

# NETWORKED PREDICTIVE CONTROL OF SYSTEMS WITH RANDOM NETWORK TRANSMISSION DELAY – A POLYNOMIAL APPROACH

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**Abstract:** The paper considers the design of networked control systems with random network transmission delay and addresses their closed-loop stability. A novel control strategy is proposed to deal with the random network delay, which is termed as networked predictive control. The key parts of the networked predictive control are the control prediction generator that provides a set of future control predictions and the network delay compensator that removes the network transmission delay. The analytical stability criteria of the closed-loop networked predictive control systems are derived for both fixed and random networked transmission delays. *Copyright © 2005 IFAC*

**Keywords:** Networked control system, predictive control, random network delay.

## 1. INTRODUCTION

With the development of network technology, more and more intelligent devices or systems have been embedded into the Internet for service, security and entertainment, including distributed computer systems, surveillance cameras, telescopes, manipulators and mobile robots. Clearly, the Internet has provided a powerful tool for distributed collaborative work. The emerging network technologies do have the potential to apply the advantages of this way of working to advanced control systems. The advantages include the following: allow remote monitoring and tuning of control systems; allow large (or global) area distributed control; and allow collaboration between skilled system designers and operators situated in geographically diverse locations. These are not achievable by the use of design methodologies for conventional control systems.

Although the notion of networked control systems (or network based distributed control systems) is relatively new and still in its infancy, it has captured the interest of many researchers worldwide (Overstreet and Tzes, 1999; Nemoto et al., 2000; Tipsuwan and Chow, 2003). Networked control systems have opened up a complete new area of real-world applications, namely tele-training, tele-

manufacturing, tele-surgery, museum guidance, traffic control, space exploration, disaster rescue, and health care. Recently, more and more attention has been paid to various issues of network based control systems, for example, the stability problem in the presence of network delays and data packet drops (Zhang *et al.*, 2001), the design and implementation problem of the networked control system (Yang *et al.*, 2003; Zhivoglyadov and Middleton, 2003) and the network traffic congestion problem (Wong and Brockett, 1999).

It is well known that the random network transmission delay on networks makes the design of networked control systems very hard. However, there is an advantage in networked control systems, which is that a set of control sequences and measurements can be transmitted from one location to another location at the same time through a network. This advantage is not available in conventional control systems. It implies that the design of networked control systems should be different from conventional control systems. Here, a new control strategy for networked control systems is proposed, which is termed as networked predictive control. This paper addresses the design and stability of networked control systems with random network transmission delay.

## 2. DESIGN OF NETWORKED PREDICTIVE CONTROL SYSTEMS

### 2.1 Networked Predictive Control Scheme

Since there is an unknown network transmission delay, a networked predictive controller is proposed. It consists of two parts: the control prediction generator and the network delay compensator. The

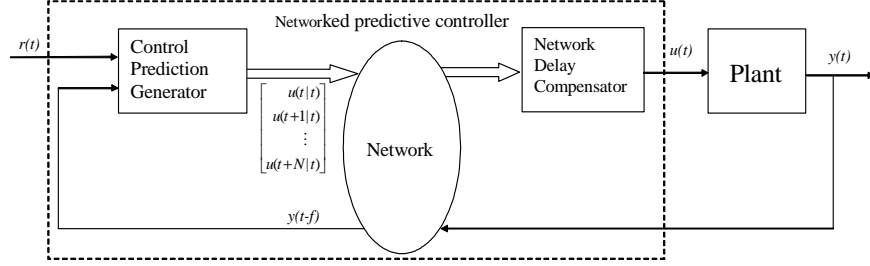


Fig. 1. The networked predictive control system.

### 2.2 Design of the Control Prediction Generator

Let  $\mathfrak{R}[z^{-1}, p]$  denote the set of polynomials in the indeterminate  $z^{-1}$  with coefficients in the field of real numbers and with the order  $p$  in a set of non-negative integer numbers. For example, the polynomial  $A_k(z^{-1}) \in \mathfrak{R}[z^{-1}, n]$ , i.e.,  $A_k(z^{-1}) = a_{k,0} + a_{k,1}z^{-1} + a_{k,2}z^{-2} + \dots + a_{k,n}z^{-n}$ . For the sake of simplicity, the following assumptions are made:

- The network time delays in the forward and backward channels are  $k$  and  $f$ , respectively;
- The number of consecutive data package drops in the forward channel on the network is not greater than the largest time delay  $N$ ;
- The network time delay  $f$  in the backward channel is constant;
- The data transmitted through network are with a time stamp.

Consider a single-input single-output discrete-time plant described by the following

$$A(z^{-1})y(t+d) = B(z^{-1})u(t) \quad (1)$$

where  $y(t)$  and  $u(t)$  are the output and control input of the plant,  $d$  is the time delay, and  $A(z^{-1}) \in \mathfrak{R}[z^{-1}, n]$  and  $B(z^{-1}) \in \mathfrak{R}[z^{-1}, m]$  are the system polynomials. If there is no network transmission delay, many control design methods are available for the plant ( $I$ ), for example, PID, LQG, MPC etc. Here, it assumes that the controller of the system without network delay is given by

$$C(z^{-1})u(t) = D(z^{-1})e(t+d) \quad (2)$$

where the polynomials  $C(z^{-1}) \in \mathfrak{R}[z^{-1}, n_c]$  and  $D(z^{-1}) \in \mathfrak{R}[z^{-1}, n_d]$  and  $e(t+d) = r(t+d) - \hat{y}(t+d)$  is the error between the future reference  $r(t+d)$  and the output prediction  $\hat{y}(t+d)$ .

To compensate the network transmission delay, the control prediction sequence  $u(t+k|t)$  at time  $t$ , for  $i=0, 1, 2, \dots, N$ , is generated by

$$C(z^{-1})u(t+i|t) = D(z^{-1})e(t+d+i|t) \quad (3)$$

former is designed to generate a set of future control predictions. The latter is used to compensate the unknown random network delay. To make use of the network advantage of transmitting data packages, a set of consecutive control predictions in the forward channel are packed and transmitted through the network at time  $t$ . So, this networked predictive control system (NPCS) structure is shown in Fig. 1.

and the error prediction  $e(t+d+i|t)$  at time  $t$  is defined as

$$e(t+d+i|t) = r(t+d+i) - \hat{y}(t+d+i|t) \quad (4)$$

where  $\hat{y}(t+d+i|t)$  is the output prediction at time  $t$  and  $r(t+d+i)$  is the future reference input. For the sake of simplicity, define the following operations:

$$x(t-i|t-i) = z^{-1}x(t-i+1|t-i+1) \quad (5)$$

for  $i=1, 2, \dots, t$

$$x(t+i|t) = z^{-1}x(t+i+1|t), \quad \text{for } i=0, 1, 2, \dots \quad (6)$$

where  $x(\cdot)$  represents  $\hat{y}(\cdot)$  and  $u(\cdot)$ .

For  $i=0, 1, 2, \dots, N$ , there exists the following Diophantine equation (Clarke, et al., 1987):

$$A(z^{-1})E_i(z^{-1}) + z^{-i-f}F_i(z^{-1}) = 1 \quad (7)$$

where the polynomials  $E_i(z^{-1}) \in \mathfrak{R}[z^{-1}, i+f-1]$  and  $F_i(z^{-1}) \in \mathfrak{R}[z^{-1}, n-1]$ . It is clear from assumption c) that the past outputs up to time  $t-f$  are available on the control prediction generator side. Combining the above and the controlled plant yields the following output predictions at  $t$ :

$$\begin{aligned} \hat{y}(t+d|t) &= F_d(z^{-1})y(t-f) + B(z^{-1})E_d(z^{-1})u(t|t) \\ \hat{y}(t+d+1|t) &= F_{d+1}(z^{-1})y(t-f) \\ &\quad + B(z^{-1})E_{d+1}(z^{-1})u(t+1|t) \\ &\quad \vdots \\ \hat{y}(t+d+N|t) &= F_{d+N}(z^{-1})y(t-f) \\ &\quad + B(z^{-1})E_{d+N}(z^{-1})u(t+N|t) \end{aligned} \quad (8)$$

which can be compacted as

$$\begin{bmatrix} \hat{y}(t+d|t) \\ \hat{y}(t+d+1|t) \\ \vdots \\ \hat{y}(t+d+N|t) \end{bmatrix} = \begin{bmatrix} F_d(z^{-1}) \\ F_{d+1}(z^{-1}) \\ \vdots \\ F_{d+N}(z^{-1}) \end{bmatrix} y(t-f) +$$

$$+ \begin{bmatrix} B(z^{-1})E_d(z^{-1})u(t|t) \\ B(z^{-1})E_{d+1}(z^{-1})u(t+1|t) \\ \vdots \\ B(z^{-1})E_{d+N}(z^{-1})u(t+N|t) \end{bmatrix} \quad (9)$$

The second term on the right side of the above can be separated into two parts: the first part contains the control sequence before time  $t$  and the second part the future control prediction sequence. So, let

$$\begin{aligned} & \begin{bmatrix} B(z^{-1})E_d(z^{-1})u(t|t) \\ B(z^{-1})E_{d+1}(z^{-1})u(t+1|t) \\ \vdots \\ B(z^{-1})E_{d+N}(z^{-1})u(t+N|t) \end{bmatrix} = \\ & = \begin{bmatrix} G_d(z^{-1}) \\ G_{d+1}(z^{-1}) \\ \vdots \\ G_{d+N}(z^{-1}) \end{bmatrix} u(t-1|t-1) + M_1 \begin{bmatrix} u(t|t) \\ u(t+1|t) \\ \vdots \\ u(t+N|t) \end{bmatrix} \end{aligned} \quad (10)$$

where the polynomial  $G_k(z^{-1}) \in \mathfrak{R}[z^{-1}, m+f+d-2]$  and the matrix  $M_1 \in \mathfrak{R}^{(N+1) \times (N+1)}$ . Thus,

$$\hat{Y}(t+d|t) = F(z^{-1})y(t-f) + G(z^{-1})u(t-1|t-1) + M_1 U(t|t) \quad (11)$$

where

$$\hat{Y}(t+d|t) = [\hat{y}(t+d|t), \hat{y}(t+d+1|t), \dots, \hat{y}(t+d+N|t)]^T \quad (12)$$

$$U(t|t) = [u(t|t), \dots, u(t+N|t)]^T \quad (13)$$

$$G(z^{-1}) = [G_d(z^{-1}), \dots, G_{d+N}(z^{-1})]^T \quad (14)$$

$$F(z^{-1}) = [F_d(z^{-1}), \dots, F_{d+N}(z^{-1})]^T \quad (15)$$

From the controller designed for the system without network delay, it is clear that the future control sequence can be expressed by

$$C(z^{-1})U(t|t) = D(z^{-1})(R(t+d) - \hat{Y}(t+d|t)) \quad (16)$$

where  $R(t+d) = [r(t+d), \dots, r(t+d+N)]^T$ . The term  $C(z^{-1})U(t|t)$  can also be separated into two parts: the first part contains the control sequence before time  $t$  and the second part the predicted future control sequence. Then, let

$$C(z^{-1})U(t|t) = H(z^{-1})u(t-1|t-1) + LU(t|t) \quad (17)$$

where  $H(z^{-1}) = [H_0(z^{-1}), H_1(z^{-1}), \dots, H_N(z^{-1})]^T$ , the polynomial  $H_i(z^{-1}) \in \mathfrak{R}[z^{-1}, \max\{n_c - i - 1, 0\}]$  and the matrix  $L \in \mathfrak{R}^{(N+1) \times (N+1)}$ . Combining (11), (16) and (17) gives

$$\begin{aligned} & H(z^{-1})u(t-1|t-1) + LU(t|t) = D(z^{-1})R(t+d) \\ & - D(z^{-1})F(z^{-1})y(t-f) - D(z^{-1})G(z^{-1})u(t-1|t-1) \\ & - D(z^{-1})M_1U(t|t) \end{aligned} \quad (18)$$

Let

$$\begin{aligned} & \Gamma(z^{-1})u(t-1|t-1) + MU(t|t) \\ & = D(z^{-1})(G(z^{-1})u(t-1|t-1) + M_1U(t|t)) \end{aligned} \quad (19)$$

where  $\Gamma(z^{-1}) = [\Gamma_0(z^{-1}), \Gamma_1(z^{-1}), \dots, \Gamma_N(z^{-1})]^T$ , the polynomial  $\Gamma_i(z^{-1}) \in \mathfrak{R}[z^{-1}, \max\{n_d + m + f + d - 2, 0\}]$  and the matrix  $M \in \mathfrak{R}^{(N+1) \times (N+1)}$ . As a result,

$$U(t|t) = (L+M)^{-1} \begin{bmatrix} D(z^{-1})R(t+d) - \\ D(z^{-1})F(z^{-1})y(t-f) - \\ (\Gamma(z^{-1}) + H(z^{-1}))u(t-1|t-1) \end{bmatrix} \quad (20)$$

Therefore, the control prediction sequence can be determined by the following predictive controller:

$$\begin{aligned} & \begin{bmatrix} u(t|t) \\ u(t+1|t) \\ \vdots \\ u(t+N|t) \end{bmatrix} = \begin{bmatrix} P_0(z^{-1}) \\ P_1(z^{-1}) \\ \vdots \\ P_N(z^{-1}) \end{bmatrix} r(t+d+N) - \\ & - \begin{bmatrix} Q_0(z^{-1}) \\ Q_1(z^{-1}) \\ \vdots \\ Q_N(z^{-1}) \end{bmatrix} y(t-f) - \begin{bmatrix} S_0(z^{-1}) \\ S_1(z^{-1}) \\ \vdots \\ S_N(z^{-1}) \end{bmatrix} u(t-1|t-1) \end{aligned} \quad (21)$$

where

$$\begin{aligned} & \begin{bmatrix} P_0(z^{-1}) & P_1(z^{-1}) & \dots & P_N(z^{-1}) \end{bmatrix}^T \\ & = (L+M)^{-1} [z^{-N} \quad z^{-N+1} \quad \dots \quad 1]^T D(z^{-1}) \\ & \begin{bmatrix} Q_0(z^{-1}) & Q_1(z^{-1}) & \dots & Q_N(z^{-1}) \end{bmatrix}^T \\ & = (L+M)^{-1} F(z^{-1}) D(z^{-1}) \\ & \begin{bmatrix} S_0(z^{-1}) & S_1(z^{-1}) & \dots & S_N(z^{-1}) \end{bmatrix}^T \\ & = (L+M)^{-1} (\Gamma(z^{-1}) + H(z^{-1})) \end{aligned}$$

and the polynomial  $P_i(z^{-1}) \in \mathfrak{R}[z^{-1}, n_d + N]$ ,

$Q_i(z^{-1}) \in \mathfrak{R}[z^{-1}, n_d + n - 1]$  and

$S_i(z^{-1}) \in \mathfrak{R}[z^{-1}, \max\{n_c - i - 1, n_d + m + f + d - 2, 0\}]$

### 2.3 Design of the Network Delay Compensator

In order to compensate the network transmission delay, a network delay compensator is proposed. A very important characteristic of the network is that it can transmit a set of data at the same time. Thus, it is assumed that all predictive control sequence at time  $t$  is packed and sent to the plant side through network. The networked delay compensator chooses the latest control value from the control prediction sequences available on the plant side. For example, if the following predictive control sequences are received on the plant side:

$$\begin{bmatrix} u(t-k_1|t-k_1) \\ u(t-k_1+1|t-k_1) \\ \vdots \\ u(t|t-k_1) \\ \vdots \\ u(t+N-k_1|t-k_1) \end{bmatrix}, \begin{bmatrix} u(t-k_2|t-k_2) \\ u(t-k_2+1|t-k_2) \\ \vdots \\ u(t|t-k_2) \\ \vdots \\ u(t+N-k_2|t-k_2) \end{bmatrix}, \dots, \begin{bmatrix} u(t-k_i|t-k_i) \\ u(t-k_i+1|t-k_i) \\ \vdots \\ u(t|t-k_i) \\ \vdots \\ u(t+N-k_i|t-k_i) \end{bmatrix} \quad (22)$$

where the control values  $u(t|t-k_i)$  for  $i=1, 2, \dots, t$ , are available to be chosen as the control input of the

plant at time  $t$ , the output of the network delay compensator will be

$$u(t) = u(t | t - \min\{k_1, k_2, \dots, k_i\}) \quad (23)$$

which is the latest predictive control value for time  $t$ .

#### 2.4 Implementation Procedure of Networked Predictive Controllers

Following the above subsections, the networked predictive control scheme can be implemented in the following steps:

- Step 1: Design a controller of the system without network transmission delay to satisfy the requirements using conventional control methods, for example, PID, LQG, model predictive control etc., *i.e.*, equation (2).
- Step 2: Formulate output predictors to predict the future outputs, based on the past outputs, the control inputs and the reference inputs, that is, equation (11).
- Step 3: Calculate the output sequence of the control prediction generator using (21)
- Step 4: Transmit the output sequence of the control prediction generator to the controlled plant through a network each time.
- Step 5: Apply the network delay compensator to choose the control input for the plant using (23).

### 3 STABILITY OF NETWORKED CONTROL SYSTEMS

The stability of a closed-loop system is the most important issue in the design of control systems. This section considers the stability of networked control systems for two cases: the first one is the case of the fixed network transmission delay and the second one is the case of the random network transmission delay.

#### 3.1. Case 1: Fixed Network Transmission Delay

It is assumed that the network transmission delays  $k$  and  $f$  in the forward and backward channels are constant. From the control prediction sequence derived in the previous section, it can be obtained that

$$u(t | t) = P_0(z^{-1})r(t+d+N) - Q_0(z^{-1})y(t-f) - S_0(z^{-1})u(t-1 | t-1) \quad (24)$$

Then

$$u(t | t) = \frac{P_0(z^{-1})r(t+d+N) - Q_0(z^{-1})y(t-f)}{1 + S_0(z^{-1})z^{-1}} \quad (25)$$

Using (21) and (25), the  $k$ -step ahead predictive control at time  $t$  is expressed by

$$\begin{aligned} u(t+k | t) &= P_k(z^{-1})r(t+d+N) - Q_k(z^{-1})y(t-f) \\ &\quad - S_k(z^{-1})u(t-1 | t-1) \\ &= \frac{P_k(z^{-1}) + P_k(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_k(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} r(t+d+N) \\ &\quad - \frac{Q_k(z^{-1}) + Q_k(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_k(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} y(t-f) \end{aligned} \quad (26)$$

As the network transmission is assumed to be fixed (say  $k$ ), the transmission delay compensator is taken as

$$u(t-i) = u(t-i | t-i-k), \text{ for } i = 0, 1, 2, \dots, m \quad (27)$$

Thus, the closed-loop system is

$$\begin{aligned} A(z^{-1})y(t+d+k) &= B(z^{-1})u(t+k) = B(z^{-1})u(t+k | t) \\ &= B(z^{-1}) \frac{P_k(z^{-1}) + P_k(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_k(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} r(t+d+N) \\ &\quad - B(z^{-1}) \frac{Q_k(z^{-1}) + Q_k(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_k(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} y(t-f) \end{aligned} \quad (28)$$

The closed-loop characteristic equation is

$$A(z^{-1})(1 + S_0(z^{-1})z^{-1}) + z^{-d-f-k} B(z^{-1})(Q_k(z^{-1}) + Q_k(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_k(z^{-1})z^{-1}) = 0 \quad (29)$$

If the roots of the above polynomial is within the unit circle, the system is stable.

#### 3.2. Case 2: Random Network Transmission Delay

It assumes that the network transmission delay  $k$  in the forward channel is random but bounded, *i.e.*,  $k \in \{0, 1, 2, \dots, N\}$ , where  $N$  is the upper bound, and the time delay  $f$  in the backward channel is constant. The plant can be written as

$$A(z^{-1})y(t) = z^{-d} B(z^{-1})u(t) = z^{-d} \sum_{i=0}^m b_i u(t-i) \quad (30)$$

Since the network transmission delay is random, to effectively compensate for this delay the networked control predictor is designed to be

$$u(t-i) = u(t-i | t-i-k_i), \text{ for } i = 0, 1, 2, \dots, m \quad (31)$$

subject to  $k_i \leq k_{i+1} + 1$ , where  $u(t-i | t-i-k_i)$  is the latest predictive control at time  $t-i$  which is available at the plant side and  $k_i \in \{0, 1, 2, \dots, N\}$  is a random number. Following (26), the predictive control  $u(t-i | t-i-k_i)$  is calculated by

$$\begin{aligned} u(t-i | t-i-k_i) &= \frac{P_{k_i}(z^{-1}) + P_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} r(t+d+N-i-k_i) \\ &\quad - \frac{Q_{k_i}(z^{-1}) + Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} y(t-i-k_i-f) \end{aligned} \quad (32)$$

As a result, the closed-loop control system is

$$\begin{aligned} A(z^{-1})y(t) &= z^{-d} \sum_{i=0}^m b_i u(t-i | t-i-k_i) \\ &= \sum_{i=0}^m b_i \frac{P_{k_i}(z^{-1}) + P_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - P_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} z^{-i} r(t+d+N-k_i) \\ &\quad - z^{-d} \sum_{i=0}^m b_i \frac{Q_{k_i}(z^{-1}) + Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}}{1 + S_0(z^{-1})z^{-1}} z^{-i-k_i-f} y(t) \end{aligned} \quad (33)$$

Therefore, the closed-loop characteristic equation is

$$A(z^{-1})(1 + S_0(z^{-1})z^{-1}) + z^{-d-f} \sum_{i=0}^m b_i (Q_{k_i}(z^{-1}) + Q_{k_i}(z^{-1})S_0(z^{-1})z^{-1} - Q_0(z^{-1})S_{k_i}(z^{-1})z^{-1}) z^{-i-k_i} = 0 \quad (34)$$

subject to  $k_i \leq k_{i+1} + 1$ . As  $k_i \in \{0, 1, 2, \dots, N\}$  is a random number, it results in a switched control system. Actually, the stability of the closed-loop system is equivalent to the stability of the following system:

$$x_{k+1} = T_k x_k \quad (35)$$

where the state vector  $x_k \in \mathfrak{R}^{\bar{n}}$  and

$$T_k = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ -p_{\bar{n}}(k) & -p_{\bar{n}-1}(k) & \dots & \dots & -p_1(k) \end{bmatrix}$$

and  $p_i(k)$ , for  $i=1, 2, \dots, \bar{n}$  are the coefficients of the closed-loop characteristic equation. So, the stability of the closed-loop system with random network delay can be determined by the switched control theory.

#### 4. SIMULATION EXAMPLES

##### 4.1 Example 1

In order to validate the proposed method, the speed control of a DC motor using networked predictive control method was simulated. The discrete model of DC motor is given by

$$G(z) = \frac{0.009201z^{-1} + 0.005709z^{-2}}{1 - 1.088z^{-1} + 0.2369z^{-2}} \quad (36)$$

A PID controller in the discrete form was designed when the communication time delay is not considered, which is given by

$$\frac{D(z)}{C(z)} = \frac{0.5 + 0.1z^{-1} + 0.16z^{-2}}{1 - z^{-1}} \quad (37)$$

The unit step response of the closed-loop PID control system without network communication delay is shown in Figure 2, which gives a good control performance.

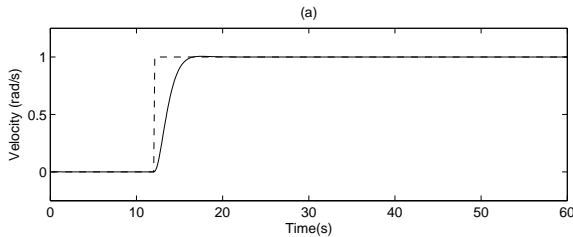


Fig. 2. The step response without network delay.

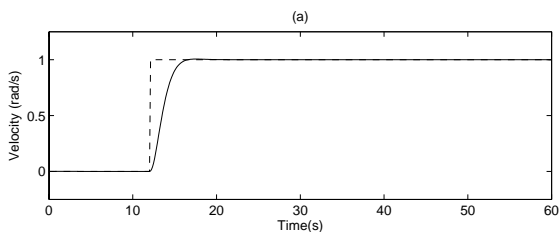


Fig. 3. The step response with 25-step constant network delay in both channels.

Now, two cases are considered: one is that there is 25-step constant communication time delay in both forward and backward channels and the other is that there is a random time delay within 25 steps in both channels. The simulation results for those two cases are shown in Figures 3 and 4. It is clear that the control performance is the same as one of the system without network time delay. So, using the proposed method, the network delays are completely compensated.

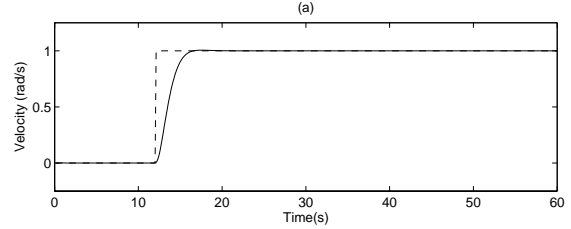


Fig. 4. The step response with random network delay within 25 steps in both channels.

Using the analytical stability criteria developed in the paper, the system stability was examined when the network delay in the forward channel changes from 0 to 30 steps and the delay in the backward channel changes from 0 to 200 steps. The maximum magnitude of the poles of the closed-loop system has been shown in Figure 5. It shows the closed-loop system is stable for all network time delays considered above.

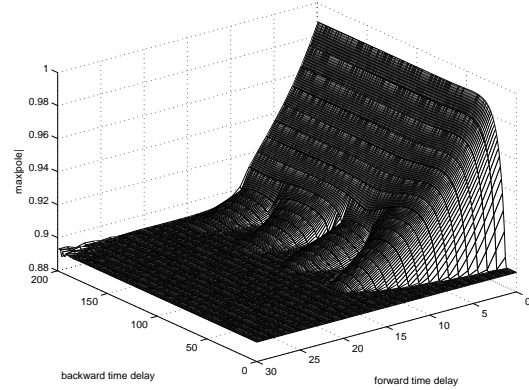


Fig. 5. The maximum magnitude of the closed-loop poles against the forward and backward delays.

##### 4.2 Example 2

Consider an unstable pendulum described by

$$\begin{aligned} & (1 - 2.099z^{-1} + z^{-2})y(t) \\ & = (-0.005041z^{-1} - 0.005041z^{-2})u(t) \end{aligned} \quad (37)$$

with the initial condition  $y(0)=0.1$  and  $y(-1)=0$ . The controller for the system without network time delay, which gives an acceptable control performance as shown in Figure 6, is designed to be

$$\frac{D(z^{-1})}{C(z^{-1})} = \frac{-5.6419 - 4.1665z^{-1} - 2.8598z^{-2}}{1 + 0.0683z^{-1} + 0.2201z^{-2}} \quad (38)$$

The control performance of the closed-loop systems using the networked predictive control scheme is shown in Figures 7-9 for the following cases:

Case a): the forward delay is 2 constant steps and the backward delay is 1 constant step;

Case b): the forward delay is 2 constant steps and the backward delay is 2 constant steps;

Case c): the forward delay is random within 2 steps and backward delay is random within 1 step.

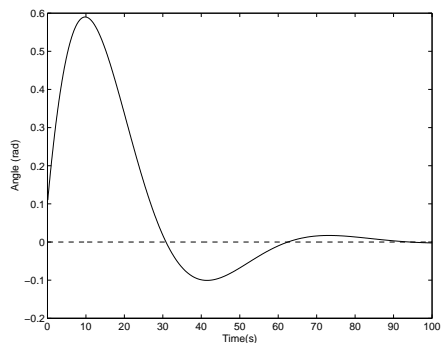


Fig. 6. Output response for the system without forward and backward time delay.

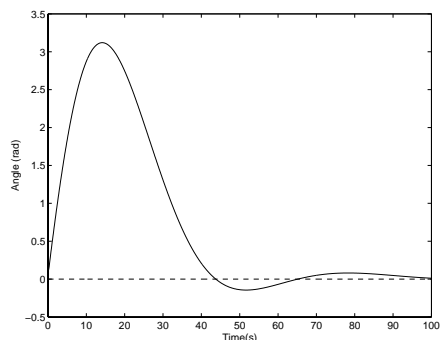


Fig. 7. Output response for Case a).

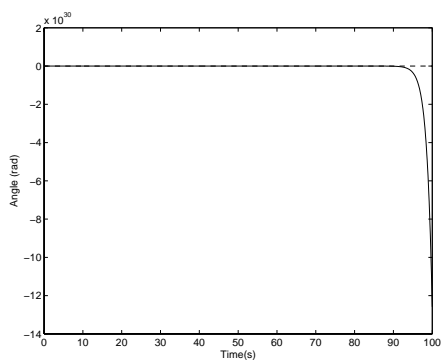


Fig. 8. Output response for Case b).

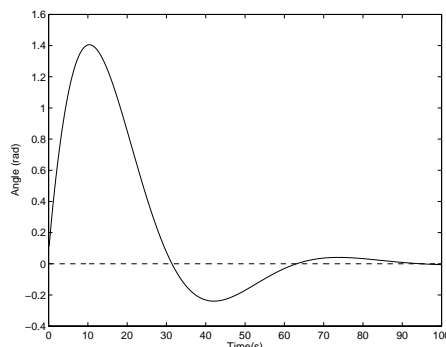


Fig. 9. Output response for Case c).

It is clear from the simulation that the closed-loop system is stable in both case a) and case c), and unstable in case b). Those are also confirmed using the closed-loop stability criteria developed in Section 3. Clearly, the simulation results are consistent with ones given by the analytical closed-loop stability criteria.

## 5. CONCLUSIONS

A new networked control scheme has been presented for networked distributed control system with random network transmission delay and also the stability of the closed-loop networked predictive control systems has been studied in this paper. The proposed networked predictive controller is comprised of the control prediction generator and the network delay compensator. The control prediction generator provides a set of future control predictions to satisfy the system performance requirements. The network delay compensator is used to overcome the random network transmission delay. The stability analysis of the closed-loop networked predictive control systems has given the analytical stability criteria for both the fixed and random network transmission delays, respectively. The paper provides a generic design procedure for networked control systems. In practice, there always exist uncertainties to a certain degree. The robustness of networked control systems with uncertainties, which has not been discussed here, will be studied in future papers.

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