpha_x \neq 0$

$$\left| \bar{v}_I(k) \right| \leq \left| E_{II} \bar{w}_{II}(k) \right|$$

Thus, analogous to indicator I, it is reasonable to construct indicator II as follows

$$I_2 = \left\| Q_2 \Gamma_{a_y,2} \Gamma_{r,2}^{+} R_{s,2,r}(k) \right\|_2 + \sigma_{2,1}^{-1/2} \left| f_{a_y,\min} \right|$$

where the definitions of $\Gamma_{r,2}^{+}, Q_2, R_{s,2,r}(k), \sigma_{2,1}$ are analogous to $\Gamma_{a_y,1}^{+}, Q_1, R_{s,1,a_y}(k), \sigma_{1,1}$, respectively, and

$$\Gamma_{a_y,2} = \begin{bmatrix} c_4 & E_{II} & & & \\ c_4\bar{A}_2 & c_4 \; -c_4 L_2 & \ddots & & \\ \vdots & \vdots & \ddots & & \\ c_4\bar{A}_2^3 & c_4\bar{A}_2^3 \; -c_4\bar{A}_2^3 L_2 & \cdots & E_{II} \end{bmatrix}$$

Note that $\Gamma_{a_y,2}$ is slightly different from Γ_2 given in (18) due to the (equivalent) direct influence of $\bar{w}_{II}(k)$ on the residual signal. As a result, the detection logic is defined as

$$\left\| Q_2 R_{s,2,a_y} \right\|_2 > I_2 \Rightarrow f_{a_y} \neq 0$$

Design of indicator III: Indicator III is used to estimate the influence of the model uncertainties on the detection signal R_{3,δ_L^*}. Due to the existence of $\bar{v}_{II}(k)$ in (16), an estimate of $\bar{v}_{II}(k)$ is, in addition to the estimate of the influence of model uncertainties on $c_6 e_{III}(k)$, necessary. Note that

$$c'_{\alpha V} \delta_L^* + Y_\beta \beta + Y_r r = m a_y$$

Hence for $f_{\delta_L^*} = 0$

$$\bar{v}_{II}(k) = -\frac{m}{c'_{\alpha V}} v_I(k)$$

This indicates that an estimate of $\bar{v}_{II}(k)$ can be achieved by an estimate of $v_I(k)$ if the latter is decoupled from $f_{\delta_L^*}$. To this end, let

$$\tilde{w}_I(k) = E_{1,w} \left(w_I(k) - \begin{bmatrix} \frac{c'_{\alpha V} K_{\phi R}}{\frac{mv}{l_V c'_{\alpha V}}} \\ I_z \end{bmatrix} f_{\delta_L^*} \right)$$
$$\tilde{v}_I(k) = v_I(k) - \frac{m}{c'_{\alpha V}} f_{\delta_L^*}$$

(11)-(12) can be re-written into

$$e_I(k+1) = \bar{A}_1 e_I(k) + \tilde{w}_I(k) + \bar{b}_1 f_{\delta_L^*}$$
$$\begin{bmatrix} R_{1,a_y}(k) \\ R_{1,r}(k) \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e_I(k) + \begin{bmatrix} \tilde{v}_I(k) \\ f_r(k) \end{bmatrix} + \begin{bmatrix} \frac{m}{c'_{\alpha V}} \\ 0 \end{bmatrix} f_{\delta_L^*}$$
$$\bar{b}_1 = E_{1,w} \begin{bmatrix} \frac{c'_{\alpha V} K_{\phi R}}{\frac{mv}{l_V c'_{\alpha V}}} \\ I_z \end{bmatrix} - L_1 \frac{m}{c'_{\alpha V}}$$

Note that $\tilde{w}_I(k), \tilde{v}_I(k)$ are independent of $f_{\delta_L^*}$. Thus, for $s = 3$, we can find a vector η so that

$$\eta \begin{bmatrix} \Gamma_I & G_{f_{\delta_L^*}} \end{bmatrix} = 0, C_I = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, E_I = \begin{bmatrix} \frac{m}{c'_{\alpha V}} \\ 0 \end{bmatrix}$$

$$\Gamma_I = \begin{bmatrix} C_I \\ \vdots \\ C_I \bar{A}_1^3 \end{bmatrix}, G_{f_{\delta_L^*}} = \begin{bmatrix} E_I & 0 & & \\ C_I \bar{b}_1 & E_I & \ddots & \\ \vdots & \ddots & \ddots & 0 \\ C_I \bar{A}_1^3 \bar{b}_1 & \cdots & C_I \bar{b}_1 & E_I \end{bmatrix}$$

which leads to

$$\eta R_{s,r,a_y}(k) = \eta \Psi \vartheta(k), \hat{v}_I(k) = \begin{bmatrix} \tilde{v}_I(k) \\ f_r(k) \end{bmatrix} \quad (22)$$

$$R_{s,r,a_y} = \begin{bmatrix} R_{1,a_y}(k-3) \\ R_{1,r}(k-3) \\ \vdots \\ R_{1,a_y}(k) \\ R_{1,r}(k) \end{bmatrix}, \vartheta(k) = \begin{bmatrix} \tilde{w}_I(k-3) \\ \hat{v}_I(k-3) \\ \vdots \\ \tilde{w}_I(k) \\ \hat{v}_I(k) \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0 \ I & & & \\ C_I \ 0 & 0 \ I & & \\ \vdots & \ddots & \ddots & \\ C_I \bar{A}_1^2 \ 0 & \cdots & C_I \ 0 & 0 \ I \end{bmatrix}$$

Note that $\vartheta(k)$ can be written as

$$\vartheta(k) = \Gamma_{\Psi}^{+} \eta R_{s,r,a_y}(k) + \left(I - \Gamma_{\Psi}^{+} \Gamma_{\Psi}\right) z \quad (23)$$
$$\Gamma_{\Psi} = \eta \Psi, \Gamma_{\Psi}^{+} = \Gamma_{\Psi}^{T} \left(\Gamma_{\Psi} \Gamma_{\Psi}^{T}\right)^{-1}$$

for some (unknown) z. It then turns out

$$\begin{bmatrix} \tilde{v}_I(k-s) \\ \vdots \\ \tilde{v}_I(k) \end{bmatrix} = V\left(\Gamma_{\Psi}^{+} \eta R_{s,r,a_y} + \left(I - \Gamma_{\Psi}^{+} \Gamma_{\Psi}\right) z\right)$$
$$V = diag\left(0 \ I \ 0, \cdots, 0 \ I \ 0\right)$$

On the other hand, it follows from (15)-(16) that the influence of $w_{II}(k) - L_3 f_r$ on $c_6 e_{III}(k)$ can be well estimated by $\left\| Q_3 \Gamma_{\delta_L^*,3} \Gamma_{r,3}^{+} R_{s,3,r}(k) \right\|_2$, where the definitions of $\Gamma_{\delta_L^*,3}, \Gamma_{r,3}^{+}, Q_3, R_{s,3,r}(k)$ are analogous to $\Gamma_{r,1}, \Gamma_{a_y,1}^{+}, Q_1, R_{s,1,a_y}(k)$, respectively. Combining it with (23), the indicator III is finally defined as

$$I_3 = \left\| Q_3 \Gamma_{\delta_L^*,3} \Gamma_{r,3}^{+} R_{s,3,r}(k) \right\|_2 + \left\| Q_3 V \Gamma_{\Psi}^{+} \eta R_{s,r,a_y} \right\|_2 + \sigma_{3,1}^{-1/2} \left| f_{\delta_L^*,\min} \right|$$

As a result, the detection logic is set to be

$$\left\| Q_3 R_{s,3,\delta_L^*} \right\|_2 > I_3 \Rightarrow f_{\delta_L^*} \neq 0$$

4. IMPLEMENTATION AND TEST RESULTS

The design scheme presented in the last section is applied to the design of an FDI system for the sensors mounted in a test car. To test the performance of this FDI system, real data collected during driving tests undertaken by the test car are used. Below is a summary of the major design and test procedure as well as the achieved results.

Identification of the model parameters of the test car: In vehicle model (1), besides of the construction data of the car, $c'_{\alpha V}, c_{\alpha H}, K_{\Phi R}$, have to be identified. It is realised using the data collected during circle driving with constant velocity, a standard programme for this purpose.

Re-modelling: As described in sub-section 3.2, a re-modelling is carried out. The systems matrices of the discrete-time models (5)-(10) are as follows:

$$A_1 = \begin{bmatrix} 0.998 & -0.451 \\ 0.010 & 0.998 \end{bmatrix}, A_3 = \begin{bmatrix} 0.992 & -0.010 \\ 1.648 & 0.992 \end{bmatrix}$$

$$A_2 = A_1, B_1 = \begin{bmatrix} \frac{0.727v - 218.3}{v^2} & \frac{57.7v - 27.67}{v} \\ -\frac{3.22v + 1.092}{v^2} & \frac{0.29v + 122.6}{v} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \frac{0.727v - 218.3}{v^2} & -\frac{0.085v + 70.65}{v} \\ -\frac{3.22v + 1.092}{v^2} & \frac{0.375v + 0.353}{v} \end{bmatrix}$$

$$B_3 = \begin{bmatrix} \frac{0.013}{v} & \frac{-0.009}{v} \\ \frac{-2.55}{v} & \frac{0.008v + 0.007}{v} \end{bmatrix}$$

Observer design: The three observers are designed by means of the pole assignment method. The poles for the three observers (11), (13) and (15) are (0.94, 0.95), (0.93, 0.95) and (0.95, 0.96), respectively. The associated observer gains are

$$L_1 = \begin{bmatrix} -0.170 \\ 0.024 \end{bmatrix}, L_2 = \begin{bmatrix} -0.127 \\ 0.115 \end{bmatrix}, L_3 = \begin{bmatrix} -0.009 \\ 0.073 \end{bmatrix}$$

Design of model uncertainties indicators: As described in sub-section 3.4, three indicators are designed. Note that the indicators depend on v. The associated matrices given below are calculated for $v = 30m/s$, corresponding to the example shown below. For the on-line implementation, a table with the associated matrices in relationship with different velocities is used.

$$\sigma_{1,1} = 0.02, f_{r,\min} = 0.05$$

$$Q_1 = \begin{bmatrix} -1.0686 & -1.8627 & -2.4069 & -2.7127 \\ 2.9836 & 2.9540 & -0.0554 & -3.1546 \\ -3.9494 & 1.3671 & 3.5324 & -2.5173 \\ -2.7854 & 4.3027 & -3.7389 & 1.4602 \end{bmatrix}$$

$$\Gamma_{r,1} \Gamma_{a_y,1}^{+} = \begin{bmatrix} -0.0208 & -0.0080 & -0.0031 & -0.0014 \\ -0.0059 & -0.0245 & -0.0097 & -0.0045 \\ -0.0027 & -0.0081 & -0.0261 & -0.0123 \\ -0.0016 & -0.0048 & -0.0110 & -0.0326 \end{bmatrix}$$

$$\sigma_{2,1} = 2.2, f_{a_y,\min} = 1.5$$

$$Q_2 = \begin{bmatrix} -0.0621 & -0.1344 & -0.1887 & -0.2176 \\ 0.2680 & 0.3242 & 0.0268 & -0.2999 \\ -0.3979 & 0.0713 & 0.3636 & -0.2459 \\ -0.3146 & 0.4077 & -0.3365 & 0.1296 \end{bmatrix}$$

$$\Gamma_{a_y,2} \Gamma_{r,2}^{+} = \begin{bmatrix} 18.6140 & -0.0367 & -0.0031 & -0.0261 \\ 1.0900 & 18.6598 & -0.0407 & -0.0601 \\ 1.0876 & 1.1359 & 18.6559 & -0.1276 \\ 1.0854 & 1.1336 & 1.1322 & 18.5095 \end{bmatrix}$$

$$\sigma_{3,1} = 0.86, f_{\delta_L^*,\min} = 0.25$$

$$Q_3 = \begin{bmatrix} -0.0019 & -0.0088 & -0.0380 & 0.4521 \\ 0.3952 & 0.6150 & 0.5615 & 0.0608 \\ -0.7192 & -0.1651 & 0.6814 & 0.0510 \\ 0.5969 & -0.7856 & 0.4379 & 0.0241 \end{bmatrix}$$

$$\Gamma_{\delta_L^*,3}\Gamma_{r,3}^{+} = \begin{bmatrix} -2.2980 & 2.5899 & -0.1274 & -0.0453 \\ 0.1557 & -2.4255 & 2.5858 & -0.2139 \\ 0.0417 & 0.1517 & -2.3170 & 2.2225 \\ -0.0646 & -0.2348 & -1.0439 & 1.4331 \end{bmatrix}$$

Tests: The test car undertook more than 600 driving tests on different roads and with different driving maneuverers. Most of these driving maneuverers are critical ones. The main test results can be summarised as follows:

All three kinds of sensor faults with different sizes could be detected and isolated. The false alarm rates for the yaw rate, lateral acceleration and steering wheel angle sensors are about $0,4\%, 0,1\%$ and $2,7\%$,respectively.

Due to the space limitation, the achieved results cannot be presented here in more details. The readers are referred to the report available at the website of the first author (publications) under http://aks.uni-duisburg.de, in which the test results are described in details and compared with a number of other FDI methods .

5. CONCLUDING REMARKS

In this paper, an advanced observer-based scheme has been developed for the detection and isolation of sensor faults in the lateral dynamics control systems. To meet the demands on a reliable FDI, a low false alarm rate and modularisation of the system structure, three observers have been designed for residual generation. In addition, three model uncertainties indicators have been developed and integrated in the FDI system. They ensure both a (satisfactorily) low false alarm rate and a high fault detectability. The developed FDI scheme has been successfully tested using real driving data. It should be pointed out that the developed FDI system is only a part of a fault tolerant monitoring system for the lateral dynamics control systems. It provides not only knowledge of a failed sensor but also an estimate for the failed sensor signal. The integration of this FDI system into the fault tolerant monitoring system is the major focus in the last project phase. Due to the space limitation, a more detailed description of the design scheme and the background of this study is not possible. Also, a great number of useful and significant publications cannot be included in References.

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Appendix: Nomenclature of the vehicle model

	Unit	Explanation
g	$[m/s^2]$	gravity constant
m	$[kg]$	total mass
l_V	$[m]$	dist. from the CG to the front axle
l_H	$[m]$	dist. from the CG to the rear axle
I_z	$[kg\,m^2]$	moment of inertia about the z-axis
$c'_{\alpha V}$	$[N/rad]$	front tire cornering stiffn.
$c_{\alpha H}$	$[N/rad]$	rear tire cornering stiffn.
$K_{\Phi R}$	-	compensation constant of rolling dynamics
v	$[m/s]$	vehicle longitude velocity
a_y	$[m/s^2]$	lateral acceleration
β	$[rad]$	vehicle side slip angle
r	$[rad/s]$	vehicle yaw rate
δ_L^*	$[rad]$	vehicle steering angle
α_x	$[rad]$	road bank angle