A MINIMUM-TIME CONTROL STRATEGY FOR TORQUE TRACKING IN PERMANENT MAGNET AC MOTOR DRIVES

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Abstract: A minimum-time torque control strategy for permanent-magnet ac motor drives is presented. The proposed solution is obtained by an ad-hoc procedure based on the computation of reachability and controllability sets which in turn requires solving iteratively a fourth-degree polynomial equation. For its efficient implementation an algorithm based on Sturm sequences is proposed. The algorithm has been implemented on a laboratory prototype and the experimental results, showing the effectiveness of the proposed technique, are included in the paper.

Keywords: minimum–time, AC motor drive, Sturm sequence, experimental.

1. INTRODUCTION AND MOTIVATION

In view of the urgent industrial demand for high-performance drives, much research effort has recently been made to improve transient torque control. Actually, in many special-purpose devices, like spindles, internal combustion engine test benches, automotive safety applications and fast robotic manipulators, prompt response to sharp speed reference changes is an essential requirement. Interior permanent magnet (IPM) synchronous motor drives are the most appropriate candidates for this job since they can deliver very high torque to low-inertia motors. Torque, and thus acceleration, can be modified by varying the currents in the stator windings Li and Xu (2001). Conventional PI control does not guarantee the best dynamics, especially during large transients where the handling of output voltage saturation phenomena is still an open problem. To this aim, voltage overmodulation techniques accounting for phase current dynamics have recently been presented Holtz and Beyer (1995).

An alternative approach is followed in this paper: precisely, a control strategy that ensures the fastest transition between two torque levels under given input and state constraints is proposed. The first studies on minimum-time techniques focused on the on-line computation of the input voltage capable of achieving the fastest transition between two system states corresponding to two torque values (*point-to-point* control) Bianchi et al. (2003). However, due to the high complexity of the equation describing the optimal trajectory, which can be solved only by recursive numerical techniques, a sub-optimal minimum-time control strategy was developed and tested on a laboratory prototype Bolognani et al. (2004b). The present contribution represents a substantial improvement over previous methods because it solves the more general problem of determining the minimum-time transition between two torque levels (*point-to-curve* control). In fact, the proposed algorithm automatically finds the time-optimal arrival state on the constant-torque locus aimed at.

The main contribution of the present paper is that of providing a feedback form solution to the point–

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to–curve time–optimal control problem based on the controllability and reachability sets without resorting to the solution of an HJB equation that would render it practically unsuitable. It is shown that such solution is achieved by computing a sequence of Sturm polynomials to determine the existence of a real root of a fourth order polynomial. A detailed description of the operations necessary for the computation of the optimal time and the control, within an assigned tolerance, is also provided.

After a brief description of the drive mathematical model in Section 2, the basic idea to determine the state-feedback minimum-time control strategy is described in Section 3. The computational details are given in Section 4 while experimental results are presented in Section 5.

2. MODEL DESCRIPTION

The IPM motor model can be described in a d, q reference frame synchronous to the rotor, with the d-axis conventionally placed along the permanent magnet (PM) flux linkage. In the following, the infinite-inertia hypothesis will be adopted, which is equivalent to considering the speed as a constant during torque transients. The dependence of stator inductances L_d, L_q on direct and quadrature currents i_d, i_q is also neglected; as a consequence, the relationships between i_d, i_q and direct and quadrature flux linkages x_1, x_2 are affine:

$$x_1 = L_d i_d + \Lambda_{mg} \qquad x_2 = L_q i_q \tag{1}$$

where Λ_{mg} is the permanent magnet flux linkage. The equations describing the direct and quadrature stator voltages u_d, u_q are:

$$\begin{aligned} u_d &= Ri_d + L_d \frac{di_d}{dt} - \omega_{me} L_q i_q \\ u_q &= Ri_q + L_q \frac{di_q}{dt} + \omega_{me} L_d i_d + \omega_{me} \Lambda_{mg} \end{aligned} \tag{2}$$

where ω_{me} denotes the electromechanical speed.

In practice, the resistive voltage drops Ri_d and Ri_q are negligible compared to the other terms in (2). By taking this fact into account, from (1) and (2) the following linear dynamic model is obtained:

$$\dot{x}(t) = \begin{bmatrix} 0 & \omega_{me} \\ -\omega_{me} & 0 \end{bmatrix} x(t) + u(t) \tag{3}$$

where $x(t) = [x_1(t) \;\; x_2(t)]^T$ and $u(t) = [u_d(t) \;\; u_q(t)]^T$.

The constraint on stator input voltage is expressed as:

$$\|u(t)\| = \sqrt{u_d^2(t) + u_q^2(t)} \leq \bar{U} \ , \ \forall t \geq 0. \tag{4}$$

The set of all input functions satisfying (4) will be denoted by $\mathscr{U}$.

The electromagnetic torque is related to the flux linkages by:

$$\tau_e = \frac{3}{2} p x_2 \left[\left(\frac{1}{L_q} - \frac{1}{L_d} \right) x_1 + \frac{\Lambda_{mg}}{L_d} \right] \tag{5}$$

where p is the number of motor pole pairs. The problem considered in this paper can be stated as follows.

Problem 1. Given the initial direct and quadrature flux linkages, the stator voltages constraint (4) and a desired electromagnetic torque τ_e, find an optimal feedback strategy $u(t) = \Phi(x(t), \tau_e) \in \mathscr{U}$ which drives the system to direct and quadrature flux linkages corresponding to the desired torque in minimum time.

3. PROBLEM SOLUTION

The considered problem falls within the framework of input-constrained optimal control with terminal constraints Sage and White (1977). Therefore, the optimal solution may be characterized either in terms of the pointwise minimizer of an ad-hoc Hamiltonian or in terms of the solution of an HJB-type equation. Unfortunately, these techniques do not lead to a simple, even implicit, representation of the optimal control law due to the presence of the terminal constraint. Although this constraint could be managed by Pontryagin's principle, a simpler and more intuitive approach is followed in the sequel, which leads to a practically implementable solution of the above-mentioned problem in feedback form. To simplify notation, let us define $\bar{x}_1 = \frac{\Lambda_{mg} L_q}{L_q - L_d}$, $\tau = \tau_e \frac{2(L_q L_d)}{3p(L_d - L_q)}$ so that (5) becomes

$$(x_1 - \bar{x}_1) x_2 = \tau \tag{6}$$

and denote by $\mathscr{L}_\tau$ the locus of all points satisfying (6) for a given τ, i.e.:

$$\mathscr{L}_\tau = \{(x_1, x_2) : \text{equation (6) is satisfied}\}. \tag{7}$$

Given the desired value of τ and initial condition $x(0) = x$, Problem 1 entails the determination of the minimum-time function

$$T(x, \tau) = \min_{u \in \mathscr{U}} \{t \ : \ x(t) \in \mathscr{L}_\tau\}. \tag{8}$$

Once this function has been found, the derivation of the optimal state-feedback control law $u = \Phi_{min}(x, \tau)$ is straightforward. Indeed, it can be obtained as the pointwise minimizer of the directional derivative:

$$\Phi_{min}(x, \tau) = \arg \min_{\|u\| \leq \bar{U}} D^+ T(x, \tau, u)$$

where $D^+ T(x, \tau, u)$ denotes the generalized Lyapunov derivative (cf., e.g., Blanchini (1999)) of the system under consideration:

$$D^+ T(x, \tau, u) \doteq \limsup_{h \to 0^+} \frac{T[x + h(Ax + u), \tau] - T(x, \tau)}{h}$$

where A is the system matrix in (3).

According to (4), the control action is confined in the disk of radius $\bar{U}$. The above-mentioned minimizer coincides with the unique vector of magnitude $\bar{U}$, aligned with the vector n normal to the level surface of T and passing through the current state x. As will be shown later on, this normal vector is well defined.

In the remaining part of this section it is shown: (i) how to compute $T(x,\tau)$, given x and τ, and (ii) how to determine the normal direction n at x.

3.1 Reachability sets

Given state x, let us define the reachability set $\mathscr{R}(x,T)$ as the set of all states $x(T)$ that can be reached from $x(0)=x$ at time T by means of a suitable input $u\in\bar{\mathscr{U}}$, and the controllability set $\mathscr{C}(x,T)$ as the set of all states $x(0)$ that can be driven to $x(T)=x$ by means of a suitable input $u\in\bar{\mathscr{U}}$. Also, let us consider the rotation matrix:

$$\Omega(\theta)=\begin{bmatrix}\cos(\omega_{me}\theta) & \sin(\omega_{me}\theta)\\ -\sin(\omega_{me}\theta) & \cos(\omega_{me}\theta)\end{bmatrix} \qquad (9)$$

so that

$$\Omega(-T)=\Omega^T(T)=\Omega^{-1}(T)\,. \qquad (10)$$

Denoting by $\mathscr{D}_{\bar{U}}$ the disk of radius $\bar{U}$ centered at the origin, the following proposition can easily be proved.

Proposition 1. The reachability and controllability sets are given by

$$\mathscr{R}(x,T)=\Omega(T)x+T\mathscr{D}_{\bar{U}} \qquad (11)$$

$$\mathscr{C}(x,T)=\Omega(T)^T x+T\mathscr{D}_{\bar{U}}\,. \qquad (12)$$

Therefore, sets $\mathscr{R}(x,T)$ and $\mathscr{C}(x,T)$ are disks of radius $T\bar{U}$ whose centers x_c lie on the circumference of radius $\|x\|$ and form angles equal to $\omega_{me}T$ and, respectively, $-\omega_{me}T$ in the clockwise direction with vector x. The minimum time to reach the constant-torque hyperbola $\mathscr{L}_\tau$ from $x(0)=x$ corresponds to the minimum time $t=T$ such that circle $\mathscr{R}(x,T)$ is tangent to either of the branches of $\mathscr{L}_\tau$ (see Fig. 1).

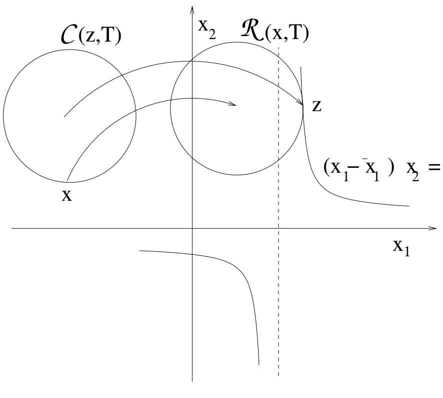

Fig. 1. Reachability set $\mathscr{R}(x,T)$ and controllability set $\mathscr{C}(x,T)$

Among the points on the target curve (see Fig. 1), the tangency point z can be reached in the shortest time. It is evident that x is on the boundary of the controllability set $\mathscr{C}(z,T)$. The controllability sets characterize the level curves of function $T(x,\tau)$ as follows. If all possible "landing" points z on both branches of the target curve are considered, then for fixed T a family of circles $\mathscr{C}(z,T)$ with the same radius $T\bar{U}$ will be obtained. The envelope of these circles are smooth curves representing the level surfaces of $T(x,\tau)$, which are the loci of the points x that can be driven to $\mathscr{L}_\tau$ in the same minimum time $T(x,\tau)=T$ (see Fig. 2).

A level surface is the envelope of circles with the same radius $T\bar{U}$. The vector n, normal to any point of the envelope, can be computed straightforwardly as the unit vector aligned with the vector that connects the center y of circle $\mathscr{C}(z,T)$ to x (see Fig. 2). It is important to note that point y can uniquely be determined from the current state x and the target curve τ, i.e. $y=\phi(x,\tau)$. On the basis of this geometric interpretation, it is possible to determine the control action in a feedback form as:

$$\Phi_{min}(x,\tau)=\frac{y-x}{\|y-x\|}\,\bar{U}=\frac{y-x}{T}\,, \qquad (13)$$

provided that y can be computed as a function of x. The control u can be obtained according to the

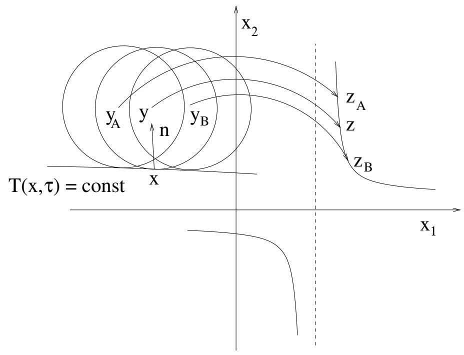

Fig. 2. Determination of the curve of constant minimum–time T to reach the target line

following procedure, consisting of two basic steps.

Procedure 3.1.

(1) Given the current state x, determine the minimum value of T such that $\mathscr{R}(x,T)$ touches $\mathscr{L}_\tau$.
(2) Compute the "landing" point z as well as the center y of the controllability circle $\mathscr{C}(z,T)$. Then the control action at x is given by (13).

After curve $\mathscr{L}_\tau$ has been reached, the state must be maintained on $\mathscr{L}_\tau$, which is possible only if the reached point is a steady–state one. Now, the admissible steady states are obtained by solving the equilibrium equations:

$$\omega_{me}x_2+u_1 \;=\; 0, \quad -\omega_{me}x_1+u_2 \;=\; 0$$

subject to constraint (4). The set of their solutions is the circle:

$$\mathscr{S}=\{(x_1,x_2):\;\sqrt{x_1^2+x_2^2}\;\le\;\frac{\bar{U}}{\omega_{me}}\}\,. \qquad (14)$$

This introduces a new constraint on the "landing" state $z=[z_1\;z_2]^T$, which takes either the form:

$$x_1^{min}(\tau,\omega_{me})\;\le z_1\;\le x_1^{max}(\tau,\omega_{me}) \qquad (15)$$

or the form:

$$x_2^{min}(\tau,\omega_{me}) \le z_2 \le x_2^{max}(\tau,\omega_{me}) \quad (16)$$

where $x_1^{min}(\tau,\omega_{me})$, $x_1^{max}(\tau,\omega_{me})$ (resp. $x_2^{min}(\tau,\omega_{me})$, $x_2^{max}(\tau,\omega_{me})$) are the coordinates of the lower and upper intersections (whose computation is trivial) of $\mathscr{L}_\tau$ with the boundary of $\mathscr{S}$.

4. CONTROL IMPLEMENTATION

The first step of Procedure 3.1 is critical, since it requires the computation of the minimum time. The equation of the circular boundary of $\mathscr{R}(x,T)$, whose center and radius are, respectfully, $x_c = \Omega(T)x$ and $T\bar{U}$, is:

$$[z_1 - \cos(\omega_{me}T)x_1 - \sin(\omega_{me}T)x_2]^2 + [z_2 + \sin(\omega_{me}T)x_1 - \cos(\omega_{me}T)x_2]^2 = T^2\,\bar{U}^2 \quad (17)$$

From (6) and (17) we get:

$$p(z_2,T) \doteq z_2^4 + [-2w_2]z_2^3 + [\|x\| + \bar{x}_1 - 2w_1x_1 - T^2\bar{U}^2]z_2^2 + \tau[\bar{x}_1 - w_1]z_2 + \tau^2 = 0 \quad (18)$$

where $w = [\,w_1\ w_2\,]^T = \Omega(T)x$ and we have exploited the fact that $\|x\| = \|w\|$.

Let us assume that the landing point z_2 belongs to the interval $[z_2^-, z_2^+]$ [2]. On the basis of the properties of the reachability sets, the following proposition can be proved

Proposition 2. A state x can be driven to the target curve in the interval $[z_2^-, z_2^+]$ in a time not exceeding T if and only if (18) admits a real solution in $[z_2^-, z_2^+]$.

By the nature of the problem, it is immediate to see that if two roots are present, then they must have the same sign while, if four roots are present, two of them are positive, and the others are negative. If there are no constraints, the presence of a double (positive/negative) root discriminates which branch (upper/lower) of the characteristic is reachable in the shortest time. With reference to the interval $[z_2^-, z_2^+]$, the optimal T corresponds either to the tangency at an internal point or to the fact that one of the two extremes is a zero of (18) and (18) is positive at all other points of $[z_2^-, z_2^+]$. From the operative point of view, a simple bisection algorithm can be followed to find the optimal T, starting from an upper bound T_r and iterating according to the following criterion: if for the current value of T polynomial (18) has a zero inside the considered interval, then T is decreased; otherwise, it is increased. The procedure is repeated until one of the above-mentioned optimality conditions is satisfied (within a reasonable tolerance). Since the formulas for finding the roots of fourth-degree polynomial equations Birkhoff and Lane (1965) are unsuitable for on-line implementation (using standard hardware), the following alternative procedure based on Sturm sequences Stoer and Bulirsch (2002) has been conceived (note, in this regard, that the roots corresponding to the current T need not be determined; only their existence must be ascertained).

To simplify notation, let us rewrite polynomial (18) as:

$$p_0(z) = z^4 + p_{03}z^3 + p_{02}z^2 + p_{01}z + p_{00} \quad (19)$$

where $p_{03} = -2w_2$, $p_{02} = \|x\| + \bar{x}_1 - 2w_1x_1 - T^2\bar{U}^2$, $p_{01} = \bar{x}_1 - w_1$, $p_{00} = \tau^2$.

Denote by $p_0(z),\ p_1(z),\ldots,p_4(z)$ the polynomials in the Sturm sequence obtained from (19). Therefore: $p_1(z) = p'(z) \doteq p_{13}z^3 + p_{12}z^2 + p_{11}z + p_{10}$, where $p_{13} = 4$, $p_{12} = 3p_{03}$, $p_{11} = 2p_{02}$, $p_{10} = p_{01}$ and the other polynomials $p_k(z)$ in the sequence are computed as the opposite of the remainder of the division of $p_{k-2}(z)$ by $p_{k-1}(z)$ (thus each of the coefficients p_{kj}, corresponding to the j^{th} power of the Sturm polynomial whose order is $4-k$, can be computed by simple linear operations on the coefficients of the preceding two polynomials).

The following theorem holds (see for instance Stoer and Bulirsch (2002)).

Theorem 4.1. Given the interval $[z^-, z^+]$ let m^- be the number of sign changes in the sequence

$$p_0(z^-),\ p_1(z^-),\ p_2(z^-),\ p_3(z^-),\ p_4(z^-)$$

and m^+ be the number of sign changes in the sequence

$$p_0(z^+),\ p_1(z^+),\ p_2(z^+),\ p_3(z^+),\ p_4(z^+)$$

then the number of roots of $p(z)$ in the interval $[z^-, z^+]$ is $m^- - m^+$.

Furthermore, the last non–zero polynomial of the sequence is the greatest common divisor of $p_0(z) = p(z)$ and $p_1(z) = p'(z)$. Therefore the polynomial has a single double root if and only if $p_{40} = 0$ and $p_{31} \neq 0$. In this case the double root is given by $z_2 = -p_{30}/p_{31}$ so that, from (6), the other state component is $z_1 = \frac{\tau}{z_2} + \bar{x}_1$. (note that the last expression is always defined because zeros in the origin are not allowed for (18)).

The above result can be easily exploited to find out whether a branch of the constant-torque hyperbola is intersected by a reachability circle. Let us assume for simplicity that positive values of z_2 are of interest, so that $z_2^- = 0$ and $z_2^+ = \infty$. In this case it is possible to consider only the signs of the constant coefficients p_{00}, p_{10}, p_{20}, p_{30}, p_{40} and those of the leading ones 1, p_{13}, p_{22}, p_{31}, p_{40}. In the opposite case, $z_2 \le 0$ we need consider the signs of 1, $-p_{13}$, p_{22}, $-p_{31}$, p_{40} and p_{00}, p_{10}, p_{20}, p_{30}, p_{40}.

Depending on the relationship between the currently selected time T and the minimum-time $T(x,\tau)$, the following situations may occur:

[2] In practice it is desirable to reach a specified branch of the target hyperbola without exceeding a given time bound, which defines an admissible landing interval. The unconstrained case is managed by assuming $[-\infty,+\infty]$

- If $T \leq T(x,\tau)$ there are no roots of $p(z_2,T)$ for $z_2 \geq 0$, so that $m^- - m^+ = 0$.
- If $T \geq T(x,\tau)$ there are roots of $p(z_2,T)$ for $z_2 \geq 0$ so that $m^- - m^+ = 2$.
- If $T = T(x,\tau)$ there exist two coincident positive roots of $p(z_2,T)$, so that $m^- - m^+ = 2$, $p_{40} = 0$. The double root is $z = -p_{30}/p_{31}$.

Thus the following algorithm to compute the optimal time can be adopted:

Procedure 4.1. **Inputs**: the current state x, the torque value τ an upper bound T_r on the reachability time, a tolerance Δ on the computation of the optimal time.

Outputs: The optimal time value $\hat{T}$ (up to the tolerance Δ) and the "landing point" z on the upper (lower) branch.

(1) Let $\underline{T} = 0$, $\overline{T} = T_r$.
(2) Set $T = (\underline{T} + \overline{T})/2$
(3) Compute $p(z_2,T)$ and the corresponding Sturm sequence.
(4) Compute $m^- - m^+$, the number of roots in the interval under consideration.
(5) If $m^- - m^+ = 0$, then $\underline{T} := T$.
(6) If $m^- - m^+ = 2$, then $\overline{T} := T$.
(7) If $\overline{T} - \underline{T} > \Delta$ go to Step 2; else Set $z_2 = -p_{30}/p_{31}$ and $z_1 = \overline{x}_1 + \frac{\tau}{z_2}$.
(8) Compute vector $y = \Omega(-T)z$ and control u as:

$$u = \frac{y - x}{\|y - x\|}\,\overline{U}. \qquad (20)$$

It is worth saying that the above procedure can be easily accommodated to incorporate constraints like (15) and (16). At each iteration, the operations to compute the Sturm sequence polynomials need be performed. Denoting by n_{flops} the corresponding number of flops, the proposed bisection procedure stops after $n_{step} = \left\lceil \log_2\left(\frac{T_r}{\Delta}\right)\right\rceil$ steps, so that the overall number of flops required at each iteration is $n_{step} n_{flops}$ plus the number of flops necessary for the computation of y and for the computation of u via (20).

5. EXPERIMENTAL RESULTS

As stated in Sect. 3, the proposed procedure requires iterating over T and calculating the coefficients of the Sturm sequence. The proposed control procedure 4.1 in Section 4 and the SVM timing calculations have been implemented on a Texas Instruments TMS320C31 Floating Point DSP, with 33.3ns instruction cycle. A slave Fixed Point TMS320P14 DSP, with 160ns instruction cycle, featuring 16 individual bit-selectable I/O and six PWM channels, with a period resolution of 40ns, handles digital I/O management and switching pattern generation. Control software routines for the slave processor have been written in

Table 1. Floating Point Operations for the procedure 4.1 (Sect.3)

	+	*	/	Table Accessing
Initialization	3	8	0	0
Init while cycle	1	2	0	0
Sturm Algorithm	33	31	4	1
End while cycle	9	8	2	3

Assembly language, for a closer link with the embedded hardware peripherals. The procedure 4.1 has been written in C language, optimized for shortest execution time. For the sake of clarity, it has been split in different phases, and the number of real time floating point operations for each step has been collected and reported in Table 1 (table accessing figures derive from trigonometrical functions look-up tables). With the available hardware, a single while cycle takes about 19μs. It is worth noticing that writing substantial portions of code directly in assembly language could yield further reduction of the cycle length. On average, it has been found that a satisfactory computation of the reachability time (condition 7, procedure 4.1) needs about seven or eight iterations, that is, 154μs. By considering a DSP load factor of 70% as a minimum requirement to let room enough for the remaining parts of the drive management algorithm, a sampling time of 245 μs has been selected, corresponding to a switching frequency of about 4 kHz. Fig. 3 reports the behavior of direct and quadrature flux linkages during a torque transient from 1% to 65% of the rated torque.

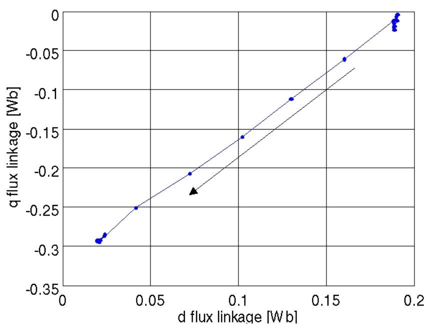

Fig. 3. Direct and quadrature flux linkages during the torque transient from 1% to 65% of rated torque

Fig. 4 shows the behavior of the direct and quadrature motor currents, which are related to the flux linkages by (1). The test has been performed with a DC bus voltage $Udc = 375V$, and the transient takes five steps (1.22ms). At steady state a conventional Proportional-Integral (PI) control is adopted, to avoid chattering usually related to hysteresis control techniques Bianchi et al. (2003); Bolognani et al. (2004b). The onset of a new torque reference step triggers the TOC strategy, which holds until the measured state falls again within a predefined area surrounding the selected landing point, where the PI control is selected again. Particular care has to be paid for a fine-tuning of the software mechanism that manages the toggle between the two control strategies, as reported in Bolognani et al. (2004b). The area around the final steady state condition has to be large enough to avoid continuous toggling between PI and TOC, but not too large, to get the maximum time saving during transient operations.

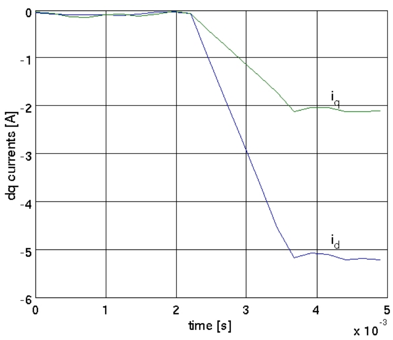

Fig. 4. Motor currents during the torque transient during the torque transient from 1% to 65% of rated torque

It is meaningful to compare the results shown in Fig. 4 with those obtainable by conventional current control techniques. A first test has been carried out by imposing to the current control the same initial and final states, which were ($i_d = i_q = -0.1A$ and ($i_d = -5.2A$, $i_q = -2.1A$) respectively. For a classical PI control and for a predictive current control Bolognani et al. (2004b) (the experimental results of this are depicted in Fig. 5) the transient times, measured as

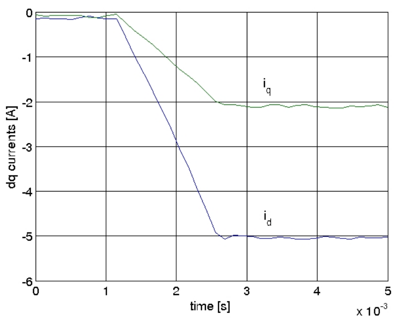

Fig. 5. Currents during the torque transient, predictive current control, final point fixed by TOC

mean value of several transients under the same conditions, were $1.61ms$ and $1.45ms$, respectively, with the landing point in the constant (final) torque curve suggested by the proposed time optimal control. Actually, it does not coincide with the final state selected by the conventional control strategies, which usually impose a maximum torque-per-ampere trajectory in the state plane. Accordingly, the final point is selected in the constant (final) torque curve by imposing the minimum current module. Under the same initial conditions, the final state was ($i_d = -2.9A$, $i_q = -3.3A$), different from the one selected by the time optimal strategy. The transient times in this case rise up to $1.87ms$ for the PI control and $1.71ms$ for the predictive control. A similar behavior has been experienced with different starting points and final torque levels.

It is worth to note that the selected switching frequency of $4081Hz$ is a good compromise between the current ripple and switching losses in the power switches. Of course, the availability of faster DSP would shorten the algorithm execution, and the switching frequency could be shifted towards the ultrasonic range, to get both a silent functioning and a ripple-free torque generation. At present, the choice of a DSP with such potentiality is out of the range for a cost-effective drive design. The growing interest towards the use of low cost, high density Field Programmable Gate Arrays (FPGA) as DSP co-processors could mark a turning point Bolognani et al. (2004a). In the next future, complex algorithms as that proposed in this paper could be implemented by HDL (hardware descriptor language) coding into an FPGA, while a less performing DSP will be dedicated to drive automation and peripheral control.

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