MULTI–INPUT SECOND ORDER SLIDING MODE CONTROL OF NONHOLONOMIC SYSTEMS

Antonella Ferrara and Luisa Giacomini

Dipartimento di Informatica e Sistemistica
Università di Pavia - Via Ferrata 1 - 27100 Pavia - Italy
e–mail: antonella.ferrara@unipv.it

Abstract: The multi–input second order sliding mode control of nonholonomic systems is addressed in this paper. The crucial point is the necessity of describing the system in new coordinates, so as to be capable of selecting a suitable sliding manifold upon which to enforce a second order sliding mode. The procedure adopted in this paper is backstepping–based. The advantage of our proposal relies on its applicability also in presence of a significant class of uncertainties, as well as on the possibility of constructing a continuous, i.e., chattering–free, control.

Keywords: Nonlinear systems, uncertain systems, sliding–mode control, variable–structure systems.

1. INTRODUCTION

The problem of controlling the so–called nonholonomic systems has attracted the attention of researchers for years (Kolmanovsky and McClamroch, 1995). The key difficulty is tied to the fact that this class of systems, very common in practical applications, see, for instance, the kinematic model of wheeled mobile robots (Jiang and Nijmeijer, 1997), does not satisfy the Brockett's necessary smooth feedback stabilization condition (Brockett, 1983).

The control problem is further complicated whenever uncertainties of various nature affect the nonholonomic system model (think of the localization errors in the case of mobile robots), or only saturated input signals are available to perform the control task (e.g., because of actuator power limits). This sort of difficulties have spurred many researchers to investigate the possibility of applying sliding mode control to this context (Guldner and Utkin, 1995; Floquet *et al.*, 2003; Ge *et al.*, 2003). Yet, when a control problem involving a nonholonomic system is faced in more general terms, that is making reference not directly to the kinematic model of a wheeled mobile robot, but to a multi–input chained form system with some kind of uncertainties, the application of sliding mode control does not seem straightforward. Some preliminary steps to transform the system into a suitable form with matched uncertainties need to be taken. Following the idea already developed in (Bartolini *et al.*, 2000*b*) with reference to nonlinear uncertain systems in some triangular feedback forms, in this paper we investigate the possibility of coupling a partial transformation of the nonholonomic system via a backstepping–based procedure, in analogy with (Jiang, 1996), with a multi–input second order sliding mode control approach (Bartolini *et al.*, 2000*a*). The advantage of our proposal relies on its applicability also in case of uncertainties (disturbances or modelling inaccuracy) affecting the system model. Moreover, the design procedure is carried out so that the control role is played by the control vector derivative: as a result, while this latter is constructed as a discontinuous signal, guaranteeing the attainment of a second order sliding mode on a pre–specified sliding manifold, the actual control is continuous and no chattering effect is present.

It should be noted that the application of the backstepping procedure to nonholonomic systems with uncertainties was already discussed, for instance, in (Jiang and Pomet, 1995). Subsequently, relying on (Jiang, 1996), and (Astolfi, 1996), the idea has been extended to perturbed nonholonomic chains of integrators (Jiang, 2000). On the other hand, also higher–order sliding modes have already been applied to the stabilization of nonholonomic perturbed systems in (Floquet *et al.*, 2003), yet requiring the knowledge of the derivative of the sliding variable, which, in contrast, in the present paper, is assumed to be inaccessible.

2. THE NONHOLONOMIC FORM

In (Jiang, 1996) a backstepping–based (Kokotović *et al.*, 1995) procedure has been designed to stabilize the multi–input chained form system

$$\begin{cases} \dot{x}_0 &= u_0 \\ \dot{x}_{i1} &= x_{i2} u_0 \\ &\vdots \\ \dot{x}_{i(n_i-1)} &= x_{in_i} u_0 \\ \dot{x}_{in_i} &= u_i, \qquad 1 \le i \le m \end{cases} \tag{1}$$

where $x^T = [x_0, x_1^T, \ldots, x_m^T] \in I\!R^n$, with $n = 1 + \sum_{i=1}^m n_i$, since $x_i^T = [x_{i1}, \ldots, x_{in_i}] \in I\!R^{n_i}$, and u_j, $j = 0, \ldots, m$, are scalar control variables. In this paper, in accordance with (Bartolini *et al.*, 2000*b*; Ferrara and Giacomini, 2000), we shall use a partial backstepping–based transformation combined with a number of steps aimed at constructing a discontinuous multi–input control strategy robust with respect to uncertainties appearing in the equations relevant to $\dot{x}_{in_i}$, $i = 1, \ldots, m$, and with respect to disturbances affecting u_0. In the next section, for the reader's convenience, the use of backstepping in the context of nonholonomic systems is briefly recalled making reference, in particular, to (Jiang, 1996).

3. BACKSTEPPING–BASED DESIGN FOR NONHOLONOMIC SYSTEMS

The backstepping design procedure in case of multi–input systems and with reference to a regulation objective, consists in the step–by–step construction of a transformed system with state

$$z_{ij} = x_{ij} - \alpha_{i(j-1)} \tag{2}$$

$i = 1, \ldots, m$, $j = 1, \ldots, n_i$, where $\alpha_{i(j-1)}$, with $\alpha_{i0} = 0$, is the so–called virtual control signal at the design step $i(j-1)$. The virtual controls for the $z_i = [z_{i1}, \ldots, z_{in_i}]^T$ subsystem are computed to drive z_i to the equilibrium point $[0, \ldots, 0]^T$. The equilibrium point is proved to be stable through a standard Lyapunov analysis. Even more, the Lyapunov functions themselves, computed at each step, are used to determine the most suitable α_{ij}.

Making reference to system (1) the following quantities are defined

$$l_1 = max(n_1 - 3, \ldots, n_m - 3)$$

l_1 being a nonnegative integer,

$$\begin{aligned} p &= max(n_1 - 1, \ldots, n_m - 1) \\ \mu_1 &= u_0^{2l_1+1} \\ \mu_k &= \frac{\dot{\mu}_{k-1}}{u_0}, \qquad 2 \le k \le p \end{aligned}$$

i.e., for instance, $\mu_2 = \frac{\dot{\mu}_1}{u_0} = (2l_1+1) u_0^{2l_1-1} \dot{u}_0$. It is easy to verify that μ_k, $k = 2, \ldots, p$, are, at least, C^1 functions provided that u_0 is a C^p function. The procedure consist of two parts.

Part one. (*Determination of* u_i):

At the i–th iteration it is considered the x_i–subsystem of (1). Then, a number of steps j are taken, $1 \le j \le n_i$. At the generic step $i(j)$, $1 \le j \le n_i - 1$, the following transformation is performed

$$z_{i(j+1)} = x_{i(j+1)} - \alpha_{ij} \tag{3}$$

having set $z_{i1} = x_{i1}$, so that

$$V_{ij} = \sum_{k=1}^{j} \frac{1}{2} z_{ik}^2 \tag{4}$$

and

$$\alpha_{ij} = -z_{i(j-1)} - c_{ij} z_{ij} \mu_1 + \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial x_{ik}} x_{i(k+1)} + \sum_{k=1}^{j-1} \frac{\partial \alpha_{i(j-1)}}{\partial \mu_k} \mu_{k+1} \tag{5}$$

where, $c_{ij} \in I\!R_+$ are design constants. Then, at step $i(n_i)$, it results that, if

$$u_i = -c_{in_i} z_{in_i} - z_{i(n_i-1)} u_0 + \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial x_{ik}} x_{i(k+1)} u_0 + \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i(n_i-1)}}{\partial \mu_k} \dot{\mu}_k \tag{6}$$

then, provided that u_0 is a smooth function of time, it yields

$$\dot{V}_{in_i} = - \sum_{k=1}^{n_i-1} c_{ik} z_{ik}^2 u_0^{2l_1+2} - c_{in_i} z_{in_i}^2 \tag{7}$$

Note that the illustrated procedure is repeated for all x_i–subsystems, $i = 1, \dots, m$.

Part two. (*Generation of* u_0):

u_0 and its derivatives can be generated, for example, as the output of a nonlinear time–varying system of order p

$$\begin{cases} \dot{y}_1 &= y_2 \\ &\vdots \\ \dot{y}_{p-1} &= y_p \\ \dot{y}_p &= -a_0 x_0 - a_1 y_1 - \dots - a_p y_p \\ & \quad +\kappa(z)\sin(t) \\ u_0 &= y_1 \end{cases} \tag{8}$$

where the coefficients a_j's are real numbers such that $s^{p+1} + a_p s^p + \dots a_1 s + a_0$ is Hurwitz, and $\kappa(z_1, \dots, z_m)$ is a smooth function equal to zero if and only if $z_i = 0$, $i = 1, \dots, m$. With this choice the k–th order derivative of u_0, $u_0^{(k)}$, is equal to y_{k+1}, $0 \le k \le p-1$, so that the α_{ij}'s defined before do not need any differentiator to be implemented (note that the term μ_{k+1} in (5) depends on the differentiation of u_0). Alternatives to (8) are also proposed in the literature, see (Jiang, 2000).

4. THE CONSIDERED PROBLEM AND THE PROPOSED MODIFIED STATE TRANSFORMATION

Let system (1) be affected by structural uncertainty appearing in the x_{in_i} equations, and by disturbances in the control input channel as in (Floquet *et al.*, 2003), i.e.

$$\begin{cases} \dot{x}_0 &= u_0 + \delta(t) \\ \dot{x}_{i1} &= x_{i2}(u_0 + \delta(t)) \\ &\vdots \\ \dot{x}_{i(n_i-1)} &= x_{in_i}(u_0 + \delta(t)) \\ \dot{x}_{in_i} &= \Delta(x,t) + g(x,t)u_i, \qquad 1 \le i \le m \end{cases} \tag{9}$$

where $\delta(t)$, $\Delta(x,t)$ and $g(x,t)$ are unknown scalar functions, bounded with their first time derivatives, and with known bounds. The function $g(x,t)$ has known constant sign: for the sake of simplicity, assume that $g(x,t) > 0$. Further, we suppose that $x \in \mathcal{X} \subset I\!R^n$. To cope with this new system, first we design $u_0 = \tilde{u}_0 - k sign\{x_0 - y_0\}$, where $k > |\delta(t)|$, and y_0 is a suitable reference signal for x_0. Then, we shall follow the procedure illustrated in Section 3 to transform the system state, but the transformation will be stopped at step $i(n_i - 2)$ of each i–th iteration in order to replace the expression of u_i in (6) with a discontinuous function based on the solution to a second order sliding mode control problem, following the line of (Bartolini *et al.*, 2000*b*; Ferrara and Giacomini, 2000). More precisely, instead of z_{in_i}, the following variables are introduced

$$w_{i1} = \tilde{u}_0 \Big(x_{in_i} + z_{i(n_i-2)} - \sum_{k=1}^{n_i-2} \frac{\partial \alpha_{i(n_i-2)}}{\partial x_{ik}} x_{i(k+1)} - \sum_{k=1}^{n_i-2} \frac{\partial \alpha_{i(n_i-2)}}{\partial \mu_k} \mu_{k+1} \Big) + c_{i(n_i-1)} z_{i(n_i-1)} \tag{10}$$

$$w_{i2} = \dot{w}_{i1} \tag{11}$$

It can be proved that, if u_i is designed in such a way that both w_{i1} and w_{i2} go to zero in a finite time, then the origin of the state space is a locally asymptotically stable equilibrium point of the overall closed–loop system (8)–(9).

5. THE ROLE OF MULTI-INPUT SLIDING MODE CONTROL

Now, consider the variables (10)–(11). Let $w_1 = [w_{1,1}, \dots, w_{m,1}]^T$, $\dot{w}_1 = w_2 = [w_{1,2}, \dots, w_{m,2}]^T \in I\!R^m$. On the basis of this definition, the following dynamic system can be written

$$\begin{cases} \dot{w}_1 = w_2 \\ \dot{w}_2 = F(x_i, \tilde{u}_0, \dots, \tilde{u}_0^{(n_i-1)}, t) + \tilde{u}_0 g(x,t) \dot{u} \end{cases} \tag{12}$$

where $u^T = [u_1, \dots, u_m]$. System (12), is a double integrator affected by the matched uncertainty terms

$$F^T(x, \mathcal{U}_0, t) = = [F_1(x_1, \mathcal{U}_0^{(n_1-1)}, t), \ \dots, F_m(x_m, \mathcal{U}_0^{(n_m-1)}, t)]$$

and $\tilde{u}_0 g(x,t) := B(\tilde{u}_0, x, t)$, with $\mathcal{U}_0^{(n_i-1)}$ being the set $\{\tilde{u}_0, \dots, \tilde{u}_0^{(n_i-1)}\}$, and $\mathcal{U}_0 := \bigcup_{i=1}^m \mathcal{U}_0^{(n_i-1)}$. Relying on the previous assumptions, it turns out that $F(x, \mathcal{U}_0, t)$ is uncertain but such that its components result in being bounded, i.e.,

$$|F_i(x_i, \mathcal{U}_0^{(n_i-1)}, t)| < \bar{F}_i \tag{13}$$

and the same holds for the elements b_{ii} of the diagonal matrix $B(\tilde{u}_0, x, t)$

$$0 < B_{1_i} < b_{ii} < B_{2_i} \tag{14}$$

having assumed, for the time being, that the sign $\tilde{u}_0$ is positive (a comment on the relaxation of this assumption will be made later). $\bar{F}_i$, B_{1_i}, and B_{2_i}, $i = 1, \dots, m$, are suitable constants.

Note that the quantity w_2 can be viewed as an unmeasurable quantity, being the first derivative of w_1 which depends on $\Delta(x,t)$ and $g(x,t)$. Then, the control problem can be reformulated as follows: given system (12), where $F(x, \mathcal{U}_0, t)$, $B(\tilde{u}_0, x, t)$ satisfy (13)–(14), and w_2 is inaccessible, design the auxiliary control signal $\dot{u}$ so as to steer w_1, w_2 to zero in finite time.

Now, observe that, regarding $w_1 = 0$ as an m–dimensional sliding manifold, the control objective corresponds to find the solution to a second–order sliding mode control problem. In fact, denoting with s the sliding variable, and setting $s := w_1$, the control vector u directly affects $\ddot{s} = \dot{w}_2$, and w_2 is not measurable. Then, the second equation of system (12) can be rewritten, component–wise, as

$$\dot{w}_{i,2} = F_i(x_i, \mathcal{U}_0^{(n_i-1)}, t) + b_{ii}\dot{u}_i \tag{15}$$

Equation (15) can be regarded as the model of an uncoupled single input system. The second–order sliding mode is attained, for instance, by means of the following algorithm (Bartolini *et al.*, 2000*a*), based on the assumption of being capable of detecting the extremal values of $w_{i,1}$ (e.g., by means of peak detectors).

Algorithm 1:

i) Set $\delta_i^* \in (0,1] \cap \left(0, \frac{3B_{1_i}}{B_{2_i}}\right)$.
ii) Set $w_{i,1_{Max}} = w_{i,1}(0)$.
Repeat, for any $t > 0$, the following steps.
iii) If $[w_{i,1}(t) - \frac{1}{2}w_{i,1_{Max}}][w_{i,1_{Max}} - w_{i,1}(t)] > 0$ then set $\delta_i = \delta_i^*$ else set $\delta_i = 1$.
iv) If $w_{i,1}(t)$ is extremal value then set $w_{i,1_{Max}} = w_{i,1}(t)$.
v) Apply the control law

$$\dot{u}_i(t) = -\delta_i U_{i_{Max}}\ sign\left\{ w_{i,1}(t) - \frac{1}{2}w_{i,1_{Max}} \right\} \tag{16}$$

Until the end of the control time interval. □

Note that in (16), according to (Bartolini *et al.*, 2000*a*),

$$U_{i_{Max}} > \max\left(\frac{\overline{F}_{1_i}}{\delta^* B_{1_i}}; \frac{4\overline{F}_{1_i}}{3B_{1_i} - \delta^* B_{2_i}}\right) \tag{17}$$

This condition ensures a contraction of the elements of the sequence $\{w_{i,1_{Max}}\}$. Moreover, it can be proved that under condition (17),

$$|w_1(t)| < |w_1(0)| + \frac{1}{2}\frac{|w_2(0)|^2}{U_{Max}} \tag{18}$$

$$|w_2(t)| \le \sqrt{|w_{Max_j}|} < \sqrt{|w_1(0)| + \frac{1}{2}\frac{|w_2(0)|^2}{U_M}} \tag{19}$$

hold, as shown in (Bartolini *et al.*, 1998), and that the convergence of $w_{i,1_{Max}}$ to zeros takes place in finite time. Clearly, if $w_{i,1_{Max}} \to 0$, then $w_{i,1} \to 0$ and $w_{i,2} \to 0$, because they are both bounded by $w_{i,1_{Max}} \to 0$.

6. THE CONTROL DESIGN PROCEDURE

Taking into account the proposed modified state transformation described in Section 5, and the possibilities offered by multi–input second order sliding mode control, the design procedure we propose to solve the control problem in question can be expressed in algorithmic form as follows.

Algorithm 2:

i) In (9), let $u_0 = \tilde{u}_0 - k sign\{x_0 - y_0\}$, where $\dot{y}_0 = \tilde{u}_0$ and $\tilde{u}_0 = y_1$ is generated as follows

$$\begin{cases} \dot{y}_0 &= y_1 \\ \dot{y}_1 &= y_2 \\ &\vdots \\ \dot{y}_{p-1} &= y_p \\ \dot{y}_p &= r(x_0, y) \\ y_1(0) &= y_{1_0} \end{cases} \tag{20}$$

where $y^T = [y_1, \ldots, y_p]$, y_{1_0} is a constant different from zero,

$$r(x_0, y) = \begin{cases} 0 & \|z\| \ge \epsilon \\ -a_0 x_0 - \sum_{k=1}^{p} -a_k y_k & \|z\| < \epsilon \end{cases} \tag{21}$$

ii) Apply Part one of the backstepping–based procedure (Jiang, 1996) recalled in (3)–(5). Stop it m times at steps $i(n_i - 1)$, $i = 1, \ldots, m$, and compute the quantities $\alpha_{i(n_i-2)}$, $z_{i(n_i-1)}$.
iii) Define the vectors

$$\begin{aligned} \tilde{z} &= [z_{1(n_1-1)}, \ldots, z_{m(n_m-1)}]^T \\ \tilde{c} &= [\tilde{c}_{1(n_1-1)}, \ldots, \tilde{c}_{m(n_m-1)}]^T \\ \tilde{\alpha} &= [z_{1(n_1-2)} - \sum_{k=1}^{n_1-2} \frac{\partial \alpha_{1(n_1-2)}}{\partial x_{1k}} x_{1(k+1)} \\ &\quad - \sum_{k=1}^{n_1-2} \frac{\partial \alpha_{1(n_1-2)}}{\partial \mu_k} \mu_{k+1}, \ldots, z_{m(n_m-2)} \\ &\quad - \sum_{k=1}^{n_m-2} \frac{\partial \alpha_{m(n_m-2)}}{\partial x_{mk}} x_{m(k+1)} \\ &\quad - \sum_{k=1}^{n_m-2} \frac{\partial \alpha_{m(n_m-2)}}{\partial \mu_k} \mu_{k+1}]^T \\ \tilde{x} &= [x_{1(n_1-1)}, \ldots, x_{m(n_m-1)}]^T \end{aligned}$$

and compute the sliding quantity $s = w_1 = \tilde{c}\tilde{z} + u_0(\tilde{x} - \tilde{\alpha})$.
iv) On the basis of the knowledge of the upper bounds of $\Delta(x,t)$ and $g(x,t)$, of the upper bounds of their first time derivatives, and of the upper bounds of the components of vector x, determinable from the knowledge of the set $\mathcal{X}$, compute the upper bounds of the uncertain terms in (12), i.e., $\overline{F}_i$, B_{1_i} and B_{2_i}, $i = 1, \ldots, m$.
v) Apply Algorithm 1 to determine each component $\dot{u}_i$ of the control vector derivative, with $U_{i_{Max}} = U_{Max}$ as in (17).

Note that, in our proposal, the generator of signal u_0 in (8) has been replaced by (20)–(21) to increase the convergence rate of vector z to zero. Further, observe that the control vector derivative plays the role of discontinuous control to generate the desired sliding mode behavior on the selected sliding manifold. In contrast, the actual control is continuous, overcoming the drawback of chattering (Bartolini *et al.*, 1998).

7. COMMENTS ON STABILITY

To analyze the stability properties of the closed loop system, let us write the expression of the first derivative of $V_{i(n_i-1)} = \frac{1}{2}\sum_{k=1}^{n_i-1} z_{ik}^2$ as

$$\dot{V}_{i(n_i-1)} = -\sum_{k=1}^{n_i-2} c_{ik} z_{ik}^2 (u_0+\delta(t))^{2l_1+2} + z_{i(n_i-1)}(u_0+\delta(t))\Big(z_{i(n_i-2)} + x_{i(n_i)} - \sum_{k=1}^{n_i-2}\frac{\partial \alpha_{i(n_i-2)}}{\partial x_{ik}} x_{i(k+1)}\mu_1 - \sum_{k=1}^{n_i-2}\frac{\partial \alpha_{i(n_i-2)}}{\partial \mu_k}\mu_{k+1}\Big) \quad (22)$$

From (10)

$$x_{i(n_i)} = \frac{w_{i1} - c_{i(n_i-1)} z_{i(n_i-1)}}{\tilde{u}_0} - z_{i(n_i-2)} + \sum_{k=1}^{n_i-2}\frac{\partial \alpha_{i(n_i-2)}}{\partial x_{ik}} x_{i(k+1)} + \sum_{k=1}^{n_i-2}\frac{\partial \alpha_{i(n_i-2)}}{\partial \mu_k}\mu_{k+1} \quad (23)$$

Moreover, the first equation of system (9), i.e., $\dot{x}_0 = \tilde{u}_0 - ksign\{x_0 - y_0\} + \delta(t)$, with the choice made for $\tilde{u}_0$ in Algorithm 2, becomes

$$\dot{e} = -ksign\{e\} + \delta(t) \quad (24)$$

Then, according to basic sliding mode control theory, having suitably selected k, $e = x_0 - y_0$ goes to zero in finite time, and, once in sliding mode, the continuous signal equivalent in Filippov's sense to $-ksign\{e\}$, namely $[ksign\{e\}]_{eq}$, is equal to $\delta(t)$ (Utkin, 1992). Thus, in sliding mode, substituting (23) into (22), using $[ksign\{e\}]_{eq}$ to replace its discontinuous counterpart, and posing $-[ksign\{e\}]_{eq} + \delta(t) = 0$, it yields

$$\dot{V}_{i(n_i-1)} = -\sum_{k=1}^{n_i-2} c_{ik} z_{ik}^2 \tilde{u}_0^{2l_1+2} + z_{i(n_1-1)} w_{i1} - c_{i(n_i-1)} z_{i(n_i-1)}^2 \quad (25)$$

that is the z_i coordinate enters a ball of radius $\frac{w_{i1}}{c_{i(n_i-1)} z_{i(n_1-1)}}$ at time t^*, and there remains for $t > t^*$. As far as w_{i1} and w_{i2} are concerned, bounds (18)–(19) apply. As for $z(t)$, during transients, one has $\frac{d}{dt}\left(\frac{1}{2}|z|^2\right) \leq -c_0|z|_2^2 - \tilde{c}|\tilde{z}|_2^2 + \tilde{z}^T w_1$, $c_0 = \min_i c_i$, and $\tilde{z} = [z_{1(n_i-2)}, \ldots, z_{m(n_m-2)}]$. Then z is bounded. Given this fact, the asymptotic stability of system (8), the fact that the $w_1(t)$, $w_2(t)$ are steered to zero in finite time as previously discussed, it follows that the point $x = 0$, $y_1 = \ldots = y_p = 0$ is a locally asymptotically stable equilibrium point of the controlled system. Clearly, it is advisable that k is sufficiently high to allow a very fast transient of x_0. Moreover, as a final comment, note that Algorithm 1 applies for $B(\tilde{u}_0, x, t) > 0$. $B(\tilde{u}_0, x, t) = \tilde{u}_0 g(x,t)$, with $g(x,t) > 0$ by assumption. As for $\tilde{u}_0$, the analysis of system (20)–(21) shows that $\tilde{u}_0 = y_1$ cannot become zero for a certain $t = \bar{t}$, and remains zero for all $t > \bar{t}$ unless $z_1 = \ldots = z_m = 0$ for all $t \geq \bar{t}$, i.e., the regulation objective is attained. So $\tilde{u}_0$ can be zero, during the transient, only in insulated time instants. This is tolerated by the control algorithm, provided that the change of sign of $\tilde{u}_0$ is taken into account by pre–multiplying the second order sliding mode control law by $sign\{\tilde{u}_0\}$.

8. APPLICATION TO A WHEELED ROBOT

As an example, we consider, for the sake of simplicity, a single input system, i.e., the kinematic model of a wheeled mobile robot (Murray and Sastry, 1993). The system equations are

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\phi} = \omega \\ \dot{\theta} = \frac{1}{l}\tan\phi\, v \end{cases} \quad (26)$$

where l is the length of the body of the mobile robot, (x, y) are the coordinates of the center of the axes between the two rear wheels, θ is the orientation of the robot with respect to an inertial coordinate system, ϕ is the wheel steering angle relative to the robot body, v is the velocity of the rear wheels, and w the steering velocity. System (26) can be transformed, with a proper change of coordinates, into the form (1). So, the overall system taken into consideration is

$$\begin{cases} \dot{x}_0 = v + 0.5\sin(10t) \\ \dot{x}_1 = x_2(v + 0.5\sin(10t)) \\ \dot{x}_2 = x_3(v + 0.5\sin(10t)) \\ \dot{x}_3 = w + 0.5\sin(t) \end{cases} \quad (27)$$

where sinusoidal terms have been added to model some input disturbances. As for the input signals, $\dot{w}$ is generated by the second order sliding mode control law previously described, while v is generated by the system

$$\begin{cases} v = \tilde{v} - 2sign\{x_0 - y_0\} \\ \tilde{v} = y_1 \\ \dot{y}_0 = y_1 \\ \dot{y}_1 = y_2 \\ \dot{y}_2 = \begin{cases} 0 & x_2 > 0.005 \\ -ax_0 - by_1 - cy_2 & otherwise \end{cases} \end{cases}$$

In Fig. 1 the results of the simulation of a parking maneuver plotted in the inertial coordinate

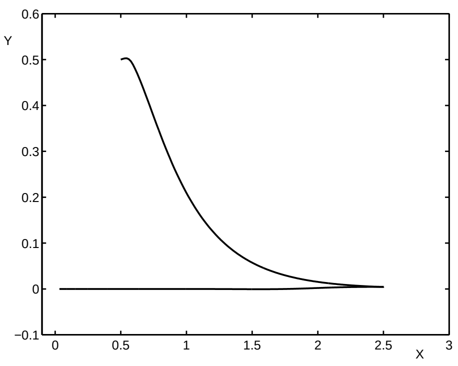

Fig. 1. A parking maneuver obtained via the proposed procedure.

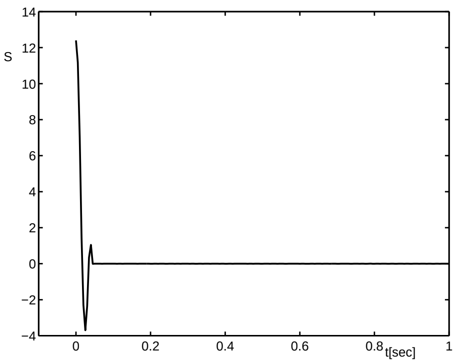

Fig. 2. The time evolution of the sliding quantity.

systems is provided, while in Fig. 2 it is reported the time evolution of the sliding quantity which results in being steered to zero quite rapidly.

9. CONCLUSIONS

The problem of controlling multi–input nonholonomic systems has been dealt with in the paper, making reference to a complicated scenario in which uncertainties affect the system equations. The key idea is to transform the original system, through a backstepping–based procedure, into a form suitable to design a sliding manifold upon which to enforce a second order sliding mode. In this way, the overall stabilization problem can be solved relying on a continuous control vector. This fact enables the application of the proposed strategy even to systems, such as mechanical ones, for which the chattering effect, which always accompanies sliding mode control, can be unacceptable.

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