PERFORMANCE TUNING FOR A NEW CLASS OF GLOBALLY STABLE CONTROLLERS FOR ROBOT MANIPULATORS [1]

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Abstract: In this paper a new class of globally stable controllers for robot manipulators is proposed. The global asymptotic stabilization is achieved by adding a nonlinear damping term to linear PID controller. By using Lyapunov's direct method and LaSalle's invariance principle, explicit conditions on controller parameters which ensure global asymptotic stability are obtained. Further, the Lyapunov function is employed for evaluation of the performance index and determination of the optimal values of controller parameters. Finally, an example is given to demonstrate the obtained results.

Keywords: robot control, global stability, performance evaluation, Lyapunov stability, nonlinear control, robot manipulators.

1. INTRODUCTION

The most industrial robots are controlled by linear PID controllers which does not require any component of robot dynamics into its control law. A simple linear and decoupled PID feedback controller with appropriate control gains achieves desired position without any steady-state error. This is the main reason why PID controllers are still used in industrial robots. However, a linear PID controller in closed loop with a robot manipulator guarantees only local asymptotic stability (Arimoto and Miyazaki, 1986; Kelly, 1995).

The first nonlinear PID controller which ensures global asymptotic stability (GAS) in closed loop with a robot manipulator is proposed in (Kelly, 1993). In this work, which was inspired by results of Tomei (Tomei, 1991), is proven that global convergence is still preserved if regressor matrix is replaced by constant matrix. Since the regressor matrix is constant, the control law can be interpreted as a nonlinear PID controller which achieve GAS by normalization nonlinearities in the integrator term of control law. Second approach to achieves GAS is the scheme of Arimoto (Arimoto, 1994) that uses a saturation function in the integrator to render the system globally asymptotically stable, just as the normalization did in (Kelly, 1993). A unified approach to both above controllers, which belong to the class of PD plus a nonlinear integral action (PD+NI) controllers, is given in (Kelly, 1998).

An alternative approach to global asymptotic stabilization of robot manipulator is "delayed PID" (PI_dD) (Loria *et al.*, 2000). PI_dD can be understood as a simple PD controller to which an integral action is added after some transient of time.

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The idea of this approach consists of "patching" a global and a local controller. The first drives the solutions to an arbitrarily small domain, while the second, yields local asymptotic stability.

Although the stability properties of globally stabile controllers for robot manipulators are well understood, there are no many results regarding to optimality and performance tuning rules, except of H_∞ optimality (Nakayama and Arimoto, 1996).

In this paper a new approach to GAS of robot manipulators is presented. In this approach GAS is achieved by adding a nonlinear damping term to linear PID controller. Explicit conditions on controller parameters which guarantee GAS are given.

Also, a performance index is evaluated using the Lyapunov function and optimal values of the parameters are obtained. The proposed approach is based on construction of a parameter dependent Lyapunov function. With the appropriate choice of the free parameter, which is not included in stability conditions, an estimation of the integral performance index is obtained. The performance index depends on controller parameters and few parameters which characterize the robot dynamics. The optimal values of the controller gains are obtained by minimization of the performance index.

This paper is organized as follows. The system description is presented in Section 2. The stability criterion based on the Lyapunov's approach is derived in Section 3. The performance tuning is presented in Section 4. In Section 5, an example is given to demonstrate results. Finally, the concluding remarks are emphasized in Section 6.

2. SYSTEM DESCRIPTION

We consider a robot manipulator with n-degree of freedom in closed loop with a nonlinear PID controller.

2.1 Dynamics of Rigid Robot

The model of n-link rigid-body robotic manipulator, in the absence of friction and disturbances, is represented by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u, \tag{1}$$

where q is the $n \times 1$ vector of robot joint coordinates, $\dot{q}$ is the $n \times 1$ vector of joint velocities, u is the $n \times 1$ vector of applied joint torques, $M(q)$ is $n \times n$ inertia matrix, $C(q,\dot{q})\dot{q}$ is the $n \times 1$ vector of centrifugal and Coriolis torques, and $g(q)$ is the $n \times 1$ vector of gravitational torques obtained as the gradient of the robot potential energy $U(q)$

$$g(q) = \frac{\partial U(q)}{\partial q}. \tag{2}$$

The following well known properties of the robot dynamics, (Arimoto, 1997; de Wit *et al.*, 1996), are important for stability analysis.

Property 1. The inertia matrix $M(q)$ is a positive definite symmetric matrix which satisfies

$$\lambda_m\{M\}\|\dot{q}\|^2 \le \dot{q}^T M(q)\dot{q} \le \lambda_M\{M\}\|\dot{q}\|^2, \tag{3}$$

where $\lambda_m\{M\}$ and $\lambda_M\{M\}$ denotes strictly positive minimum and maximum eigenvalues of $M(q)$, respectively.

Property 2. The matrix $S(q,\dot{q}) = \dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric, i.e.,

$$z^T S(q,\dot{q})z = 0, \quad \forall z \in \mathbb{R}^n. \tag{4}$$

This implies

$$\dot{M}(q) = C(q,\dot{q}) + C(q,\dot{q})^T. \tag{5}$$

Property 3. The Coriolis and centrifugal terms $C(q,\dot{q})\dot{q}$ satisfies

$$\|C(q,\dot{q})\dot{q}\| \le k_c\|\dot{q}\|^2, \tag{6}$$

for some bounded constant $k_c > 0$.

Property 4. There exists some positive constant k_g such that gravity vector satisfies

$$\|g(x) - g(y)\| \le k_g\|x - y\|, \quad \forall\, x, y \in \mathbb{R}^n. \tag{7}$$

Property 5. There exist positive diagonal matrix K_P such that the following two inequalities with specified constant $k_1 > 0$ are satisfied simultaneously

$$\tilde{q}^T K_P \tilde{q} + \tilde{q}^T(g(q) - g(q_d)) \ge k_1\|\tilde{q}\|^2, \tag{8}$$

$$\frac{1}{2}\tilde{q}^T K_P \tilde{q} + \bar{U}(\tilde{q}) \ge \frac{1}{2}k_1\|\tilde{q}\|^2, \tag{9}$$

where

$$\bar{U}(\tilde{q}) = U(q) - U(q_d) - \tilde{q}^T g(q_d), \tag{10}$$

$$k_1 = \lambda_m\{K_P\} - k_g \ge 0. \tag{11}$$

2.2 Nonlinear PID Controller

The nonlinear PID control law is given by

$$u = -\Psi_P(\tilde{q})\tilde{q} - \Psi_D(\tilde{q})\dot{q} - K_I\nu, \tag{12}$$

$$\dot{\nu} = \tilde{q}, \tag{13}$$

where $\tilde{q} = q - q_d$ is joint position error, $\Psi_j(\tilde{q})$, $j = P, D$, are $(n \times n)$ positive definite diagonal matrix functions which can be written in next form

$$\Psi_j(\tilde{q}) = K_j + \bar{K}_j \bar{\Psi}_j(\tilde{q}), \tag{14}$$

where K_P, K_D, K_I, $\bar{K}_P$ and $\bar{K}_D$ are constant positive-definite diagonal matrix, and $\bar{\Psi}_j(\tilde{q})$, $j = P, D$, are $(n \times n)$ positive definite diagonal matrix function

$$\bar{\Psi}_j(\tilde{q}) = \mathrm{diag}\{\bar{\psi}_j(\tilde{q}_1), ..., \bar{\psi}_j(\tilde{q}_n)\}. \tag{15}$$

The function $\Psi_D(\tilde{q})$ will be determined to ensure global asymptotic stability, and function $\Psi_P(\tilde{q})$ will be used for performance tuning. The function $\bar{\Psi}_P(\tilde{q})$ satisfies additional conditions

$$0 \leq \bar{\Psi}_P(\tilde{q}) \leq I, \;\; \bar{\Psi}_P(0) = I, \;\; \lim_{\tilde{q} \to \pm\infty} \bar{\Psi}_P(\tilde{q}) = 0,$$

that prevent high control jump during the transient response, because of large error at the beginning of control action, $u(0) \approx -K_P \tilde{q}(0) = K_P q_d$.

The following properties of functions $\Psi_j(\tilde{q})$, $j = P, D$, are important for stability analysis.

Property 1. Functions $\Psi_j(\tilde{q})$, $j = P, D$, are lower bounded and satisfy next inequalities

$$\begin{aligned} z^T \Psi_j(\tilde{q}) z &\geq (\lambda_m\{K_j\} + \lambda_m\{\bar{K}_j\} \|\bar{\psi}_j(\tilde{q})\|) \|z\|^2 \geq \\ &\geq \lambda_m\{K_j\} \|z\|^2, \quad \forall z \in \mathbb{R}^n. \end{aligned} \tag{16}$$

Property 2. The following integrals are positive-definite functions

$$\int_0^z \bar{\psi}_{Di}(\xi) \xi d\xi \geq 0, \quad \forall z \in \mathbb{R}, \tag{17}$$

$$0 \leq \int_0^z \bar{\psi}_{Pi}(\xi) \xi d\xi \leq \frac{1}{2} z^2, \quad \forall z \in \mathbb{R}. \tag{18}$$

for $i = 1, ..., n$.

3. STABILITY ANALYSIS

The stability analysis is based on Lyapunov's direct method, and can be divided in four parts. First, error equations for closed loop system (1), (12), (13) is determined. Second, Lyapunov function (LF) candidate is proposed. Then, a global stability criterion on system parameters is established to guarantee the asymptotic stability. Finally, LaSalle invariance principle is invoked to guarantee the asymptotic stability.

The stationary state of the system (1), (12), (13) is $\tilde{q} = 0$, $\dot{q} = 0$, $\nu = \nu^*$, and ν^* satisfies $g(q_d) = -K_I \nu^*$. If a new variable $z = \nu - \nu^*$ is introduced, then system (1), (12), (13) becomes

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) - g(q_d) = u, \tag{19}$$

$$u = -\Psi_P(\tilde{q})\tilde{q} - \Psi_D(\tilde{q})\dot{q} - K_I z, \tag{20}$$

$$\dot{z} = \tilde{q}. \tag{21}$$

3.1 Construction of Lyapunov function

First, an output variable $y = \dot{q} + \alpha \tilde{q}$ with some $\alpha > 0$ is introduced, and inner product between (19) and y is made, resulting in a nonlinear differential form which can be separated in the following way

$$\frac{dV(\tilde{q}, \dot{q}, z)}{dt} = -W(\tilde{q}, \dot{q}), \tag{22}$$

where $V(\tilde{q}, \dot{q}, z)$ is Lyapunov function candidate.

For easier determination of conditions for positive-definiteness of function V and W, the following decompositions are made: $V(\tilde{q}, \dot{q}, z) = V_1(\tilde{q}, \dot{q}) + V_2(\tilde{q}, z)$ and $W(\tilde{q}, \dot{q}) = W_1(\tilde{q}, \dot{q}) + W_2(\tilde{q})$, where

$$\begin{aligned} V_1(\tilde{q}, \dot{q}) &= \frac{1}{2}\dot{q}^T M(q)\dot{q} + \alpha \sum_{i=1}^n \bar{K}_{Di} \int_0^{\tilde{q}_i} \bar{\psi}_{Di}(\xi)\xi d\xi + \\ &+ \frac{1}{2}\alpha \tilde{q}^T K_D \tilde{q} + \alpha \tilde{q}^T M(q)\dot{q}, \end{aligned} \tag{23}$$

$$\begin{aligned} V_2(\tilde{q}, z) &= \frac{1}{2}\alpha z^T K_I z + \tilde{q}^T K_I z + \frac{1}{2}\tilde{q}^T K_P \tilde{q} + \\ &+ \sum_{i=1}^n \bar{K}_{Pi} \int_0^{\tilde{q}_i} \bar{\psi}_{Pi}(\xi)\xi d\xi + \\ &+ U(q) - U(q_d) - \tilde{q}^T g(q_d), \end{aligned} \tag{24}$$

and

$$\begin{aligned} W_1(\tilde{q}, \dot{q}) &= -\alpha \dot{q}^T M(q)\dot{q} + \dot{q}^T \Psi_D(\tilde{q})\dot{q} + \\ &+ \alpha \tilde{q}^T (\dot{M}(q) - C(q, \dot{q}))\dot{q}, \end{aligned} \tag{25}$$

$$W_2(\tilde{q}) = -\tilde{q}^T (K_I - \alpha \Psi_P(\tilde{q}))\tilde{q} + \tag{26}$$

$$+ \alpha \tilde{q}^T (g(q) - g(q_d)). \tag{27}$$

3.2 Stability criterion determination

First, we consider function V_1 which can be rearranged to be of the form

$$\begin{aligned} V_1 &= \frac{1}{2}(\dot{q} + \alpha\tilde{q})^T M(q)(\dot{q} + \alpha\tilde{q}) - \frac{1}{2}\alpha^2 \tilde{q}^T M(q)\tilde{q} + \\ &+ \frac{1}{2}\alpha \tilde{q}^T K_D \tilde{q} + \alpha \sum_{i=1}^n \bar{K}_{Di} \int_0^{\tilde{q}_i} \bar{\psi}_{Di}(\xi)\xi d\xi, \end{aligned} \tag{28}$$

and using properties (16) and (3) we get

$$V_1 \geq \frac{1}{2}\alpha(\lambda_m\{K_D\} - \alpha\lambda_M\{M\})\|\tilde{q}\|^2 \geq 0, \tag{29}$$

that is positive-definite if the following condition is satisfied

$$\frac{\lambda_m\{K_D\}}{\lambda_M\{M\}} > \alpha. \tag{30}$$

Further, we consider function V_2 which can be rearranged to be of the form

$$V_2 = \frac{1}{2}\left(\sqrt{\alpha}z + \frac{1}{\sqrt{\alpha}}\tilde{q}\right)^T K_I \left(\sqrt{\alpha}z + \frac{1}{\sqrt{\alpha}}\tilde{q}\right) + \\ + \frac{1}{2}\tilde{q}^T K_P \tilde{q} + U(q) - U(q_d) - \tilde{q}^T g(q_d) + \\ + \sum_{i=1}^{n} \bar{K}_{Pi} \int_0^{\tilde{q}_i} \bar{\psi}_{Pi}(\xi)\xi d\xi - \frac{1}{2\alpha}\tilde{q}^T K_I \tilde{q}. \tag{31}$$

If we apply properties (9) and (16) than

$$V_2 \geq \frac{1}{2}\left(k_1 - \frac{1}{\alpha}\lambda_M\{K_I\}\right)\|\tilde{q}\|^2, \tag{32}$$

that is positive-definite if the following condition is satisfied

$$\alpha > \frac{\lambda_M\{K_I\}}{k_1}. \tag{33}$$

Comparing (33) with (30) we obtain

$$k_1\lambda_m\{K_D\} > \lambda_M\{K_I\}\lambda_M\{M\}. \tag{34}$$

Note that in above condition the unspecified positive constant α is eliminated.

The following step is condition which ensure that time derivative of LF is negative definite function, i.e., $W \geq 0$. First, we consider function W_1. Applying properties (3), (5), (6) and (16) we get

$$W_1 \geq (\lambda_m\{K_D\} + \lambda_m\{\bar{K}_D\}\|\bar{\psi}_D(\tilde{q})\|)\|\dot{q}\|^2 - \\ - \alpha\lambda_M\{M\}\|\dot{q}\|^2 - \alpha k_c\|\tilde{q}\|\|\dot{q}\|^2 \geq 0, \tag{35}$$

that is positive-definite if the following condition is satisfied

$$\frac{\lambda_m\{K_D\} + \lambda_m\{\bar{K}_D\}\|\bar{\psi}_D(\tilde{q})\|}{\lambda_M\{M\} + k_c\|\tilde{q}\|} > \alpha. \tag{36}$$

Further, we consider function W_2. Using property (8) we get

$$W_2 \geq (\alpha k_1 - \lambda_M\{K_I\})\|\tilde{q}\|^2, \tag{37}$$

that is positive-definite if we have

$$\alpha > \frac{\lambda_M\{K_I\}}{k_1}. \tag{38}$$

Comparing (36) with (38) the following condition is obtained

$$\frac{\lambda_m\{K_D\} + \lambda_m\{\bar{K}_D\}\|\bar{\psi}_D(\tilde{q})\|}{\lambda_M\{M\} + k_c\|\tilde{q}\|} > \frac{\lambda_M\{K_I\}}{k_1}.$$

Also, in above condition the unspecified positive constant α is eliminated. The above condition can be rearranged such that includes condition (34),

$$k_1\lambda_m\{\bar{K}_D\}\|\bar{\psi}_D(\tilde{q})\| - k_c\lambda_M\{K_I\}\|\tilde{q}\| + S_M > 0, \\ S_M = k_1\lambda_m\{K_D\} - \lambda_M\{K_I\}\lambda_M\{M\} > 0. \tag{39}$$

The global asymptotic stability will be guarantied if conditions (39) are satisfied for all $\tilde{q} \in \mathbb{R}^n$.

This conditions can be satisfied for different choices of the function $\bar{\psi}_D(\tilde{q})$. We will consider the most simple form of function $\bar{\psi}_{Di}(\tilde{q}_i)$ which satisfies the conditions (39).

If we make the following choice

$$\bar{\psi}_{Di}(\tilde{q}_i) = |\tilde{q}_i| = \tilde{q}_i \text{sign}(\tilde{q}_i), \tag{40}$$

condition (39) will become

$$(k_1\lambda_m\{\bar{K}_D\} - k_c\lambda_M\{K_I\})\|\tilde{q}\| + S_M > 0, \tag{41}$$
$$S_M = k_1\lambda_m\{K_D\} - \lambda_M\{K_I\}\lambda_M\{M\} > 0, \tag{42}$$

that will be satisfied when

$$\lambda_m\{\bar{K}_D\} > \frac{k_c\lambda_M\{K_I\}}{k_1}, \tag{43}$$
$$\lambda_m\{K_D\} > \frac{\lambda_M\{M\}\lambda_M\{K_I\}}{k_1}. \tag{44}$$

Further, the following choice

$$\lambda_m\{\bar{K}_D\} = \frac{k_c\lambda_m\{K_D\}}{\lambda_M\{M\}}, \tag{45}$$

will satisfied condition (43).

4. PERFORMANCE TUNING

The Lyapunov function V and its time derivative $\dot{V} = -W$ contain free parameter $\alpha > 0$ that is not included in stability condition. This fact can be employed for the evaluation of the following performance index

$$I = I_1 + \tau^2 I_2 = \int_0^\infty \|\tilde{q}\|^2 dt + \tau^2 \int_0^\infty \|\dot{q}\|^2 dt, \tag{46}$$

with respect to $\lambda_m\{K_D\}$ and $\lambda_M\{K_I\}$, where the constant τ^2 is the weighting factor. Also, in this section, because of compactness, the following shortened notation is introduced
$k_{jm} = \lambda_m\{K_j\}$, $k_{jM} = \lambda_M\{K_j\}$, $\bar{k}_{jm} = \lambda_m\{\bar{K}_j\}$, $\bar{k}_{jM} = \lambda_M\{\bar{K}_j\}$, $\bar{m} = \lambda_M\{M\}$,

$$\mu_j = \frac{\lambda_M\{K_j\}}{\lambda_m\{K_j\}}, \quad \bar{\mu}_j = \frac{\lambda_M\{\bar{K}_j\}}{\lambda_m\{\bar{K}_j\}}, \tag{47}$$

where $j = P, I, D$ and $w_p = \frac{1}{p}\|q_d\|_p^p, \quad p = 2, 3.$

The performance index (46) can be evaluated using Lyapunov function (23), (24) and their time derivative. From the equation (22) we can get

$$V(0) = \int_0^\infty W(\tilde{q}(s), \dot{q}(s))ds, \tag{48}$$

where we used $V(\infty) = 0$. Putting (35) and (37) in (48) we get

$$V(0) \geq (k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1 + + (\bar{k}_{Dm} - \alpha k_c)\int_0^\infty \|\tilde{q}\|\|\dot{q}\|^2 dt. \tag{49}$$

The third term on the right side of above expression is positive, $\bar{k}_{Dm} - \alpha k_c > \bar{k}_{Dm} - \frac{k_{IM}}{k_1}k_c > 0$, where we used (38) and (43), so that

$$V(0) \geq (k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1. \tag{50}$$

The following step is estimation of upper bounds on V(0). We have $\tilde{q}(0) = -q_d$, $\dot{q}(0) = 0$, $z(0) = -\nu^* = K_I^{-1}g(q_d)$, so that $V(0)$ satisfies the following expression

$$V(0) = U(0) - U(q_d) + \frac{1}{2}q_d^T K_P q_d + + \frac{1}{2}\alpha q_d^T K_D q_d + \frac{1}{2}\alpha g(q_d)^T K_I^{-1} g(q_d) + + \sum_{i=1}^n \bar{K}_{Pi}\int_0^{-q_{di}} \bar{\psi}_{Pi}(\xi)\xi d\xi + + \alpha\sum_{i=1}^n \bar{K}_{Di}\int_0^{-q_{di}} \bar{\psi}_{Di}(\xi)\xi d\xi. \tag{51}$$

So, we can estimate upper bounds

$$V(0) \leq \frac{1}{2}(k_{PM} + \bar{k}_{PM} + \alpha k_{DM})\|q_d\|^2 + + \frac{1}{2}\alpha k_{IM}^{-1}\|g(q_d)\|^2 + \frac{1}{3}\alpha\bar{k}_{DM}\|q_d\|_3^3. \tag{52}$$

Because of (7) and $\lambda_M\{K_I^{-1}\} = 1/\lambda_m\{K_I\}$ we have

$$V(0) \leq w_2\left[k_{PM} + \bar{k}_{PM} + \alpha\left(k_{DM} + \frac{k_g^2}{k_{Im}}\right)\right] + + w_3\alpha\bar{k}_{DM}. \tag{53}$$

Finally, comparing (50) and (53) we have

$$(k_{Dm} - \alpha\bar{m})I_2 + (\alpha k_1 - k_{IM})I_1 \leq w_3\alpha\bar{k}_{DM} + + w_2\left[k_{PM} + \bar{k}_{PM} + \alpha\left(k_{DM} + \frac{k_g^2}{k_{Im}}\right)\right]. \tag{54}$$

From above mentioned expression we can get integral terms I_1 and I_2 on the following way. If we put $\alpha \to (k_{Dm}/\bar{m})_-$ in expression (54) we get

$$I_1 \leq \frac{w_3}{S_M}k_{Dm}\bar{k}_{DM} + \frac{w_2}{S_M}(k_{PM} + \bar{k}_{PM})\bar{m} + + \frac{w_2}{S_M}\left(k_{DM} + \frac{k_g^2}{k_{Im}}\right)k_{Dm}. \tag{55}$$

Similarly, if we put $\alpha \to (k_{IM}/k_1)_+$ in expression (54) then

$$I_2 \leq \frac{w_3}{S_M}k_{IM}\bar{k}_{DM} + \frac{w_2}{S_M}(k_{PM} + \bar{k}_{PM})k_1 + + \frac{w_2}{S_M}\left(k_{DM} + \frac{k_g^2}{k_{Im}}\right)k_{IM}. \tag{56}$$

Finally, if we put expressions (55) and (56) in (46) including (45) and (47) we get

$$I \leq \hat{I} = \frac{1}{S_M}(k_P^* + A(k_{Dm}^2 + \tau^2 k_{Dm}k_{IM})) + + \frac{B}{S_M}\left(\frac{k_{Dm}}{k_{IM}} + \tau^2\right), \tag{57}$$

where $\hat{I}$ is the estimation of the upper bounds of the performance index (46), and

$$A = w_2\mu_D + w_3 k_c\bar{m}^{-1}, \quad B = w_2\mu_I k_g^2,$$
$$k_P^* = w_2(\bar{m} + \tau^2 k_1)(k_{PM} + \bar{k}_{PM}).$$

Expression (57) can be employed to find the optimal values of the controller gains

$$\frac{\partial\hat{I}}{\partial k_{Dm}} = 0, \qquad \frac{\partial\hat{I}}{\partial k_{IM}} = 0. \tag{58}$$

The solution of above set of equation is the following set of polynomial equation regarding to variables k_{Dm} and k_{IM}

$$a_D k_{Dm}^2 - b_D k_{Dm} - c_D = 0, \tag{59}$$

$$a_I k_{IM}^2 + b_I k_{IM} - c_I = 0, \tag{60}$$

where $a_D = k_1 A$, $b_D = 2\bar{m}Ak_{IM}$, $b_I = 2\bar{m}Bk_{Dm}$, $c_I = k_1 B k_{Dm}^2$, and

$$c_D = \bar{m}(A\tau^2 k_{IM}^2 + B) + k_1(k_P^* + B\tau^2),$$
$$a_I = \bar{m}(k_P^* + B\tau^2) + A(\bar{m} + k_1\tau^2)k_{Dm}^2.$$

We can rewrite the above set equations on the following way

$$k_{Dm} = \frac{1}{2a_D}\left(b_D + \sqrt{b_D^2 + 4a_D c_D}\right), \tag{61}$$

$$k_{IM} = \frac{1}{2a_I}\left(-b_I + \sqrt{b_I^2 + 4a_I c_I}\right). \tag{62}$$

On this way, we guarantee positivity of the parameters k_{Dm}, k_{IM} for any value of coefficients in expressions (59) and (60). The solution of above set of nonlinear algebraic equations can be found by applying simple iterative procedure.

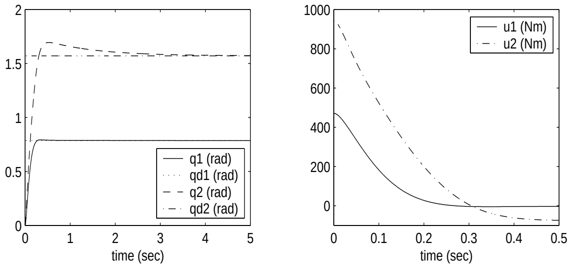

Fig. 1. The transient response for positions and control torque for $K_P = \text{diag}\{600\}$, $\bar{K}_P = \text{diag}\{0\}$ and for the optimal values $K_D = \text{diag}\{53.1\}$ and $K_I = \text{diag}\{492.7\}$.

A possible choice for the nonlinear proportional gain is

$$\psi_{Pi}(\tilde{q}_i) = K_{Pi} + \bar{K}_{Pi} \exp(-\frac{\tilde{q}_i^2}{2\sigma_P}). \qquad (63)$$

In this way, we ensure high proportional gain $\Psi_P(\tilde{q}) \approx K_P + \bar{K}_P$, when the system state is near the stationary state, $\tilde{q} \approx 0$, preventing a large overshoot in the transient response. On the other side, for large error, $\tilde{q} \approx -q_d$, we have small gain $\Psi_P(\tilde{q}) \approx K_P$, what prevent high control jump during the transient response. The parameter σ_P defines a bandwidth around stationary state $\tilde{q}_i = 0$ with high proportional gains influence. So, the maximal value of proportional gain K_P is determined by the maximal allowed control variable $u_{\max}$, $k_{PM} \leq |u_{\max}/q_{d,\max}|$, where $q_{d,\max}$ is the maximal value of q_d.

5. SIMULATION EXAMPLE

The manipulator used for simulation is a two revolute jointed robot (planar elbow manipulator) with numerical values of robot parameters which have been taken from (Kelly, 1995).

In Fig. 1-2 we can see comparison between controller with $\bar{K}_P = 0$ and controller with $\bar{K}_P \neq 0$. To make the comparison fair, the value of $\lambda_M\{\Psi_P(\tilde{q})\}$ will be same in both cases. We can see that for almost same quality of the transient response, controller in Fig. 2. has not a high jump of the control variable which can be seen for the controller in Fig. 1.

6. CONCLUSION

In this paper has been presented a new class of globally stable controllers for robot manipulators. Also, a new approach to performance tuning is proposed which provide fast transient response without oscillations and large overshoots, overcoming undesirable effect of high control jumps which is characteristic for conventional linear PID controllers. The performance tuning rule involve only few parameters which characterize the robot dynamics.

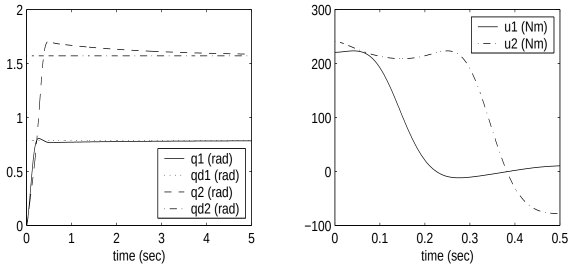

Fig. 2. The transient response for positions and control torque for $K_P = \text{diag}\{150\}$, $\bar{K}_P = \text{diag}\{450\}$ and for the optimal values $K_D = \text{diag}\{35.9\}$ and $K_I = \text{diag}\{265.3\}$.

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