FORMATION STABILIZATION OF NONLINEAR VEHICLES BASED ON DYNAMIC INVERSION AND PASSIVITY

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Abstract: This paper proposes a hierarchical formation stabilization method for vehicles having nonlinear dynamics. Supposing that the formation control problem is already solved for the case of linear vehicle dynamics, the method proposes a dynamic inversion based low-level control, which linearizes, at least approximately, the original vehicle dynamics so that the formation control can be applied. In this way a hierarchical control system is obtained, which is then completed with a passivity based stabilization procedure for the stability of the entire system can be guaranteed.

Keywords: formation control, dynamic inversion, passivity

1. INTRODUCTION

In the last years the increased computational capabilities of computer systems and the rapid development in the communication and sensor technologies have made it possible to build highly autonomous unmanned vehicles, which can perform, individually or cooperating in a group, complex tasks (traffic on an automated highway, motion in a platoon formation, etc.) in the presence of uncertainty and disturbances (e.g. (Li *et al.*, 2002)). The control of autonomous vehicles is generally hierarchical, where the components on the lower levels are local, in the sense that they depend on the particular - and generally nonlinear - vehicle dynamics. These local controllers modify the original vehicle dynamics so that the dynamic behavior of the closed loop can be modelled by a linear system. This linear model, which can be the same for different vehicles, is then used in the design of the higher-level control components, which depend on the prescribed cooperative tasks. Therefore it is sufficient to solve the complex, task-dependent control problem for the case of linear systems only, and the obtained controllers will be independent from the particular vehicles (for high-level control design see e.g. (Borrelli *et al.*, 2004), which proposes a model predictive control method or (Tanner *et al.*, 2003), which uses artificial potential functions).

It is clear that the stability of the entire hierarchically controlled formation is a key issue in the controller design. Despite of this, the cooperative control literature concentrates mainly on the high-level control and the stability problem of the coupled system remains open in most cases.

This paper proposes a dynamic inversion based low-level control, which fits to the most high-level control framework and it can be completed with a passivity based stabilization procedure, which guarantees the stability of the entire formation.

2. PROBLEM FORMULATION AND THE OUTLINE OF SOLUTION

Suppose there is given a formation control problem, which has to be solved by vehicles moving on a 2D plane (in 3D a similar argument can be applied.) This control problem generally a prescribed cooperative motion, e.g. a geometric formation shaping, obstacle avoidance, cooperative trajectory tracking, or a simple collective motion called 'flocking'. We do not need to specify this problem more precisely, since in this paper we assume that this control problem has already been solved for the special case when every vehicle has the following simple double-integrator dynamics

$$\dot{q}_i = p_i \quad \dot{p}_i = v_i \tag{1}$$

where $q_i, p_i, v_i \in \mathbb{R}^2$ denote the vector of position, velocity and acceleration of vehicle i defined in a fixed coordinate frame K_0. In most cases the formation control problem is formulated in a moving coordinate system K, the orientation and the origin of which are given by the time-varying functions $\varphi_K(t)$ and $q_K(t)$ respectively. The equations (1) in K can be expressed in the following form:

$$\begin{aligned}
\tilde{q}_i &= R_{-\varphi_d}(q_i - q_d) \\
\dot{\tilde{q}}_i &= \dot{R}_{-\varphi_d}(q_i - q_d) + R_{-\varphi_d}(p_i - p_d) := \tilde{p}_i \\
\dot{\tilde{p}}_i &= \ddot{R}_{-\varphi_d}(q_i - q_d) + 2\dot{R}_{-\varphi_d}(p_i - p_d) \\
&\quad + R_{-\varphi_d}(v_i - \ddot{q}_d)
\end{aligned} \tag{2}$$

By setting

$$\begin{aligned}
v_i = \ddot{q}_d + R^{-1}_{-\varphi_d}\Big[&-\ddot{R}_{-\varphi_d}(q_i - q_d) \\
&-2\dot{R}_{-\varphi_d}(p_i - p_d) + \tilde{v}_i\Big]
\end{aligned} \tag{3}$$

we get the equivalent of dynamics (1) in K:

$$\dot{\tilde{q}} = \tilde{p} \quad \dot{\tilde{p}} = \tilde{v} \tag{4}$$

where $\tilde{q} = [\tilde{q}_1, \ldots, \tilde{q}_N]$, $\tilde{p} = [\tilde{p}_1, \ldots, \tilde{p}_N]$, and $\tilde{v} = [\tilde{v}_1, \ldots, \tilde{v}_N]$. Due to our assumption the formation control problem is solved, i.e. there exists a formation controller $\tilde{v} = \tilde{v}_c(\tilde{q}, \tilde{p})$, generated by an appropriate, but arbitrary control algorithm, s.t. the coupled system (4) is asymptotically stable with appropriate Lyapunov function $\mathcal{V}(\tilde{q}, \tilde{p})$, and the trajectories of (4) converge to a state $(\tilde{q}^*, \tilde{p}^*)$ representing the prescribed formation.

Our purpose is to apply the above formation control method for vehicles having nonlinear dynamics and to ensure the asymptotic stability of the coupled dynamics obtained.

Suppose that the dynamics of the vehicles can be given in the following form:

$$\begin{aligned}
y &= x_1 \\
\dot{x}_1 &= h(Cx_2) \\
\dot{x}_2 &= A(\rho(t))x_2 + B(\rho(t))u
\end{aligned} \tag{5}$$

where $x_1, \dot{x}_1$ represent the position- and velocity vector of the vehicle in a fixed coordinate system; both are supposed to be measured by appropriate inertial and/or GPS sensors. The output map $h(\cdot)$ represents an invertible coordinate transformation and the matrix $C \in \mathbb{R}^{k_1 \times k_2}$, $k_1 < k_2$ is constant. We assume that the time-varying parameters collected in the vector $\rho(t)$ are also available for measurement. The class of systems determined by (5) is able to describe the dynamics of the most aerial and road vehicles (Kiencke and Nielsen, 2000), that is why it was chosen in our work.

To solve the formation stabilization problem formulated above a two level hierarchical control framework is proposed. In this framework the formation control designed for the simplified vehicle dynamics forms the high-level controller. On the low-level a dynamic inversion based control is applied to transform the nonlinear dynamics (5) to the required double integrator form (Sec.3) It will be seen that this linearization can be solved approximately only, i.e. the resulted double integrator will be coupled with the dynamics of the estimation error of the unmeasured states in (5). Due to the presence of this additional error dynamics the simple connection of the high-level and the low-level controllers does not necessarily yields globally stable closed loop behavior. Thus, in the second step of the design procedure (Sec. 4), we design a passivity-based external controller, which stabilizes the entire system. The obtained control structure is then tested on a formation control problem of road vehicles (Sec. 5).

3. DYNAMIC INVERSION BASED LOW-LEVEL CONTROLLER DESIGN

In this section a dynamic inversion based linearizing low-level controller is applied to transform, at least approximately, the nonlinear vehicle dynamics into simple double integrator. (The detailed discussion of inversion can be found e.g. in (Szabó *et al.*, 2003)). Following the control design procedure discussed in detail in (Péni and Bokor, 2004*b*) we first apply the following state transformation in order to separate the output-related, measurable states from the others:

$$\Phi : \begin{bmatrix} x_1 = y \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} z_1 = x_1 = y \\ z_2 = \dot{x}_1 = h(Cx_2) \\ z_3 = C^{\perp}x_2 \end{bmatrix} \tag{6}$$

where $\begin{bmatrix} C \\ C^{\perp} \end{bmatrix}$ is invertible. The original system can be rewritten in z coordinates as

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= JCA\tilde{C}h^{-1}(z_2) + JCA\tilde{C}^{\perp}z_3 + JCBu \\
&= Jf_2(z_2) + JA_2z_3 + JB_2u \\
\dot{z}_3 &= C^{\perp}A\tilde{C}h^{-1}(z_2) + C^{\perp}A\tilde{C}^{\perp}z_3 + C^{\perp}Bu \\
&= f_3(z_2) + A_3z_3 + B_3u
\end{aligned} \tag{7}
$$

where $\begin{bmatrix} C \\ C^{\perp} \end{bmatrix}^{-1} = \begin{bmatrix} \tilde{C} \; \tilde{C}^{\perp} \end{bmatrix}$ and $J = \frac{\partial h}{\partial C x_2}$. It is important to keep in mind that the matrices in the above equations are not constant, they depend on the time-varying parameter ρ and/or the state variables $\tilde{x}_2 = Cx_2$.

By inverting the dynamics (7) the linearizing dynamic controller can be obtained in the following form :

$$
\begin{aligned}
u_c &= B_2^{-1}J^{-1}(-Jf_2(z_2) - JA_2z_{3c} + v) \\
\dot{z}_{3c} &= f_3(z_2) + A_3z_{3c} + B_3u_c - w = \\
&= (A_3 - B_3B_2^{-1}A_2)z_{3c} + f_3(z_2) \\
&\quad - B_3B_2^{-1}f_2(z_2) - B_3B_2^{-1}J^{-1}v - w \\
&= (A_3 - B_3B_2^{-1}A_2)z_{3c} + u_c^*
\end{aligned} \tag{8}
$$

where v and w are additional control inputs derived later on. For the applicability of u_c it has to be bounded, which follows if u_c^* is bounded and the internal dynamics

$$
\dot{z}_{3c} = (A_3 - B_3B_2^{-1}A_2)z_{3c} \tag{9}
$$

– which is, in fact, the zero dynamics ((Isidori, 1995)) of the original system (7) – is stable. From now on we suppose that the stability of (9) holds.

Applying the controller (8) to the system (7) we get the following closed-loop dynamics

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= JA_2(z_3 - z_{3c}) + v \\
\dot{z}_3 - \dot{z}_{3c} &= A_3(z_3 - z_{3c}) + w
\end{aligned} \tag{10}
$$

Denoting the position z_1 and the velocity z_2, as before, by q and p the dynamics of the controlled vehicles can be written as

$$
\begin{aligned}
\dot{q}_i &= p_i \\
\dot{p}_i &= v_i + J_iA_{2,i}e_i \\
\dot{e}_i &= A_{3,i}e_i + w_i
\end{aligned} \tag{11}
$$

which is, apart from the dynamics of the approximation error, equivalent to a double-integrator.

4. PASSIVITY BASED FORMATION STABILIZATION

Now, being in possession of the high-level formation and the low-level linearizing controllers we can build up the hierarchical control structure. For this, let us substitute $\tilde{v}_i^c$ into (11) and apply (2) to get the vehicle dynamics in K:

$$
\begin{aligned}
\dot{\tilde{q}}_i &= \tilde{p}_i \\
\dot{\tilde{p}}_i &= \tilde{v}_i^c(\tilde{q}, \tilde{p}) + R_{-\varphi_d}J_iA_{2,i}e_i \\
\dot{e}_i &= A_{3,i}e_i + w_i
\end{aligned} \tag{12}
$$

If $\tilde{q} = [\tilde{q}_1, \ldots, \tilde{q}_N]$, $\tilde{p} = [\tilde{p}_i, \ldots, \tilde{p}_N]$, $\tilde{A}_2 = \mathrm{diag}(R_{-\varphi_d}J_iA_{2,i})$ and $\tilde{A}_3 = \mathrm{diag}(A_{3,i})$ the equations above take the following more compact form:

$$
\begin{aligned}
\dot{\tilde{q}} &= \tilde{p} \\
\dot{\tilde{p}} &= \tilde{v}_c(\tilde{q}, \tilde{p}) + \tilde{A}_2e \\
\dot{e} &= \tilde{A}_3e + w
\end{aligned} \tag{13}
$$

The system we got is the coupled dynamics of the formation, which has to be asymptotically stabilized by appropriate external control input w.

Notice that the equations (13) realizes a partial interconnection of the following subsystems

1. $\dot{e} = \tilde{A}_3e + w$
2. $\dot{\tilde{q}} = \tilde{p}$, $\dot{\tilde{p}} = \tilde{v}_c(\tilde{q}, \tilde{p})$

We solve the stabilization problem by using passivity-based technique in the following way: first new inputs and outputs are chosen for the subsystems with respect to which they will be passive. Then the control input w is set so that the dynamics (13) realizes a negative feedback interconnection of the subsystems, which consequently will be asymptotically stable (van der Schaft, 2000).

Since we have supposed in section SEC that the subsystem 2 is asymptotically stable with Lyapunov function $\mathcal{V}(\tilde{q}, \tilde{p})$, then by calculating the time derivative of $\mathcal{V}$ we get hints for the choose of input u_2 and output y_2:

$$
\begin{aligned}
\frac{d\mathcal{V}}{dt} &= \underbrace{\frac{\partial \mathcal{V}(\tilde{q}, \tilde{p})}{\partial \tilde{q}}\tilde{p} + \frac{\partial \mathcal{V}(\tilde{q}, \tilde{p})}{\partial \tilde{p}}\tilde{v}_c}_{<0} \\
&\quad + \underbrace{\frac{\partial \mathcal{V}(\tilde{q}, \tilde{p})}{\partial \tilde{p}}\tilde{A}_2}_{y_2^T} \underbrace{e}_{u_2} \leq y_2^T u_2
\end{aligned} \tag{14}
$$

i.e. the subsystem 2 is passive with storage function $\mathcal{V}$. In order to carry out a similar input/output selection procedure for the subsystem 1, it is needed to be asymptotically stable. For this we assume that the dynamics of the approximation error $\dot{e}_i = A_{3,i}e_i$ is quadratically stable for all i with Lyapunov functions $\mathcal{W}_i = \frac{1}{2}e_i^TW_ie_i$. Thus the coupled error dynamics is also quadratically stable with Lyapunov function $\mathcal{W}(e) = \frac{1}{2}e^TWe$, where $W = \mathrm{diag}(W_i)$. By deriving the Lyapunov function we can chose appropriate input and output, i.e.

$$\frac{d\mathcal{W}}{dt} = \underbrace{e^T W \tilde{A}_3 e}_{<0} + \underbrace{e^T}_{y_1^T} \underbrace{Ww}_{u_1} \leq y_1^T u_1 \quad (15)$$

So, the subsystem 1 is also passive with respect to the chosen input u_1 and output y_1 with storage function $\mathcal{W}(e)$.

Notice that the partial interconnection of subsystem 1 and 2, coming from the original structure (13), can be expressed by the following relation $u_2 = y_1$. (The interconnected structure is depicted in Fig. 1) In order to achieve the negative feedback interconnection we have to set $u_1 = -y_2$ as it can be seen in Fig. 1. This means that the external control input w has to be chosen as follows

$$w = -W^{-1} \tilde{A}_2^T \frac{\partial \mathcal{V}(\tilde{q}, \tilde{p})}{\partial \tilde{p}} \quad (16)$$

To prove the asymptotic stability of the entire system we prove first that the interconnected system is passive with storage function $\mathcal{V}(\tilde{q}, \tilde{p}) + \mathcal{W}(e)$ and then we will see that this function can serve as Lyapunov function in our special case. Let us introduce two new, external inputs denoted by u_{e1} and u_{e2} respectively according to Fig. 1. By calculating the time-derivative of $\mathcal{V}(\tilde{q}, \tilde{p}) + \mathcal{W}(e)$:

$$\frac{d}{dt}\left\{\mathcal{V}(\tilde{q}, \tilde{p}) + \mathcal{W}(e)\right\} = \underbrace{\frac{\partial \mathcal{V}}{\partial \tilde{q}} \tilde{p} + \frac{\partial \mathcal{V}}{\partial \tilde{p}} \tilde{v}_c}_{<0} + \underbrace{e^T W \tilde{A}_3 e}_{<0}$$

$$+ y_2^T u_{2e} + y_1^T u_{1e} \leq \begin{bmatrix} y_1^T & y_2^T \end{bmatrix} \begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix} \quad (17)$$

i.e. the interconnected system is passive with respect to input $\begin{bmatrix} u_{1e} \\ u_{2e} \end{bmatrix}$ and output $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ with storage function $\mathcal{V}(\tilde{q}, \tilde{p}) + \mathcal{W}(e)$. In our case the external inputs u_{e1} and u_{e2} are 0 thus $\dot{\mathcal{V}}(\tilde{q}, \tilde{p}) + \dot{\mathcal{W}}(e) \leq 0$. Since $\dot{\mathcal{V}}(\tilde{q}, \tilde{p}) + \dot{\mathcal{W}}(e) = 0$ only if $\tilde{q} = \tilde{q}^*$, $\tilde{p} = \tilde{p}^*$ and $e = 0$ the asymptotic stability of the prescribed formation $(\tilde{q}, \tilde{p}, e) = (\tilde{q}^*, \tilde{p}^*, 0)$ follows.

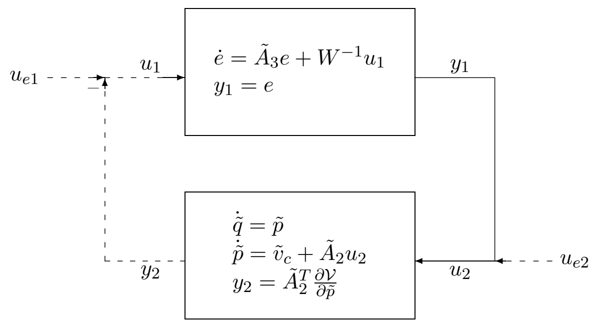

Fig. 1. Interconnection of passive subsystems

5. FORMATION CONTROL OF ROAD VEHICLES

As an illustrative example for the presented method we solve in this section a formation stabilization problem of road vehicles. Suppose that there is given 5 vehicles in an arbitrary configuration on the 2D plane and we intend to steer them to a straight row, which is perpendicular to a time-varying spatial trajectory $q_d(t)$ prescribed for the entire group to follow. Suppose the vehicle dynamics is described by the nonlinear single-track model, given in the form of (5) as follows:

$$\begin{aligned} y &= x_1 \\ \dot{x}_1 &= \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} v\cos(\beta+\psi) \\ v\sin(\beta+\psi) \end{bmatrix} = h(\beta+\psi, v) \\ &= h(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x_2) = h(Cx_2) \end{aligned} \quad (18a)$$

$$\dot{x}_2 = \begin{bmatrix} \dot{\beta}+\dot{\psi} \\ \dot{v} \\ \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{a_{11}}{v} & \frac{a_{12}}{v^2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{a_{11}}{v} & \frac{a_{12}}{v^2} - 1 \\ 0 & 0 & a_{21} & \frac{a_{22}}{v} \end{bmatrix} \begin{bmatrix} \beta+\psi \\ v \\ \beta \\ r \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{b_1}{v} & 0 \\ 0 & 1 \\ \frac{b_1}{v} & 0 \\ b_2 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \alpha \end{bmatrix} = A(v)x_2 + B(v)u \quad (18b)$$

where (x, y) denotes the position of the vehicle on the plane in a fixed coordinate frame K_0 and v, β, r, ψ are the velocity, slideslip angle, yaw rate and orientation respectively. The control inputs are the steering angle (δ) and acceleration (α). As outputs the position coordinates x and y were chosen, both are are supposed to be measured by appropriate inertial and/or GPS sensors. Throughout the paper we moreover assume that the length (v) and the arc $(\beta+\psi)$ of the velocity vector are also measured and available. The remaining parameters of the model are constant and supposed to be known. Notice that in (18) the time-varying parameter – denoted by ρ in (5) – equals to a state variable (v), so the dynamics (18b) represents a *quasi-LPV* system.

High-level control design In order to solve the formation control problem formulated above, we have to solve it first for the case of vehicles having double integrator dynamics. For this, we first rewrite the position, velocity and acceleration of the vehicles in a moving coordinate frame K, which is fixed to the formation so that at time t its origin is $q_d(t)$ and its axes are assigned by the vectors $p_d(t)$ and $p_d^{\perp}(t)$ (see Fig. 2). In K the position, velocity and acceleration of vehicle i are denoted as before by $\tilde{q}_i$, $\tilde{p}_i$ and $\tilde{u}_i$ respectively. In

K the formation is a fixed straight line coinciding with the vertical axis. The problem of finding a control input, which steers the vehicles in a row along the vertical axis can be easily solved by considering the vehicles as simple point masses and constructing an artificial potential field having minimum at the desired configuration. It can be easily checked that the following function is a possible candidate to determine the potential field:

$$V(\tilde{q}) = \sum_{i=1}^{N} \left(\mu(\delta(\tilde{q}_i)) + \sum_{j,j\neq i} \mu(d - \|\tilde{q}_i - \tilde{q}_j\|) \right) \tag{19}$$

where d denotes the prescribed inter-vehicle distance and $\delta(\tilde{q}_i)$ is the distance of vehicle i from the formation defined as in Fig. 2 i.e.

$$\delta(\tilde{q}_i) = \begin{cases} \left\| \tilde{q}_i - \begin{bmatrix} 0 \\ \mathrm{sign}(\tilde{q}_{i,y}) \cdot 2d \end{bmatrix} \right\| & \text{if } |\tilde{q}_{i,y}| > 2d \\ \tilde{q}_{i,x} & \text{if } |\tilde{q}_{i,y}| \leq 2d \end{cases}$$

and $\mu(\cdot) : \mathbb{R} \to \mathbb{R}^+$ is an appropriately constructed continuous scaling function satisfying the following conditions: $\mu(x) = 0$ if $x \leq 0$ and $\mu'(x) > 0$ if $x > 0$. In this paper

$$\mu(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ \frac{1}{2} m x^2 & \text{if } 0 \leq x \leq \frac{M}{m} \\ Mx - \frac{1}{2}\frac{M^2}{m} & \text{if} \quad \frac{M}{m} < x \end{cases} \tag{20}$$

If the total energy of the point-mass system is chosen as Lyapunov function for the formation i.e.

$$\mathcal{V}(\tilde{q}, \tilde{p}) = V(\tilde{q}) + \frac{1}{2}\|\tilde{p}\|^2 \tag{21}$$

the control input

$$\tilde{v} = -\frac{\partial V(\tilde{q})}{\partial \tilde{q}} - k\tilde{p} \quad k > 0 \tag{22}$$

i.e. $\tilde{v}_i = -\frac{\partial V(\tilde{q}_i)}{\partial \tilde{q}_i} - k\tilde{p}_i$ will stabilize the formation by rendering the time derivative of $\mathcal{V}(\tilde{q}, \tilde{p})$ negative:

$$\begin{aligned} \dot{\mathcal{V}}(\tilde{q}, \tilde{p}) &= \frac{\partial V}{\partial \tilde{q}}\tilde{p} - \tilde{p}^T \frac{\partial V}{\partial \tilde{q}} - k\tilde{p}^T\tilde{p} \\ &= -k\|\tilde{p}\|^2 \leq 0 \end{aligned} \tag{23}$$

Low-level control design. Expressing the vehicle dynamics in z-coordinates, we get

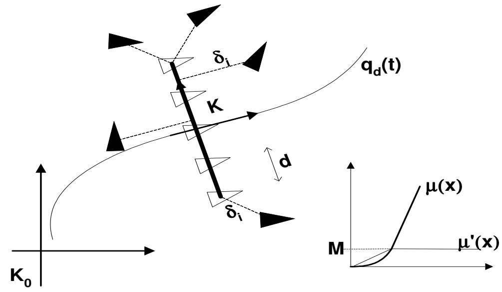

Fig. 2. Intended formation and calculation of $\delta(\cdot)$ and $\mu(\cdot)$

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= J \begin{bmatrix} \frac{a_{11}}{v} & \frac{a_{12}}{v^2} \\ 0 & 0 \end{bmatrix} z_3 + J \begin{bmatrix} \frac{b_1}{v} & 0 \\ 0 & 1 \end{bmatrix} u \\ &= JA_2 z_3 + JB_2 u \\ \dot{z}_3 &= \begin{bmatrix} \frac{a_{11}}{v} & \frac{a_{12}}{v^2} - 1 \\ a_{21} & \frac{a_{22}}{v} \end{bmatrix} z_3 + \begin{bmatrix} \frac{b_1}{v} & 0 \\ b_2 & 0 \end{bmatrix} u \\ &= A_3 z_3 + B_3 u \end{aligned} \tag{24}$$

where $J = \partial h = \begin{bmatrix} -v\sin(\beta+\psi) & \cos(\beta+\psi) \\ v\cos(\beta+\psi) & \sin(\beta+\psi) \end{bmatrix}$ In this case the dynamic controller (8) can be given by:

$$\begin{aligned} u_c &= B_2^{-1} J^{-1}(-JA_2 z_{3c} + v) \\ \dot{z}_{3c} &= (A_3 - B_3 B_2^{-1} A_2) z_{3c} + B_3 B_2^{-1} J^{-1} v - w \end{aligned} \tag{25}$$

For the boundedness of u_c the stability of the dynamics

$$\begin{aligned} \dot{z}_{3c} &= (A_3 - B_3 B_2^{-1} A_2) z_{3c} \\ &= \begin{bmatrix} 0 & -1 \\ a_{21} - \frac{b_2 a_{11}}{b_1} & \frac{1}{v}(a_{22} - \frac{b_2 a_{12}}{b_1}) \end{bmatrix} z_{3c} \end{aligned} \tag{26}$$

has to be checked. Notice that (26) is not the zero-dynamics now, since the zero output involves zero velocity, where the vehicle model (5) is not valid.

It was already shown in (Péni and Bokor, 2004a) that the dynamics (26) is globally quadratically stable independently from the actual values of the parameters a_i, a_{ij}.

Formation stabilization. Applying the proposed passivity based stabilizing procedure the following external input w can be obtained:

$$\begin{aligned} w &= -W^{-1}\tilde{A}_2^T \frac{\partial V}{\partial \tilde{p}} \\ &\Downarrow \\ w_i &= -W_i^{-1}(R_{-\varphi_d} J_i A_{2,i})^T \tilde{p}_i \end{aligned} \tag{27}$$

where $\mathcal{W}_i = \frac{1}{2}e_i^T W_i e_i$ is an appropriate Lyapunov function proving the stability of the LPV error dynamics

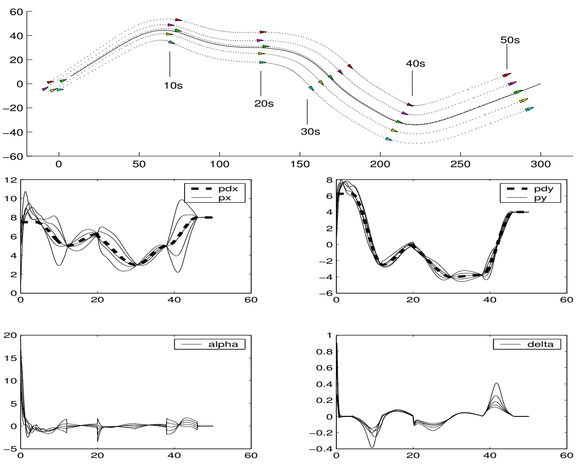

Fig. 3. Simulation results. The motion of the vehicles along the prescribed trajectory (top), tracking of the velocity reference signal $p_d(t) = [p_{d,x}(t)\ p_{d,y}(t)]$ (centre) and control inputs δ and α (bottom).

$$\dot{e}_i = A_{3,i} e_i = \begin{bmatrix} \frac{a_{11,i}}{v} & \frac{a_{12,i}}{v^2} - 1 \\ a_{21,i} & \frac{a_{22,i}}{v} \end{bmatrix} e_i \qquad (28)$$

Simulation results The formation control was tested by numerical simulation. The vehicles in the formation have the following identical modelling parameters obtained by identifying a heavy-duty vehicle: (Rödönyi, 2003):

$$a_{11} = -147.1481 \ a_{12} = 0.0645 \ a_{21} = 0.0123$$
$$a_{22} = -147.1494 \ b_1 = 66.2026 \ b_2 = 31.9835$$

If in this case $1 \leq v \leq 25$ the dynamics (28) is quadratically stable with Lyapunov function

$$\mathcal{W}_i = e_i^T \begin{bmatrix} 246.7608 & -4.7350 \\ -4.7350 & 247.7231 \end{bmatrix} e_i \ \ \forall i$$

The common trajectory $q_d(t)$ was constructed as a concatenation of polynomials of first and second order, which were defined so that the trajectory fits smoothly to the following reference points:

$$\begin{bmatrix} s_1 \ldots s_7 \end{bmatrix} = \begin{bmatrix} 0 & 60 & 100 & 150 & 180 & 220 & 300 \\ 0 & 50 & 30 & 30 & -10 & -40 & 0 \end{bmatrix}$$

The simulation results in case of controller parameters $M = 6$, $d = 4$, $m = 2$, $k = 2$ can be seen in Fig.3. It can be seen that the vehicles follow the prescribed trajectory in the intended formation while the control inputs remain in a realizable range.

6. CONCLUSIONS AND FURTHER WORK

This paper proposed a hierarchical formation stabilization method for vehicles having nonlinear dynamics. The method makes it possible to design the high- and the low-level control algorithms independently, while the stability of the entire system is guaranteed. Through an application example we showed that the nonlinear single-track model of road vehicles satisfies the requirements of the control method so it suits well to this control framework. It is important to keep in mind that the availability of a precise vehicle model was assumed during the controller design. For the practical applicability the presented methods, especially the low-level controller, are expected to be robust against disturbances and modelling uncertainties. However the increase of robustness of the dynamic inversion based controllers is generally a complex problem, which requires further researches.

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