

# INTERVAL OBSERVERS FOR NON MONOTONE SYSTEMS. APPLICATION TO BIOPROCESS MODELS

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Abstract: We develop an interval observer working in closed loop, for the state variables estimation of systems whose modelling is uncertain, as it is generally the case for bioprocess models. The considered class of systems consists in a linear component and a nonlinear part. The nonlinear part contains the uncertainty but does not necessarily fulfil the monotonicity conditions required to design an interval observer. We rewrite this term as the sum of an increasing and of a decreasing function, and then propose a closed loop interval observer whose error dynamics is a cooperative system. A bundle of observers is then generated by appropriately varying the observer gains. A reinitialisation procedure allows to improve the interval estimation and convergence rate. The observer performance is illustrated with a two dimensional model for a wastewater treatment process.

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## 1. INTRODUCTION

Biotechnological processes play an important role in the nowadays industry. Applications of biotechnology can be found *e.g.* in food industry, pharmaceutical, depollution processes (Stephanopoulos *et al.*, 1998). In contrast with other kinds of processes that are perfectly described by physical laws -like mechanical or electrical systems- biotechnological processes are dealing with living organism. As a consequence, their mathematical models are uncertain and are known to have a lower aptitude to accurately match experimental results.

On the other hand, online monitoring of biotechnological processes is a very difficult task. The general difficulty to measure chemical and biological variables is one of the main limitations in the improvement of monitoring and optimisation

of bioreactors (Bernard and Gouzé, 2002). The lack of hardware sensors to perform monitoring tasks has forced the implementation of complicated, and not reliable methods. This problem becomes of great importance in more complex systems like anaerobic wastewater treatment plants, where critical instability of the process must be avoided, making the monitoring of the system variables an important issue (Dochain and Vanrolleghem, 2001; Mailleret *et al.*, 2004).

As an efficient solution for the inherent problem of monitoring in biotechnological processes, the internal state reconstruction can be achieved by formulating observers, also called software sensors.

Many types of observers have been proposed and extensively studied, even for nonlinear biological systems (Bastin and Dochain, 1990; Bernard and

Gouzé, 2002) and the choice of the design method depends on the kind of available models. Indeed, the quality of the used model is a factor of great importance when choosing an observation strategy. For instance, if a good model is available (a correctly identified and validated model), a high gain observer (Gauthier *et al.*, 1992) may perform good estimations of the internal state. If we have to deal with large uncertainties in model parameters, inputs and measurements, interval estimations should give us the best approach (Gouzé J.L. and Hadj-Sadok, 2000; Rapaport and Gouzé, 2003), since it provides an estimate of the quality of the estimate.

Interval Observers work under the formulation of two observers: An upper observer, which produces an upper bound of the state vector, and a lower observer producing a lower bound, providing by this way a bounded interval in which the state vector is guaranteed to lie. For the formulation of the interval observer, it is necessary to know bounds of the uncertainties in the model (*i.e.* uncertainties in model parameters, input variables, etc.). The theoretical basement of an interval observer is based on theory of cooperative systems (Smith, 1995). Such cooperative systems have the property to keep the order between 2 trajectories, and they can thus provide bounds of the behaviour of the state vector, depending on the bounds of the uncertainties in the model.

The design of such observers for non monotone systems is much less straightforward. The goal of this paper is thus to extend the classical approaches to derive interval observers even when the initial system is not cooperative. Then, taking benefit of the available measurements to close the loop, we propose a guaranteed interval estimation involving an observation gain. We run then several observers in parallel, obtaining a bundle of observers (Bernard and Gouzé, 2004), and we take the best estimate provided by the bundle. This concept has been successfully applied in (Bernard and Gouzé, 2004) and basically takes advantage of the various interval observers behaviours associated to appropriate gain values.

This paper is organised as follows. In section 2 the considered system is presented, and the proposed strategy to design the closed loop interval observer is described. Section 3 is devoted to running many observers in parallel to obtain the observer bundle. The example of the anaerobic wastewater treatment process is studied in Section 4 where an observer for this system is derived and simulation results are shown and discussed.

## 2. INTERVAL OBSERVERS: CLOSED LOOP APPROACH

We consider a class of nonlinear systems whose dynamics are expressed as follows (Rapaport and Gouzé, 2003):

$$(S_2) : \begin{cases} \dot{X}(t) = AX(t) + \psi(X, y); X(0) = X_0 \\ y = CX(t) \end{cases} \quad (1)$$

where  $X \in \Omega \subset \mathbb{R}^n$  is the state vector of the system,  $A \in \mathcal{M}^{n \times n}$ ,  $C \in \mathcal{M}^{1 \times n}$  and  $y \in \mathbb{R}$  is the system output. This specific mathematical structure is composed by a linear term and an uncertain nonlinear function  $\psi(X, y)$ . We will assume in the sequel that  $\psi(X, y)$  is a  $\mathcal{C}^1$  Lipschitz function.

Note that the same principle of observer design straightforwardly applies to a system output of dimension  $p$ . For sake of simplicity we focus here on the single output case.

An example of such a structure can be found in the classical mass balance models for bioprocesses as proposed by (Bastin and Dochain, 1990). Indeed the general model of a bioreactor working in continuous or chemostat mode can be written as follows:

$$\dot{X} = -DHX + Kr(X) + DX_{in} - Q(X) \quad (2)$$

In this model, the state vector  $X = (X_1, X_2, \dots, X_n)^t$  is the vector of all the process concentrations and biomasses. The matrix  $K$  contains the stoichiometric coefficients, also known as yield coefficients of the model. The vector  $r(X) = (r_1(X), r_2(X), \dots, r_k(X))^t$  is a vector of reaction rates (or conversion rates) representing the microbial activity. The diagonal matrix  $H$  stands for the fraction of biomass or substrates in the liquid phase. The influent feeding concentration is represented by the positive vector  $X_{in}$ . The dilution rate  $D > 0$  is the ratio of the influent flow rate and of the volume of the fermenter. Finally  $Q(X)$  represents the gaseous exchange with the outside of the fermenter.

With such a classical modelling we have *e.g.*:

$$A = -DH \text{ and } \psi(X) = Kr(X) + DX_{in} - Q(X)$$

The bacterial kinetics model ( $r(X)$ ) is generally a rough approximation and is highly uncertain. Moreover, for wastewater treatment process where the influent concentrations are generally not accurately estimated, the vector  $X_{in}$  is not accurately measured.

The objective of this paper is to derive an interval observer for the class of systems (1). The idea is to develop an interval observer based on the cooperative system theory that we briefly recall hereafter.

*Definition 1.* Cooperative Matrix (Smith, 1995). A matrix  $P$  is said to be cooperative if and only if all of the components out of its diagonal are non negative.

$$P \text{ is cooperative} \iff p_{ij} \geq 0 \quad \forall i, j \leq n, i \neq j$$

The main properties of a cooperative system defined by

$$\dot{X} = PX + b$$

where  $X, b \in \mathbb{R}^n$ , is that it keeps the (partial) order of the trajectories. If we consider two initial conditions  $x^1(0)$  and  $x^2(0)$  such that  $x^1(0) \geq x^2(0)$ , then  $x^1(t, x^1(0)) \geq x^2(t, x^2(0)), \forall t \geq 0$  (note that the inequality  $x^1 \geq x^2$  means that for all the components we have  $x_i^1 \geq x_i^2$ ).

An interval observer is proposed in (Rapaport and Gouzé, 2003) for system (1) provided that  $\psi$  is a monotone function. Here we want to extend these results in the case where  $\psi$  is not monotone. For this purpose we will use the following property.

*Property 1.* The Lipschitz function  $\psi$  can be rewritten as the difference of  $f$  and  $g$  which are two increasing functions of  $X$ :

$$\psi(X, y) = f(X, y) - g(X, y)$$

As a consequence

$$X^- \leq X \leq X^+ \Rightarrow \bar{\psi}(X^+, X^-, y) \leq \psi(X, y) \leq \bar{\psi}(X^-, X^+, y) \quad (3)$$

where  $\bar{\psi}(X^1, X^2, y) = f(X^2, y) - g(X^1, y)$ .

**Proof:** Let us consider  $f(X) = \gamma X$  and  $g(X, y) = \gamma X - \psi(X, y)$ , where  $\gamma \geq 0$  is the Lipschitz constant. It is clear that  $g(X, y)$  is increasing since:

$$\frac{\partial g}{\partial X_i} = \gamma - \frac{\partial \psi}{\partial X_i} \geq 0$$

Remark that a more pragmatic way of decomposing  $\psi$  for a broad range of classical functions is illustrated in the example.

*Hypothesis 1.* We assume that the imprecisely known function  $\psi$  can be bounded by a lower and an upper Lipschitz function. As a consequence, from Property 1 there exists two known functions  $\psi^+(X^1, X^2, y)$  and  $\psi^-(X^1, X^2, y)$ , increasing with respect to  $X^1$  and decreasing with respect to  $X^2$  such that  $\forall (X^1, X^2, y) \in \Omega \times \Omega \times \mathbb{R}$ :

$$\psi^-(X^1, X^1, y) \leq \psi(X^1, y) \leq \psi^+(X^1, X^1, y)$$

and

$$X^1 \leq X \leq X^2 \Rightarrow \psi^-(X^1, X^2, y) \leq \psi(X, y) \leq \psi^+(X^2, X^1, y) \quad (4)$$

Now we can propose the interval observer for system (1) in the same spirit as for the classical Luenberger approach:

*Lemma 1.* (Closed Loop Interval Observer). If there exists gain vectors  $\Theta_1$  and  $\Theta_2$  such that matrix  $A + \Theta C$  is cooperative, then the following system is an interval observer for (1), provided that  $X^-(0) \leq X(0) \leq X^+(0)$ :

$$\frac{d\zeta}{dt} = \left( \begin{array}{c|c} A + \Theta_1 C & 0 \\ \hline 0 & A + \Theta_2 C \end{array} \right) \zeta - \begin{pmatrix} \Theta_1 \\ \Theta_2 \end{pmatrix} y + \tilde{\psi}(\cdot) \quad (5)$$

where  $\zeta = (X^+, X^-)^t$ ,  $\tilde{\psi}(X^-, X^+, y) = (\psi^+(X^+, X^-, y), \psi^-(X^-, X^+, y))^t$  and  $\Theta_i = (\theta_1^i, \dots, \theta_n^i)^t$  (for  $i = 1, 2$ ) are two gain vectors.

The vectors  $X^-, X^+ \in \mathbb{R}^n$  corresponds respectively to the lower and upper bounds of the state  $X$ . The gains  $\Theta_1$  and  $\Theta_2$  are the correcting gains between the observer predictions and the real system output.

**Proof:** Let us denote  $\mathcal{P}$  the cooperative matrix  $\mathcal{P} = \left( \begin{array}{c|c} A + \Theta_1 C & 0 \\ \hline 0 & A + \Theta_2 C \end{array} \right)$ . Let us denote  $\tilde{e} = (e^+, e^-)^t$  the error vector, where  $e^+ = X^+ - X$  and  $e^- = X - X^-$ . We have thus the following dynamics for the error:

$$\frac{d\tilde{e}}{dt} = \mathcal{P}\tilde{e} + \tilde{\phi}(X^+, X^-, y) \quad (6)$$

Function  $\tilde{\phi}(X^+, X^-, y)$  is defined as follows.

$$\tilde{\phi}(X^+, X^-, y) = \begin{pmatrix} \psi^+(X^+, X^-, y) - \psi(X, y) \\ \psi(X, y) - \psi^-(X^-, X^+, y) \end{pmatrix} \quad (7)$$

By initial condition hypothesis,  $\tilde{e}(0) \geq 0$ .

Now let us apply the standard argues for positive systems. Let us consider the first time instant  $t_0$  when one of the component of vector  $\tilde{e}$  is equal to zero (e.g. the  $k^{th}$  component  $\tilde{e}_k$ ). We have for this error component (the components of matrix  $\mathcal{P}$  are denoted  $\rho_{ij}$ ):

$$\frac{d\tilde{e}_k}{dt} = \rho_{kk}\tilde{e}_k + \sum_{i \neq k}^{2n} \rho_{ki}\tilde{e}_i + \tilde{\phi}_k(X^+, X^-, y)$$

and thus at time instant  $t_0$ :

$$\frac{d\tilde{e}_k}{dt} \Big|_{t=t_0} = \sum_{i \neq k}^{2n} \rho_{ki}\tilde{e}_i + \tilde{\phi}_k(X^+, X^-, y) \geq 0$$

As a consequence  $\tilde{e}_k$  will stay non negative and finally  $\tilde{e}$  will remain non negative for any time  $t$ .

*Remark 1.* The previous observer is an interval observer in the sense that it provides bounds of the state variables. However these bounds may tend toward infinity. If we want to guarantee the boundness of the interval provided by the observer, we must then add some stability constraint to matrix  $A + \Theta C$ , together with some boundness properties for  $\psi$ .

However, as it will be detailed in the sequel, we will consider the best estimate among a set of interval observers running in parallel. Thus a single stable observer guarantees the bounding of the observer envelope. In the example of section 3 we use unstable observers to improve the transient interval estimate.

Therefore we do not need all the observers provided by the various choices of  $\Theta$  to be bounded, but at least one of them. In other words, we will consider a vector  $\Theta$  for which  $A + \Theta C$  is both cooperative and stable such that the interval is bounded (Rapaport and Gouzé, 2003).

In the general class of bioprocesses considered in equation (2), we have  $A = -DH$  a diagonal matrix. Therefore the choice  $\Theta = 0$  provides a matrix  $\mathcal{P}$  both stable and cooperative. Since bioprocesses models as represented by the mass balance model (2) are bounded for bounded inputs, this proves the boundness of the trivial open loop interval observer for  $\Theta = 0$ .

### 3. BUNDLE OF OBSERVERS AND REINITIALISATION

As we have seen, the cooperativity concept applied to the closed loop observer provides us with a guaranteed interval for the state vector. It is worth noting that the closed loop approach introduces some degrees of freedom since the observer convergence can be adjusted by tuning of the observer gains. Taking advantage of this last feature, now we run simultaneously several observers with different values for the gain vectors  $\Theta_1$  and  $\Theta_2$ , satisfying always the cooperativity condition. Thus we use both stable and unstable observers. In this way, some observers will provide a better estimate during their transitory response and other will have better estimated for the steady state behaviour. Considering different upper [resp. lower] estimations we will take the lower [resp. upper] envelope provided by the minimum [resp. maximum] values reached by this bundle of observers (Bernard and Gouzé, 2004).

The bundle can be even improved with a reinitialisation process. Reinitialisation consist in the restarting of the whole bundle of observers with the best estimation performed at the time of reinitialisation  $t_r$ .

$$[X_0^-(t_r), X_0^+(t_r)] = [\max\{X_{\Theta_i}^-\}(t_r), \min\{X_{\Theta_i}^+\}(t_r)]$$

where  $X_{\Theta_i}^-$ ,  $X_{\Theta_i}^+$  are the lower and upper envelopes, *i.e.* the best estimates at time  $t_r$  of  $X^-$  and  $X^+$  respectively for the set of considered gain vectors  $\Theta_i$ . It is already investigated that reinitialisation of the observers can dramatically improve the interval of estimation (Bernard and Gouzé, 2004).

## 4. APPLICATION TO AN ANAEROBIC DIGESTION MODEL

### 4.1 Introduction

Anaerobic digestion is a wastewater treatment process used to remove organic carbon from water using a mixture of bacteria. We will here focus on a very simple model where only a single bacterial population is considered, leading to a two dimensional model (Mailleret *et al.*, 2004) where a biomass of bacteria  $x$  is growing in a bioreactor consuming the polluting organic substrate  $s$ . A part of the biomass is attached in the reactor

and thus we consider the fraction of biomass in the liquid phase expressed by the heterogeneity factor  $\alpha \in [0, 1]$ . We assume that the biomass is measured:  $y = x$  (it is worth noting that this case, also a bit theoretical, is much more interesting to illustrate our method than the classical case when substrate is measured).

The associated model is then the following:

$$\begin{cases} \dot{x} = r(X, t) - \alpha Dx \\ \dot{s} = k_1 r(X, t) + D(s_{in} - s) \end{cases} \quad (8)$$

where  $s_{in}$  is the influent substrate, and  $k_1$  is a yield coefficient.

This model has the form (2) with:

$$H = \begin{pmatrix} -\alpha & 0 \\ 0 & -1 \end{pmatrix}, \quad K = \begin{pmatrix} 1 \\ k_1 \end{pmatrix}, \quad (9)$$

$$X_{in} = \begin{pmatrix} 0 \\ s_{in} \end{pmatrix}, \quad Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

As most of the time in biotechnology, the bacterial growth rate  $r(X)$  is a complex function of the state which is generally badly known. We have to define an analytical expression for this term. We consider  $r(s, x) = \mu(s)x$ , where  $\mu(s)$  corresponds to the growth rate function. In particular we will use the classical Haldane expression which is a non monotone function of the substrate.

### 4.2 The Haldane model

The Haldane growth rate model is defined by the following equation:

$$\mu_H(s) = \mu_0 \frac{s}{s + k_s + s^2/k_i} \quad (10)$$

This model assumes that the growth is enhanced by the substrate until a maximum growth rate value  $\mu_{max}$  is reached

$$\mu_{max} = \mu(\sqrt{k_s k_i})$$

Then, for higher substrate values, the growth is inhibited and  $\mu_H(s)$  is decreasing. The parameters  $k_s$  and  $k_i$  represent respectively the saturation and inhibition constants.

### 4.3 Considered uncertainty

- The Haldane kinetics is uncertain, and thus we consider that  $\mu_0^- \leq \mu_0 \leq \mu_0^+$
- The influent substrate concentration to be processed in the bioreactor is not accurately measured and thus:  $s_{in}^- \leq s_{in} \leq s_{in}^+$ .

Now the uncertainty can be considered in the vector  $\psi$ , with the same notations than for system (1):

$$\psi(s, x) = \begin{pmatrix} \mu(s)x \\ Ds_{in} - k\mu(s)x \end{pmatrix} \quad (11)$$

*Remark 2.* Since the system is not monotone the design of even a classical high gain observer is not straightforward.

#### 4.4 Observer design

Since the function  $\mu(s)$  is non monotone with respect to  $s$  we will first propose to bound it with a function of two variables, which is monotone with respect to the two variables. Several mathematical expressions for the upper and lower bounds of the function  $\psi(s, x)$  could be proposed. We propose an expression which is not too conservative in order to keep a good accuracy for the interval observer. Here we use the following bounding for the Haldane non monotone growth rate (for  $s^- \leq s \leq s^+$ ):

$$\mu_0 \bar{\rho}(s^-, s^+) \leq \mu(s) \leq \mu_0 \bar{\rho}(s^+, s^-)$$

where

$$\bar{\rho}(s^-, s^+) = \frac{s^-}{s^- + k_s + s^- s^+ / k_i}$$

Then we derive  $\psi^+(s^+, s^-, x)$  and  $\psi^-(s^-, s^+, x)$

$$\begin{aligned} \psi^+(s, x) &= \begin{pmatrix} \mu_0^+ \bar{\rho}(s^+, s^-) x \\ -\mu_0^- \bar{\rho}(s^-, s^+) x + D s_{in}^+ \end{pmatrix} \\ \psi^-(s, x) &= \begin{pmatrix} \mu_0^- \bar{\rho}(s^-, s^+) x \\ -\mu_0^+ \bar{\rho}(s^+, s^-) x + D s_{in}^- \end{pmatrix} \end{aligned} \quad (12)$$

The observer gain is  $\Theta = (\theta_1, \theta_2)^t$  and since biomass is measured  $C = (1, 0)$ .

The matrix  $A + \Theta C$  is expressed by:

$$A + \Theta C = \begin{pmatrix} -\alpha D + \theta_1 & 0 \\ \theta_2 & -D \end{pmatrix}$$

Cooperativity condition of matrix  $\mathcal{P}$  is fulfilled for  $\theta_2 \geq 0$ . The eigenvalues of matrix  $A + \Theta C$  are  $\lambda_1 = -\alpha D + \theta_1$  and  $\lambda_2 = -D$ . Since  $\psi$  is bounded for a bounded  $s_{in}$ , the boundness of the observer is guaranteed for  $\theta_1 < \alpha D$ . The interval observer equations can be written as follows:

$$\begin{cases} \frac{ds^+}{dt} = D(s_{in}^+ - s^+) + \frac{-k\mu_0^- s^- x}{s^- + k_s + s^+ s^- / k_i} + \theta_2(x^+ - x) \\ \frac{ds^-}{dt} = D(s_{in}^- - s^-) + \frac{-k\mu_0^+ s^+ x}{s^+ + k_s + s^+ s^- / k_i} + \theta_2(x^- - x) \\ \frac{dx^+}{dt} = -\alpha D x^+ + \frac{-k\mu_0^- s^- x}{s^- + k_s + s^+ s^- / k_i} + \theta_1(x^+ - x) \\ \frac{dx^-}{dt} = -\alpha D x^- + \frac{-k\mu_0^+ s^+ x}{s^+ + k_s + s^+ s^- / k_i} + \theta_1(x^- - x) \end{cases} \quad (13)$$

#### 4.5 Observer positivity

Now we will slightly modify our observer in order to keep a positive lower bound of the estimate. Indeed, the variables of the original system (13) (and more generally of the biotechnological model variables (Bernard, 2002)) stay positive, and therefore a natural trivial bound is  $X \geq 0$ . As we can see from the observer equation for  $s^- = 0$ :

$$\left. \frac{ds^-}{dt} \right|_{s^-=0} = D s_{in}^- - \frac{k\mu_0^+ s^+ x}{s^+ + k_0} + \theta_2(x^- - x)$$

which can be of any sign, showing thus that the face  $s^- = 0$  is not repulsive. Assuring positive

values of  $s^-$  will be useful to avoid division by zero in the upper observer equation and also to improve the final interval of estimation (because of the positivity of  $s$ ). To do this, we first remark that there exists a positive real  $\epsilon$  such that  $s$  is always much larger than  $\epsilon$ . Indeed, we can find any  $\epsilon$  such that  $ds/dt|_{s=\epsilon}$  is as close as wanted to  $D s_{in} > 0$ , proving that we will never go under the value  $\epsilon$  (we assume that initial conditions verify  $s(0) > \epsilon$ ).

As a consequence, the real system verifies  $\epsilon < s < s^+$ . We could then take  $\epsilon$  into account in the set of lower observer when computing the upper envelope. However it is even more convenient to have a meaningful behaviour of each lower observer, and therefore we introduce the term  $g(s) = \frac{s}{s+\epsilon}$  in the equation of the lower bound, obtaining then the following equations:

$$\begin{cases} \frac{ds^-}{dt} = D(s_{in}^- - s^-) + \dots \\ \dots g(s^-) \left( \frac{-k\mu_0^+ s^+ x}{s^+ + k_s + s^+ s^- / k_i} + \theta_2(x^- - x) \right) \end{cases} \quad (14)$$

This new equation does exactly match the previous observer for  $s^- \gg \epsilon$  since  $g(s^-) \simeq 1$ . When  $s^- \rightarrow 0$  then it ensures that  $s^-$  stays positive (since  $g(0) = 0$ ).

#### 4.6 Numerical Application

A bundle of estimations with reinitialisation is obtained, by considering in parallel 77 observers working with various gains. Specifically, we run the observers considering  $-10 \leq \theta_1 \leq 2$  and  $0 \leq \theta_2 \leq 100$ .

Figure 1 shows the dilution input and the influent substrate with the considered uncertainties for this input. Bounds for  $s_{in}$  and  $\mu_0$  have been fixed in a  $\pm 5\%$  of their real values. Observer initial conditions have been fixed in a very large interval in order to fulfil the initial condition hypothesis ( $s^+(0) = 100$  and  $s^-(0) = 0$ ).

Figure 2 presents the estimates for the substrate. Parameters of the system considered in this application are:  $k = 42.14$ ,  $\mu_0 = 0.74$ ,  $k_s = 9.28 \text{ mmol/l}$ ,  $k_i = 256 \text{ mmol/l}$  and  $\alpha = 0.5$ .

The observers were reinitialised more frequently (each 0.5 days) after a change of value of the dilution input, reinitialising every 2.5 days in steady state conditions.

## 5. CONCLUSIONS

A closed loop observer has been proposed especially for non monotone systems. The main idea consists in transforming the  $n$  dimensional non monotone mapping  $\psi$  in a  $2n$  monotone mapping. Then the observer is such that the equation error is cooperative, guarantying the bounding of the state variables.

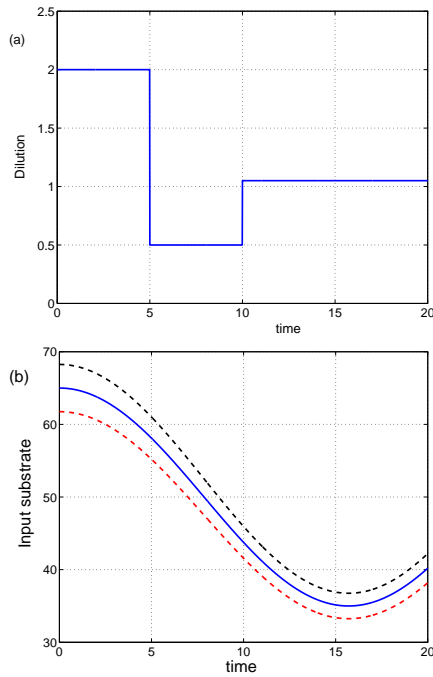


Fig. 1. (a) Dilution Input. (b) Input substrate and bounds .

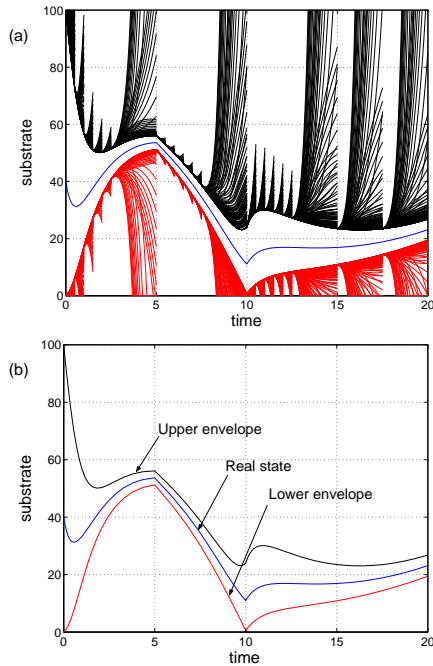


Fig. 2. (a) Bundle of estimations with reinitialisation. (b) Final upper and lower envelopes.

We remark that function  $\psi$  is computed using both values of the upper and lower observers. This way, the observer involves a strong coupling between the upper bound and the lower bound. The performance of the observer may then rapidly be degraded if one of these bounds turns out to be too loose. This is the reason why considering the observer bundle with regular reinitialisation maintains both bounds in a reasonable range. Note that we could consider a bundle of observers issued from several observers based on various

bounding functions  $\psi^+$  and  $\psi^-$ . Thus we take benefit from the fact that in some regions some bounding are better than others, and that it may change with respect to the regions. On the other hand, noise in the measurements can be faced with a straightforward bounding - interval analysis.

The approach here developed is illustrated with a model of an anaerobic digestion process where the Haldane function is a classical function that represents growth rate inhibition at high concentration, and for which the design of a classical observer is still an open problem.

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