FILTERING STRUCTURAL MODES IN AIRCRAFT: NOTCH FILTERS VS KALMAN FILTERS

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Abstract: The disturbances to an airframe's sensors caused by the flexibility of the airframe is called structural coupling. Notch filters are currently used to attenuate the effect that the structural modes have on the aircraft's inertial flight control sensors. However, this approach is becoming more problematic with modern combat aircraft. A complex model which is representative of an agile combat aircraft has been developed, and a Kalman filter has been designed to reduce the effects of structural coupling. This work forms part of an ongoing project in conjunction with BAE Systems. *Copyright © 2005 IFAC*

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1. INTRODUCTION

Structural Coupling is one of the main concerns of the designers of modern flight control systems (FCS) for agile combat aircraft. Currently sets of notch filters are used to attenuate the feedback signals from the problematic airframe flexible modes. The main difficulty is that these introduce phase lag, and for the demanding requirements of modern aircraft the effects are minimised by careful selection of the location of the notch filters in the sensor and actuator paths, a process generally undertaken by experienced structural coupling specialists. It has been proposed (Pearson, et al., 2000) that a Kalman filter could replace the notch filters in the FCS, which offers advantages in terms of the phase lag introduced into the primary flight control loops. The concept has been demonstrated on an experimental rig (Halsey, et al., 2002).

This paper presents a complex model of the rigid and flexible modes of an aircraft to represent the structural coupling problem. It demonstrates the benefits of the approach compared with the notch filter solution, and discusses some results looking at the tolerance of the Kalman filter to errors in the model, and the level of detail required by the filter. The paper has used a time-invariant Kalman-Bucy filter (Kalman and Bucy, 1961), and the analysis has been carried out using a continuous time model.

2. STRUCTURAL COUPLING

The phrase Structural Coupling (also known as Aeroservoelasticity) is used to describe the interactions between the structural dynamics, the aerodynamics and the Flight Control System (FCS)

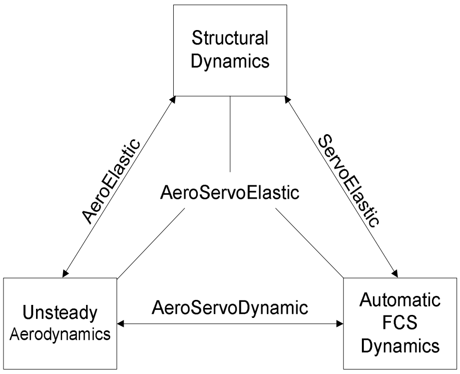

Figure 1 Airframe Interactions

of an aircraft (Felt, et al., 1979). These interactions are shown in Figure 1.

It can cause the propagation of high frequency signals (e.g. at the frequencies of the airframe's flexible modes) through the control system, and lead to undesirable oscillation of the control surfaces.

There are a number of different ways to minimise the effects of structural coupling, including the placement of sensors away from the nodes of key flexible modes, and through the design of the FCS. If these approaches do not work, then notch filters can be added to the control loop.

Structural coupling can affect any aircraft, but it is most critically noticed on combat aircraft. These aircraft are designed to be highly agile, and due to stringent performance requirements tend to have very tight stability margins. The phase lag that the notch filters introduce limits their use because of these tight stability limits, and adds significantly to the design difficulties.

2.1. Notch Filter Solution

The notch filters are strategically placed in the FCS control loop, as shown in Figure 2, in such a way to maximise their effectiveness, whilst minimising the inherent phase lag effects.

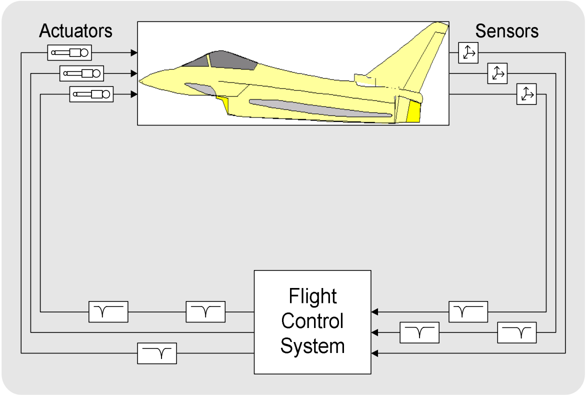

Figure 2 Current Notch Filter Solution

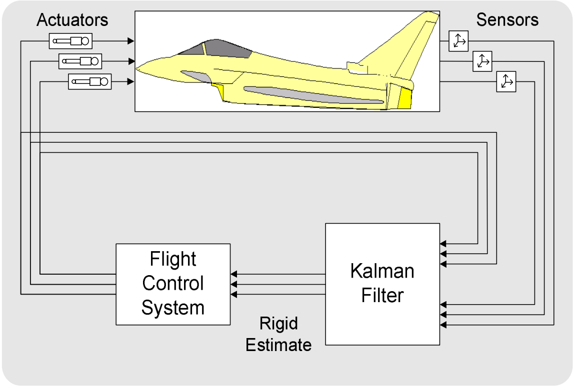

Figure 3 Proposed Kalman Filter Solution

The desire for lighter more agile aircraft and higher bandwidth control systems has made the use of notch filters less practical, and requiring an ever more complex design solution: whereas the Jaguar fly-by-wire aircraft required a single notch filter, and the forerunner of Eurofighter (the Experimental Aircraft Programme - EAP) required 7 notch filters in the pitch axis alone (Caldwell, 1996).

2.2. Kalman Filter Solution

The Kalman filter solution, as shown in Figure 3, has the potential to provide comparable performance to the notch filter solution in terms of attenuation of the structural modes, but with a reduction in the phase lag introduced, and a simpler design process. The basic principle is that the Kalman filter uses a model of the aircraft and the FCS outputs to calculate what the sensors would measure with a purely rigid aircraft, thereby removing the effects of the flexible components from the control loop. The key issues are to determine the appropriate complexity for the Kalman filter model, and also to ensure its robustness to parameter variations.

3. STRUCTURAL COUPLING MODEL

In order to provide a good representation of the structural coupling problem, a model has been developed in conjunction with BAE Systems. The model arrangement is shown in Figure 4.

This model concentrates on the longitudinal dynamics of an agile combat aircraft, and comprises a number of flexible modes and 3 rigid modes, where each mode ($flexi_{1\text{-}n}$, $rigid_{1\text{-}3}$) is represented by a row in the state space model. The model is derived from the matrix differential equation (Pratt, et al., 1994):

$$\mathbf{A}\ddot{q} + (\mathbf{D} + \sigma V\mathbf{B})\dot{q} + (\mathbf{E} + \sigma V^{2}\mathbf{C})q = 0 \tag{1}$$

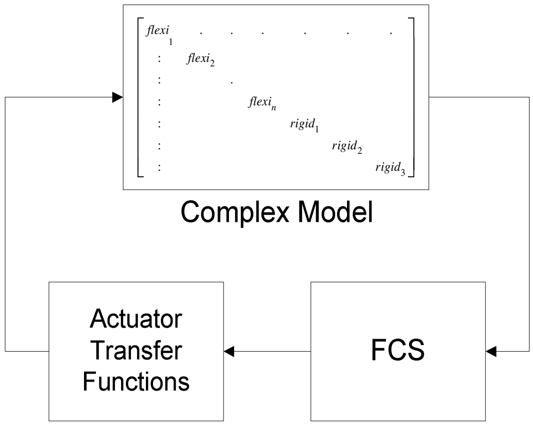

Figure 4 Complex Model

where V is the true airspeed, σ is the relative air density, and q is the vector of generalised coordinates. The matrices A, B, C, D and E are respectively the inertia, the aerodynamic damping and stiffness, and the structural damping and stiffness. Equation 1 can be converted to state space form, with the **a** matrix represented by:

$$\mathbf{a} = \begin{bmatrix} [\mathbf{0}] & [\mathbf{I}] \\ \left[-\left(\dfrac{\mathbf{E}+\sigma V^2\mathbf{C}}{\mathbf{A}}\right)\right] & \left[-\left(\dfrac{\mathbf{D}+\sigma V\mathbf{B}}{\mathbf{A}}\right)\right] \end{bmatrix} \qquad (2)$$

The model also includes a controller, which provides a linearised representation of an FCS, and a set of actuator transfer functions.

3.1. Model Size

In order to assess the trade-off between the accuracy of the Kalman filter and its processing requirements, the number of flexible modes incorporated into the aircraft model used in the Kalman filter can be altered. The model provided by finite element analysis of the aircraft has 80 flexible airframe modes, but this would need a 160 state model. Since this would take a significant amount of processing in the Kalman filter, a smaller number of modes (40 or 20) have been used for comparison purposes.

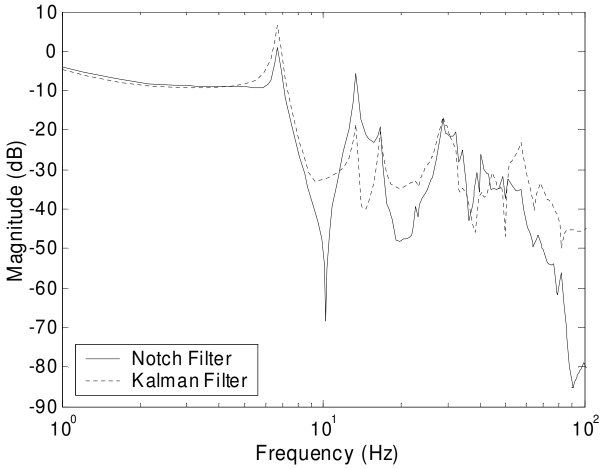

Figure 5 Initial Filter Comparison

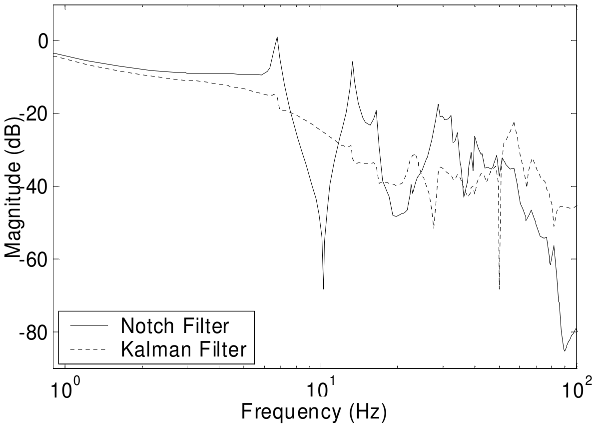

Figure 6 Decoupled Filter Comparison

3.2. Kalman Filter Design Parameters

In addition to the system model, the Kalman filter also requires the specification of the measurement and process noise in the system. In this case, the measurement parameters have been represented using typical aircraft sensor noise specifications. Initial values for the process noise have been calculated from the covariance of the states of the model in response to a sine wave input at the frequency of the first flexible mode.

3.3. Mode Decoupling

An initial comparison of the system with and without the Kalman filter indicated a potential problem with the filter design.

The Kalman filter removes the flexible modes by setting the estimate of each mode to zero as the output is calculated. Unfortunately, although this approach worked during the earlier stages of the project, the coupling between the rigid and flexible modes in the complex model prevented this from working as well in later stages.

The frequency response (magnitude), shown in Figure 5, indicates that, although some modes (e.g. 2nd mode at 11Hz) are attenuated, others (e.g. 1st mode at 6Hz) are not attenuated as well as the notch filter solution. Since the notch filter solution uses the minimum filtering possible, the poorer performance will be an issue.

$$\begin{bmatrix} [f] & [f-r] \\ [r-f] & [r] \end{bmatrix} \qquad (3)$$

Equation 3 shows the arrangement of the modes within the Kalman filter state matrix. The flexible modes are represented by matrix [f], the rigid modes are represented by [r], and the coupling between the modes by [f-r] and [r-f].

The poor performance has been overcome by decoupling the rigid and flexible modes in the Kalman filter, but this leads to an undesirable situation where the residuals of the Kalman filter will always be non-zero. The final solution adopted was to augment the Kalman filter with a second set of rigid modes: the first set is coupled and used internally to improve the estimation process; the second set is decoupled and used to create the output.

$$\begin{bmatrix} [f] & f-r & [0] \\ [r-f] & [r] & [0] \\ [0] & [r'] & [0] \end{bmatrix} \tag{4}$$

Equation 4 shows the structure of the state matrix after the Kalman filter has been augmented. The decoupled rigid modes are represented by [r'].

Figure 6 shows the impact upon the frequency response: decoupling has significantly improved the effectiveness of the Kalman filter; the frequency response of all the flexible airframe modes of interest has been attenuated; the peaks and troughs have been replaced by a much smoother curve.

4. RESULTS

The initial aim of the project was to find a replacement for the classical notch filter approach. It is therefore sensible that the filters are compared before any further study is discussed.

The structural coupling engineer is tasked to ensure that the frequency response meets certain stability margin requirements. These exclusion regions have been plotted as grey boxes on the Nichols charts, and can be seen in Figure 7. Note that these include not only the familiar exclusion region around the -180° point, but also the other regions at -540° and -900° that potentially contribute to instability. Figure 7 shows the comparison between the notch filter and Kalman filter solutions, and the unfiltered response is also included for comparison purposes.

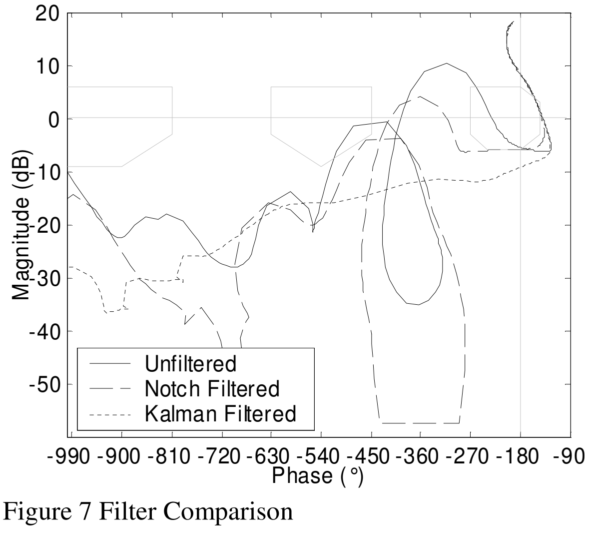

Figure 7 Filter Comparison

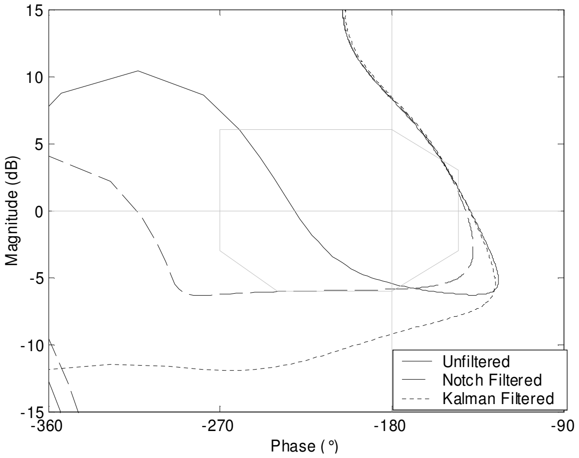

Figure 8 First Exclusion Region Frequency Response

The unfiltered (solid) response shows the need for the filtering, as one mode passes through both the first (around -180°) and second (-540°) exclusion regions. The notch filter solution (dashed) provides enough attenuation to clear both regions, but in the process adds a small amount of phase lag at low frequencies.

The Kalman filter response (dotted) provides more than adequate attenuation, filtering out most flexible modes effects.

Figure 8 shows the same Nichols chart, but zoomed in to show the stability margins around the first exclusion region. These stability margins are listed in Table 1.

Table 1 Complex Model Stability Margins

	Gain Margin	Phase Margin
Unfiltered	5.5 dB	41.0°
Notch Filter	5.9 dB	38.5°
Kalman Filter	9.2 dB	40.5°

These margins show that the Kalman filter provides 2° more phase margin than the notch filter. It also provides a significant improvement to the gain margin. Some of this gain margin could be traded for a higher phase margin, which is often of more use to FCS designers, leading to better system performance (this would be achieved by re-tuning the characteristics, but that has not been done here). A useful guide is that every dB of gain margin can in principle be converted to 5° of phase margin.

4.1. Filter Fidelity

The Kalman filter's primary disadvantage is that it is reliant on a mathematical model of the airframe. The more flexible modes that are included in this model, the higher the processing requirements; with limited resources this would result in lower sample rates. This section looks at finding a compromise solution between the two requirements. A comparison between different model sizes is shown in Figure 9.

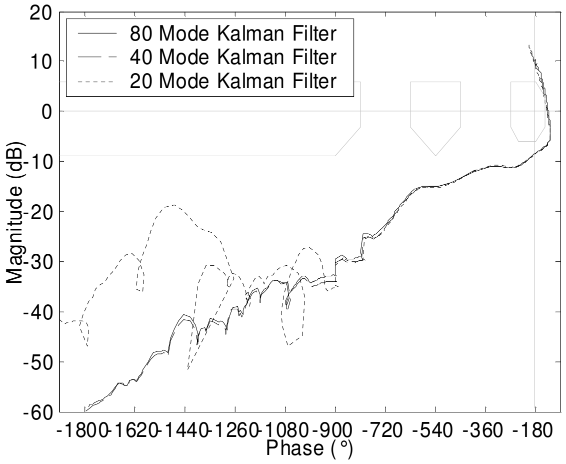

Figure 9 Kalman Filter Model Size Comparisons

Three Kalman filter configurations were designed: the first contained 80 flexible airframe modes, the second 40 and the third just 20 modes. From Figure 7 it can be seen that at lower frequencies (around the -180° exclusion region) there is little difference between the performance of the different filters. It can be seen that the frequency response for the 40 mode model (dashed) is very similar throughout the frequency range. Although the 20 mode model (dotted) deviates significantly beyond about -1000° of phase, but since the response does not come closer to the exclusion region than 10 dB, then these deviations are very unlikely to pose a problem. Hence it was decided to use the 20 mode model, which includes the effects of the flexible modes up to around 30Hz.

4.2. Filter Robustness

The flexible modes will alter as the configuration of the aircraft (e.g. stores, fuel content, or flight conditions) change. The Kalman filter will need to be able to handle these changes in some manner; either it has to be robust to changes in the aircraft configuration or the filter will need to be scheduled in some way with these changes (at present the notch filters are not scheduled).

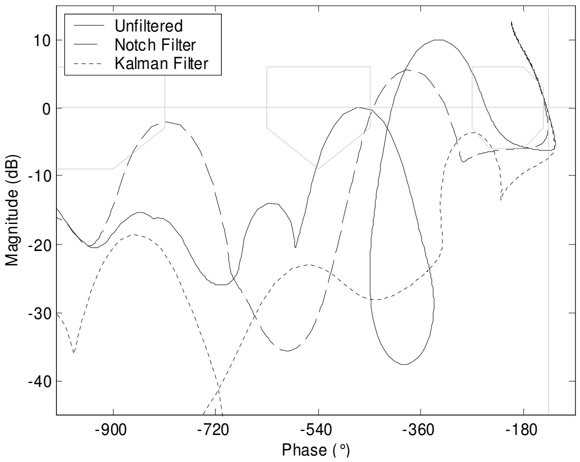

Figure 10 Effects of 7.5% Frequency Reduction

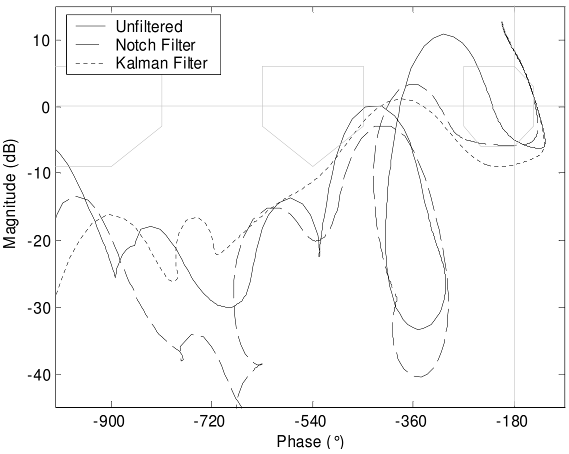

Figure 11 Effect of 7.5% Frequency Increase

These changes can be approximated by a ±7.5% change in the frequency of the flexible modes (typical effect of aircraft configuration changes).

This can be modelled in the system by changing the stiffness of the flexible component of the model by ±15%. The filter remains unchanged, and designed for the nominal case. Figure 10 shows the frequency response of the Kalman filter, compared with the unfiltered and notch filtered systems, when the flexible mode frequencies are reduced by 7.5%.

These responses can be compared to the nominal responses shown in Figure 7. There is a very small change to the unfiltered response, resulting in a slightly smaller encroachment into the first exclusion region, and a slightly larger encroachment in to the second region.

The changes have a small effect on the notch filter around the first region, but there is no encroachment. The notch filter response just touches the second region, but now slightly encroaches into the third region around -800°.

The Kalman filter solution doesn't filter the first mode as well as it did in the nominal situation, but although it just touches the first exclusion region, there is no encroachment.

Figure 11 shows the frequency response of the Kalman filter, compared with the unfiltered and notch filtered systems, when the flexible mode frequencies are increased by 7.5%.

With an increase in the frequency of the system, the unfiltered system response now has a larger impact on the first exclusion region, and a lower encroachment into the second region. It also has a significant effect on the third region.

The notch filter solution just clears both the first and second exclusion regions, although it does touch both of them. The impact of the unfiltered response on the third region is removed by the notch filters.

There is obviously some reduction in the performance of the Kalman filter solution, as is to be expected, although there are no encroachments into any of the exclusion regions. Around the first two regions, the Kalman filter performs in a similar manner to the notch filter, although there is an improvement in the gain margin. The problem mode around the third region also shows an improvement, although the effect is not as good as that achieved by the notch filter.

Overall however, although there are differences in the detail, the robustness of the notch and Kalman filter solutions are very similar

5. CONCLUSIONS

This paper shows that a model-based approach to structural coupling based upon Kalman filtering can offer a more effective solution than the conventional notch filters. It will be a little more complicated in a computational sense than the notch filters, but the filter can be implemented as a single software block, whereas the notch filters are ‘distributed’ around the FCC software. In principle the design process is simpler, because the physical understanding that the structural coupling specialist uses for notch filter design is embedded within the mathematical rigour of the model-based approach. The design problem for the Kalman filter becomes one of tuning its performance via the process covariance matrix.

Work is continuing to include expanding the complex model to cover a wider envelope of flight conditions, and examining both the effects and methods of tuning the Kalman filter to improve its performance.

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