POLE PLACEMENT IN INFINITE DIMENSIONS AND FUNCTIONAL MODELS OF LINEAR OPERATORS

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Abstract: Starting from a continuous time linear time-invariant system, a linearly similar variant of the Nagy–Foias model of the main operator of the system is constructed. A pole placement result, expressed in terms of invertibility of a certain Toeplitz operator, is obtained by applying this model. A numerical example of pole placement is given.

Keywords: Pole assignment, complete controllability, spectral analysis, feedback stabilization, state space models

1. INTRODUCTION

In many problems of the linear control theory, the spectral analysis of the main operator of the system is necessary. In particular, the spectral theorem for self-adjoint and normal operators, the theory of linear semigroups and other tools are applied, see (Curtain and Zwart, 1995). However, a comprehensive spectral theory of general linear operators has not been constructed by now. The Nagy–Foias theory of dissipative operators and contractions has been recognized to be one of the most useful tools for understanding the spectral structure (we will comment later on this theory).

The close connection between the Nagy-Foias theory and the linear control theory has been noted long ago and is developed in many works. For instance, there is an intimate connection between the H^∞ control, interpolation problems and the commutant lifting by Nagy and Foias, see (Foias and Frazho, 1990) and references therein.

In (Yakubovich, 2004), a linearly similar variant of the Nagy-Foias model was constructed, which differs from the original Nagy-Foias model in several points. Here this model will be described on the language of the control theory. A comparison between these two models will be made, and an application to the spectrum assignment will be given.

2. INPUT-SPACE AND SPACE-OUTPUT MAPS. EXACT CONTROLLABILITY AND EXACT OBSERVABILITY

Consider the linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.1}$$

where A is a generator of a C_0 semigroup on the state space X, $u(t) \in U$ is the input and $y(t) \in Y$ is the output. Spaces X, U, Y are supposed to be Hilbert. Take any point $\lambda_0 \notin \sigma(A)$. Define formally a Hilbert space $X_{-1}(A) = (A - \lambda_0)X$, equipped with the norm $\|(A-\lambda_0)x\|_{-1} = \|x\|_X$. It is larger than X and contains X as a dense subset. Assume that B is a bounded linear operator from U to $X_{-1}(A)$ (or equivalently, that $(A-\lambda_0)^{-1}B$ is bounded from U to X). It will be assumed that C is bounded from $\mathcal{D}(A)$ (with the graph norm) to Y. This setting is close to the theory of well-posed systems, see (Staffans, 2005).

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For a real γ, put

$$\Pi_-^\gamma = \{z : \operatorname{Re} z < \gamma\}, \qquad \Pi_+^\gamma = \{z : \operatorname{Re} z > \gamma\}.$$

Let some γ such that the spectrum $\sigma(A)$ is contained in $\operatorname{clos} \Pi_-^\gamma$ be fixed. Consider weighted spaces

$$L^2_\gamma(\mathbb{R}_\pm, U) = \{f = f(t) = e^{-t\gamma} g(t) : g \in L^2(\mathbb{R}_\pm, U)\}.$$

Introduce the **input-state map**

$$u \in L^2_\gamma(\mathbb{R}_-, U) \mapsto \mathcal{C}_{A,\,B} u \stackrel{\text{def}}{=} x(0) \in X, \qquad (2.2)$$

and the **state-output map**

$$a = x(0) \in X \mapsto \mathcal{O}_{A,\,C}\, a \stackrel{\text{def}}{=} y|[0, +\infty) \in L^2_\gamma(\mathbb{R}_+, Y). \qquad (2.3)$$

The control system (A, B) is called γ-*admissible* if the input-state map (defined initially for smooth functions u with compact support) extends continuously to $L^2_\gamma(\mathbb{R}_-, U)$. If the extended map $\mathcal{C}_{A,\,B}$ is onto, then the system (A, B) is called *infinite time γ-exactly controllable*. These notions depend on γ, but sometimes in these expressions, γ will be omitted.

The observation system (A, C) is called γ-*admissible* if the state-output map is continuous. If for some positive constants c_1, c_2, the inequality

$$c_1\|a\| \le \|\mathcal{O}_{A,C}\, a\|_{L^2} \le c_2\|a\|, \qquad a \in X$$

holds, then system (A, C) is called *exactly observable.*

Exact controllability and exact observability have been studied in detail for many classes of linear systems, arising in mathematical physics, see (Zuazua, 1998).

3. TWO FUNCTIONAL MODELS

3.1 Control model

The Laplace transform

$$\hat{u}(z) = \mathcal{L}u(z) = \frac{1}{\sqrt{2\pi}} \int e^{-tz} u(t)\, dt$$

maps (isometrically) $L^2_\gamma(\mathbb{R})$ onto $L^2(\partial\Pi_-^\gamma)$, and maps the corresponding subspaces $L^2_\gamma(\mathbb{R}_\pm)$ onto the Hardy spaces $H^2_\pm = H^2(\Pi_\pm^\gamma)$ of functions, analytic in $\Pi_\pm^\gamma$. The same is true for vector valued spaces $L^2_\gamma(\mathbb{R}_\pm, Y)$ and $H^2_\pm(Y)$ with values in a Hilbert space Y.

For an admissible system, define the controllability map

$$\widehat{\mathcal{C}}_{A,\,B} : H^2_-(U) \to X$$

by taking the composition map in the diagram

$$H^2_-(U) \underset{\mathcal{L}^{-1}}{\longrightarrow} L^2_\gamma(\mathbb{R}_-) \underset{\mathcal{C}_{A,\,B}}{\longrightarrow} X.$$

It is easy to see that $\widehat{\mathcal{C}}_{A,\,B}$ transforms the multiplication operator by z on $H^2_-(U)$ into the operator A; more exactly,

$$\widehat{\mathcal{C}}_{A,\,B}[q(z)\hat{u}(z)] = q(A)\big(\widehat{\mathcal{C}}_{A,\,B}\hat{u}\big) \qquad (3.4)$$

for any rational scalar function $q \in H^\infty_- = H^\infty(\Pi_-^\gamma)$ (for these functions, $q(A)$ is bounded).

Denote by $B(H, H')$ the set of all bounded linear operators between Hilbert spaces H and H', and put $B(H) = B(H, H)$.

Definitions. An operator-valued function $\delta \in H^\infty_-\big(\mathcal{L}(Y, U)\big)$ is called *admissible* if there is some constant $c > 0$ such that for a.e. $z \in i\mathbb{R}$, the inequality $\delta(z)$ is invertible and $\|\delta(z)y\| \ge c\|y\|$ holds for all $y \in Y$. It is called *two-sided admissible* if there is some constant $c > 0$ such that for a.e. $z \in \partial\Pi_-^\gamma$, $\delta(z)$ is invertible and $\|\delta(z)^{-1}\| \le c$.

Function δ is called *two-sided inner* if $\delta(z)$ is an isometric isomorphism of Y onto U for a.e. $z \in \partial\Pi_-^\gamma$.

We make the following simplifying assumption:

(*) *the set $\sigma(A) \cap \partial\Pi_-^\gamma$ has zero length.*

By applying (3.4) and the Beurling-Lax-Halmos theorem, see for instance (Nikolski, 2002), one gets that

$$\ker \widehat{\mathcal{C}}_{A,\,B} = \delta H^2_-(Y) \qquad (3.5)$$

for a Hilbert space Y and an admissible function δ. It may happen, in particular, that $\ker \widehat{\mathcal{C}}_{A,\,B} = 0$.

Definition. Suppose that the system (A, B) is infinite time exactly controllable and (3.5) holds. Then δ will be called *a generalized characteristic function (G. Ch. F.) of the system (A, B)* (and of the operator A).

Theorem 1. (Yakubovich, 2004), see also (Grabowski and Callier, 1986), (Staffans and Weiss, 2002) *Suppose that the system (A, B) is infinite time exactly controllable and that (*) holds. Let δ be a generalized characteristic function of system (A, B). Then δ is two-sided admissible, and A is similar to the (possibly, unbounded) operator of multiplication by the independent variable on the quotient space $H^2_-(U)/\delta H^2_-(Y)$. This operator is given by the formula*

$$f + \delta H^2_-(Y) \mapsto zf + \delta H^2_-(Y),$$

and it domain is $\mathcal{D} = \big\{f + \delta H^2_-(Y)$:

$$f = f(z) \in H^2_-(U),\ zf \in H^2_-(U)\big\}.$$

If δ is a generalized characteristic function of the system (A, B), then $\delta\psi$ also is, whenever $\psi, \psi^{-1} \in H^\infty_-\big(B(Y)\big)$.

The model operator obtained is exactly the Sz.-Nagy–Foias model operator (Sz.-Nagy and Foias, 1970) in the case of a semiplane. Therefore every exactly controllable system gives rise to a Nagy–Foias type functional model of A up to similarity, and all the techniques of the Nagy–Foias functional model can be applied. This fact was discussed in detail in (Yakubovich, 2004). The original Nagy–Foias model is up to the unitary isomorphism.

3.2 Observation model

For an admissible observation system (A, C), define the controllability map

$$\widehat{\mathcal{O}}_{A,\,C} : X \to H^2_+(Y)$$

by taking the composition map in the diagram

$$X \underset{\mathcal{O}_{A,\,C}}{\longrightarrow} L^2_\gamma(\mathbb{R}_+) \underset{\mathcal{L}}{\longrightarrow} H^2_+(Y).$$

Suppose that δ is a two-sided admissible H^∞ function on Π^γ_- with values in $B(Y, U)$. Consider *the observation model space:*

$$\mathcal{H}(\delta) \overset{\mathrm{def}}{=} \big\{ f \in H^2_+(Y) : \quad \delta \cdot f \big| \partial\Pi_- \in H^2_-(U) \big\}.$$

Theorem 2. (observation model). *If the system (A, C) is exactly observable then there exists an auxiliary Hilbert space U and a two-sided admissible function δ on Π^γ_- with values in $B(Y, U)$ such that $\widehat{\mathcal{O}}_{A,\,C}$ is an isomorphism from X onto $\mathcal{H}(\delta)$ that transforms A into the "backward shift" M^T_z on $\mathcal{H}(\delta)$*

$$M^T_z : \qquad f \in \mathcal{H}(\delta) \mapsto zf - (zf)(\infty).$$

Remarks. If the conclusion of the Theorem holds, then δ will be called *the generalized characteristic function of the observation system* (A, C). Operator M^T_z can be called *the main operator in the observation model.* It is unbounded, and its domain is

$$\mathcal{D}(M^T_z) = \big\{ f \in \mathcal{H}(\delta) : \exists\, c \in Y : zf - c \in \mathcal{H}(\delta) \big\}.$$

The converse statement to the assertion of the Theorem also is true: if $\widehat{\mathcal{O}}_{A,\,C}$ is an isomorphism that transforms A into M^T_z, then the system (A, C) is exactly observable.

3.3 Isomorphism between the two models

Suppose a two-sided admissible $B(Y, U)$-valued function δ on Π^γ_- is given. Then the control model space $H^2_-(U)/\delta H^2_-(Y)$ and the observation model space $\mathcal{H}(\delta)$, as well as the corresponding model quotient operator M_z and model operator M^T_z are defined. These two model operators are always similar.

Definition. An operator A generates *a Nagy–Foias type semigroup* if it generates a C_0 semigroup $T(t)$, $t \geq 0$, such that for some real γ, the semigroup $\big\{ e^{-\gamma t} T(t) \big\}$ is similar to a contractive semigroup and has the property that $e^{-\gamma t}T(t)$ and $e^{-\gamma t}T^*(t)$ strongly tend to 0 as $t \to \infty$.

The results by Jacob, Zwart and Le Merdy, see (Le Merdy, 2000) show that there exist Hilbert space C_0 semigroups that are not of Nagy–Foias type.

Proposition 1. *Suppose condition (*) for operator A holds. Then the following are equivalent.*

1) A generates a Nagy–Foias type semigroup;

2) There exists B such that (A, B) is exactly controllable;

3) There exists C such that (A, C) is exactly observable.

For any two-sided admissible function $\delta \in H^\infty_-(B(Y, U))$, its *spectrum* $\sigma(\delta)$ can be defined, see (Nikolski, 2002).

Proposition 2. (see (Nikolski, 2002)). *If A generates a Nagy–Foias type semigroup and A is similar to its observation or control model with generalized characteristic function δ, then the spectrum of A coincides with the spectrum of δ.*

In the hypotheses of this Proposition, if A has a compact resolvent, then the spectrum of δ is just the set of points $\lambda \in \Pi^\gamma_-$ such that $\delta(\lambda)$ is not invertible.

4. THE NAGY-FOIAS CASE

Suppose $\frac{1}{i}A$ is a maximal dissipative operator. Then A is a generator of a C_0 semigroup (Sz.-Nagy and Foias, 1970). Assume for simplicity that $A = -A_r + iA_i$, where A_r and A_i are self-adjoint, $A_r \geq 0$, $A_i \geq 0$, and $\mathcal{D}(A_r) \subset \mathcal{D}(A_i)$. Put $C = (2A_r)^{1/2}$, $Y = \operatorname{Range} C$, and $\gamma = 0$. More general cases can be treated by applying the results of (Solomyak, 1992), (Arov and Nudelman, 1996).

Proposition 3. *1) The control system (A, C) is exact; moreover, $\|\widehat{\mathcal{C}}_{A,\,C}x\| = \|x\|$ for all x in X.*

2) If Theorem 1 is applied to the control system (A, C), then one of the generalized characteristic functions obtained coincides with the characteristic function in the sense of Nagy and Foias.

5. EXAMPLES OF OPERATORS ADMITTING FUNCTIONAL MODELS

Apart from dissipative operators, the following examples were given in (Yakubovich, 2004).

1) Any generator A of a C_0 group is a generator of a Nagy–Foias type semigroup;

2) Any differentiation operator A on an interval $[0, L]$ with non-dissipative boundary conditions:

$$\begin{aligned} A\varphi &= -\varphi', \\ \mathcal{D}(A) &\stackrel{\text{def}}{=} \big\{\varphi \in W^{1,2}\big([0,L],\mathbb{C}^n\big) : \\ &\varphi(0) - \int d\beta(x)\varphi(x) = 0\big\}. \end{aligned} \tag{5.6}$$

Here $W^{1,2}\big([0,L],\mathbb{C}^n\big)$ is the vector Sobolev class of functions whose first derivative is in L^2, and β is any $n\times n$ matrix complex measure on $[0, L]$ such that $\beta(\{0\}) = 0$. In this case, the model can be deduced by considering the exact control operator

$$C\varphi = \varphi(0), \qquad \varphi \in \mathcal{D}(A). \tag{5.7}$$

One of generalized characteristic function of the observation system (A, C) is

$$\delta(z) = e^{zL} I_{n\times n} - \int e^{z(L-x)} d\beta(x),\ z \in \Pi^{\gamma}_{-}, \tag{5.8}$$

where γ is a suitable real number depending on β.

3) Generators of C_0 semigroups, related with linear neutral systems, are generators of Nagy-Foias type C_0 semigroups and fit into this scheme, see (Lunel and Yakubovich, 1997), (Yakubovich, 2004).

In all these examples, an exact control operator B and exact observation operator C, as well as an explicit formula for generalized characterictic functions were given.

6. DISCUSSION

In the Nagy–Foias theory, the model of an operator is constructed up to the unitary equivalence and is unique. In our setting, the model of an operator A is far from unique.

One can restrict himself only to generalized characteristic functions that are two-sided inner. Then the uniqueness of δ will be restored both for the observation model and for the control model.

There are many examples when one only has an explicit expression for some generalized characteristic function and no explicit expression for its inner part.

A Nagy–Foias type model in a rather general domain in $\mathbb{C}$, and not only in a half-plane was constructed in (Yakubovich, 2004). For operators A, close to self-adjoint ones, one can construct such model in a suitable parabolic domain.

7. POLE PLACEMENT

7.1 Problem statement

Roughly speaking, the problem is as follows.

Consider a system with a feedback

$$\dot{x}(t) = Ax(t) + Ku(t) \tag{7.9}$$

$$u(t) \quad = Cx(t), \tag{7.10}$$

where the main operator A and the output operator C are fixed and the control operator K is a parameter. The closed loop system (at least formally) has the form

$$\dot{x}(t) = A_1 x(t), \quad \text{where } A_1 = A + KC. \tag{7.11}$$

It is required to describe all possible closed loop spectra $\sigma(A_1)$ in terms of operators A and C.

The answer for finite dimensional systems is a well-known Rosenbrock theorem, see for instance (Ball *et al.*, 1990): if the system (A, C) is observable (which is a necessary condition), the spectrum of A_1 can be an arbitrary subset of the complex plane of no more than $\dim X$ points. In finite dimensions, the observability requirement is very natural: if it is not fulfilled, there are parts of the spectrum of A which cannot be moved.

Let us pass to the exact setting of the problem.

Assume (*) is fulfilled. The main hypothesis is

(**) System (A, C) is exactly observable.

Definition. It will be said that operator K in the control law (7.9) is admissible and gives rise to a closed loop system (7.11) if the following holds:

1) operator A_1 (as well as A) generates a C_0 semigroup;

2) K is a continuous operator from U to $X_{-1}(A_1)$;

3) for any $x \in \mathcal{D}(A)$, equality

$$A_1 x = Ax + KCx$$

understood as an equality in the Hilbert space $X_{-1}(A_1)$, holds.

In infinite dimensions, an answer to the pole placement problem is known for the case when A is a Riesz spectral operator, see (Russell, 1978), (Xu and Sallet, 1996); in these papers condition (**) is not assumed.

The notion of admissibility of K permits one to treat the internal stability of the closed loop system. See (Oostveen, 2000), (Staffans, 2005) for a modern treatment of strong stabilization.

This notion might seem to be complicated, but it becomes natural if one considers a reciprocal system to (A, K, C) in the sense of R. Curtain.

7.2 Sufficient condition for pole placement

Let G be a bounded function on $\partial\Pi_-^\gamma$, whose values are operators on Y. Define the vector Toeplitz operator T_G on $H_-^2(Y)$ by

$$T_G f = P_-(G \cdot f), \qquad f \in H_-^2(Y),$$

where $P_\pm$ stands for the orthogonal projection of $L^2(\partial\Pi_-^\gamma, Y)$ onto $H_\pm^2(Y)$.

Theorem 3. *Suppose that A generates a Nagy–Foias type C_0 semigroup, and $\delta \in H^\infty(\Pi_-^\gamma, B(Y,U))$ is a generalized characteristic function of the observation system (A, C). Suppose that $\delta_1 \in H^\infty(\Pi_-^\gamma, B(Y,U))$ is another two-sided admissible function. If δ_1 is such that*

$$T_{\delta_1^*\delta} \quad \text{is an isomorphism,} \tag{7.12}$$

then there exists an admissible operator K, which gives rise to a closed loop system $\dot{x}(t) = A_1 x(t)$ such that δ_1 is a G. Ch. F. of A_1. In this case, in particular, $\sigma(A_1) = \sigma(\delta_1)$.

One has a similar formulation for the case when K is fixed and C is varying.

Corollary 1. *Suppose that $\|\delta^{-1}(z)\| \le C_1$ for a.e. $z \in \partial\Pi_-^\gamma$. Let α be any function in $H^\infty(\Pi_-^\gamma, B(Y,U))$ such that*

$$\|\alpha\|_\infty = \sup_{z\in\Pi_-^\gamma} \|\alpha(z)\| < C_1^{-1},$$

and put

$$\delta_1 = \delta + \alpha.$$

Then there exists an admissible feedback operator K such that δ_1 is a generalized characteristic function of the closed loop operator $A_1 = A + KC$.

Corollary 2. *Let A, C be operators given by (5.6), (5.7). Let A_1 be another operator of the form (5.6). Then operator A can be transformed into an operator similar to A_1 by applying a closed loop control (7.9), (7.11).*

See (Olbrot, 1978), (Watanabe, 1988), (Jugo and de la Sen, 2002), and references therein for related results.

Corollary 3. *Suppose that in the hypotheses of Theorem 3, Y is finite dimensional and for some $\lambda \in \Pi_+^\gamma$, $zP_+(\dfrac{\delta_1^{-1}\delta}{z-\lambda})c \in \mathbb{C} + H_+^2(Y)$ for all constant vectors $c \in Y$. Then the operator K can be chosen to be bounded.*

The proof of Theorem 3 consists in applying Theorem 2 and the techniques of orthogonal projections of one coinvariant subspace onto another. See (Hruščev *et al.*, 1981) and (Nikolski, 2002), Vol. 2, where these techniques were applied to the classical problem about bases of exponential functions over an interval.

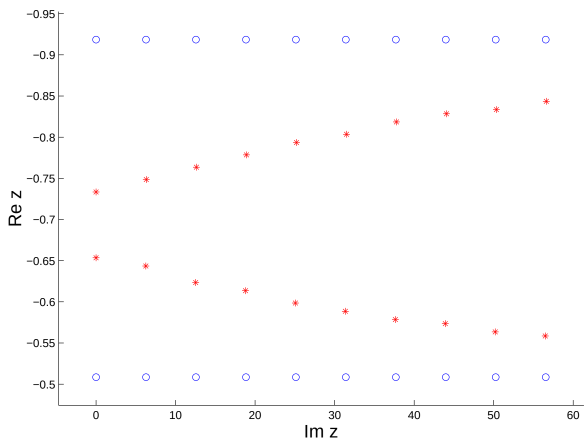

Fig. 1. Spectra of the open loop system (marked with "o") and the closed loop system (marked with "*"). Only the spectrum with positive imaginary part is shown.

Theorem 3 only gives a sufficient condition for the possibility of the spectrum assignment. The advantage of this theorem is that it deals with rather general operators A. See (Assawinchaichote and Nguang, 2004) for another approach to the problem setting and (Ball and Vinnikov, 1999), (Gurvits *et al.*, 1991), (Hermida-Alonso, 2003) for different algebraic settings. In fact, the literature on the pole placement is very vast, and these references cannot be complete.

7.3 Example

Just to illustrate the use of Corollary 1 of Theorem 3, consider a very particular case of operator (5.6) with $n = 1$, $L = 2$ and with boundary conditions

$$\varphi(0) - \varphi(1) + 0.24\,\varphi(2) = 0.$$

The generalized characteristic function of system (A, C), obtained from (5.8) is

$$\delta(z) = e^{2z} - e^z + 0.24 = (e^z - 0.4)(e^z - 0.6).$$

Operator A generates a Nagy–Foias type semigroup with $\gamma = 0$, that is, it is a semigroup similar to a contractive one. To shift the spectrum, one can take in Corollary 1 any function $\alpha \in H^\infty(\Pi_-^0)$ of norm less than 0.24. Figure 1 shows the spectra of the open-loop system (two arithmetic progressions parallel to the imaginary axis) and the results of computer calculation of the closed-loop system for $\alpha(z) = -0.7/(z - 72)$. For this choice of α, the feedback operator K is bounded.

8. CONCLUSIONS

Nagy–Foias type C_0 semigroups $T(t)$, such that for some real γ, $\{e^{-\gamma t}T(t)\}$ is similar to a contractive semigroup have been singled out. For this class of semigroups, a notion of the generalized characteristic function was discussed. In a series of examples, this function can be computed explicitly. A pole placement criterion in terms of generalized characteristic functions of the open loop and the closed loop systems, as well as a numerical example of the pole placement were given.

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