ROBUST ADAPTIVE CONTROL OF NONLINEAR TRANSLATING BEAMS WITH A VARYING SPEED

Kyung-Jinn Yang*, Keum-Shik Hong**, and Fumitoshi Matsuno*

** Department of Mechanical Engineering and Intelligent Systems, University of Electro-Communications
1-5-1, Chofugaoka, Chofu, Tokyo 182-8585, Japan. Tel/Fax: +81-424-43-5403,
Emails: {jinnky, matsuno}@hi.mce.uec.ac.jp
** School of Mechanical Engineering, Pusan National University, San 30 Jangjeon-dong Gumjeong-gu, Busan,
609-735, Korea. Tel.: +82-51-510-2454, Fax: +82-51-514-0685, Email: kshong@pusan.ac.kr*

Abstract: In this paper, the investigational results for a robust adaptive vibration control of a translating tensioned beam with a varying traveling speed are presented. The dynamics of the moving beam are modeled as an Euler-Bernoulli beam, in which the tension applied to the beam is given as a nonlinear spatiotemporally varying function. The moving beam span is divided into two parts, i.e., a controlled span and an uncontrolled span, by a hydraulic touch-roll actuator that is located in the middle section of the beam. The transverse vibration of the controlled span part is suppressed by the actuator under unknown bounded disturbances exerted from the uncontrolled span. *Copyright © 2005 IFAC*

Keywords: Axially moving beam, robust adaptive control, uniform ultimate boundedness, nonlinear spatiotemporally varying tension, Lyapunov method.

1. INTRODUCTION

The control problem of axially moving continua occurs in such high performance mechanical systems as cranes, strips in a thin metal-sheet production line, and high-speed magnetic tapes. Dynamics analysis and control of axially moving beams have received particular and growing attention due to the emergence of new applications for deployable robotic manipulators (Romero and Vignjevic, 2002). Diverse research on the dynamics, stability, and/or active/passive controls for axially moving systems has been conducted in the last few decades (Yang *et al.*, 2005; Choi *et al*, 2004; Chen *et al.*, 2003; Li *et al.*, 2002; Li and Rahn, 2000; Zhu and Li, 2000; Lee and Mote, 1999; Wickert and Mote, 1989). However, in practical situations, to achieve better control performance of the axially moving system, a novel point-wise controller incorporating spatiotemporally varying tension, time-varying traveling velocity, and unknown disturbances from the adjacent span should be investigated.

Fig. 1 shows a typical schematic of a control strategy for axially moving continua using a hydraulic touch-roll actuator. The axially moving beam is divided into two spans, i.e., a controlled span and an uncontrolled span, by a transverse force actuator as shown in Fig. 1. The main objective is to suppress the transverse vibration in the controlled span despite of unknown bounded disturbances from the uncontrolled span.

The contributions of this paper are the following. The dynamics of the considered moving beam is modeled as an Euler-Bernolli beam, in which the tension applied to the beam is given as a nonlinear spatiotemporally varying function due to the varying traveling speed. To control the traveling beam through a hydraulic touch-roll actuator at the middle point, a robust adaptive vibration suppression scheme is proposed using the Lyapunov method. Since the proposed control law depends on the displacement and slope measurements on the controlled side of the actuator, the vibration suppression of the axially

moving beam can be successfully implemented. Further, the control scheme proposed can be directly applied to axially moving strings

2. PROBLEM FORMULATION

In Fig. 1, the roll at $x = 0$ is assumed to be fixed. Also, assume that there is the eccentricity of the support roll at $x = l_T$ that causes a periodic excitation at the right boundary of the beam. The two touch rolls, where the control input (force) is exerted from the hydraulic actuator, are located at $x = l$ in the middle section of the beam. Note that the vibration energy of the uncontrolled span part cannot converge to zero due to the periodic excitation at $x = l_T$, however, uniform ultimate boundedness can be concluded, which will be proved in Section 3 following. Such undesired vibration of the uncontrolled span gives an effect to the hydraulic actuator like an external force. The unknown but bounded external force applied to the actuator can then be treated as a right boundary disturbance on the controlled span part.

Let t be the time, x be the spatial coordinate along the longitude of motion, $v_s(t)$ be the varying axial speed of the beam, $v_s(t) > 0$ for all t, $w(x,t)$ be the transversal displacement of the beam at spatial coordinate x and time t, and l be the length of the controlled span part. Also, let ρ be the mass per unit length, A be the cross-sectional area, E be the coefficient of elasticity, I be the moment of inertia of the beam cross section, and $T_s(x,t)$ be the spatiotemporally varying tension applied to the beam. Let the mass and damping coefficients of the hydraulic actuator be m_c and d_c, respectively. The control force $f_c(t)$ is applied to the touch rolls to suppress the transverse vibrations of the axially moving beam. $d(t)$ denotes the unknown but uniformly bounded external disturbance force exerting on the actuator due to the transverse vibration of the uncontrolled span.

Now, Hamilton's principle for translating continua (Yang *et al.*, 2005) is utilized and then the following nonlinear equation of motion of the Euler-Bernoulli beam traveling at a varying velocity $v_s(t)$ between two support rolls separated by a distance l is obtained:

$$\rho A\left(w_{tt} + \dot{v}_s w_x + 2v_s w_{xt} + v_s^2 w_{xx}\right) - \left((T_s w_x)_x + \frac{3EA}{2}w_x^2 w_{xx}\right)$$
$$+ EIw_{xxxx} = 0. \qquad (1)$$

The boundary conditions are given as
$$w(0) = 0, \ w_x(0) = 0, \ w_{xx}(l) = 0, \text{ and} \qquad (2)$$
$$m_c w_{tt}(l) + d_c w_t(l) + T_s(l)w_x(l) + \frac{EA}{2}w_x^3(l) - EIw_{xxx}(l) - d(t) = f_c(t). \ (3)$$

The term $(T_s + 3EAw_x^2/2)$ in (1) is often called a nonlinear tension.

Remark 1: In this paper, $T_s(x,t)$ in (1) is given as

$$T_s(x,t) = T_0 - \rho A(l_T - x)(eg - \dot{v}_s), \qquad (4)$$

where $e = 0$ for the horizontally translating beam, $e = 1$ for the vertically translating beam, and g and T_0 denote the gravitational acceleration and the initial tension applied to the beam, respectively.

Remark 2: Provided that there is no big disturbance in the system, $T_s(x,t)$ can be assumed to be continuous and uniformly bounded, as follows:

$$0 < T_{s,\min} \le T_s(x,t) \le T_{s,\max}, \ \left|(T_s)_t\right| \le (T_s)_{t,\max},$$
$$\text{and } \left|(T_s)_x\right| \le (T_s)_{x,\max}, \qquad (5)$$

for all $x \in [0,l]$, $t \ge 0$, and some *a priori* known constants $T_{s,\min}$, $T_{s,\max}$, $(T_s)_{t,\max}$, and $(T_s)_{x,\max}$, where $(T_s)_t = \rho A(l_T - x)\ddot{v}_s$ and $(T_s)_x = \rho A(eg - \dot{v}_s)$ from (4). Considering practical situations such as a high-tensioned beam under axial transport processing, it can be assumed that the lower bound $T_{s,\min}$ is larger than both $(T_s)_{t,\max}$ and $(T_s)_{x,\max}$ due to the high tension limit. However, for some visco-elastic materials such as synthetic rubber and synthetic fiber, in which such a high tension is not required, the fluctuating $v_s(t)$ might not guarantee $T_{s,\min} > \max[(T_s)_{t,\max}, (T_s)_{x,\max}]$.

In order to provide a specific idea regarding how a boundary control works, the dynamics of the moving beam with both fixed right and left boundaries is first analyzed. The vibration energy $E_{beam}(t)$ of the beam is given by

$$E_{beam}(t) = \frac{\rho A}{2}\int_0^l (w_t + v_s w_x)^2 dx + \frac{1}{2}\int_0^l T_s w_x^2 dx$$
$$+ \frac{EA}{8}\int_0^l w_x^4 dx + \frac{EI}{2}\int_0^l w_{xx}^2 dx. \qquad (6)$$

The time derivative of $E_{beam}(t)$ in (6) is now evaluated by applying the one-dimensional transport theorem of moving material (see Yang et al., 2005) as follows:

$$\dot{E}_{beam} = -EIv_s w_{xx}^2(0) + EIv_s w_{xx}^2(l) + \frac{1}{2}\int_0^l \{(T_s)_t + v_s(T_s)_x\} w_x^2 dx. \ (7)$$

Thus, it is concluded that for the traveling beam, the beam bending moment or beam force occurring on the right boundary $x = l$ and the time rate of the change of tension $T_s(x,t)$ should be properly handled in order to decrease the vibration energy $E_{beam}(t)$

3. ROBUST ADAPTIVE CONTROL

In this section, robust adaptive control laws that suppress the transverse vibration of the beam governed by (1)-(3) is derived, and the stability of the closed-loop system under the control law is proven.

Lemma 1: The beam vibration energy E_{beam} in (6) and the following function are equivalent:

$$V_{beam} = \alpha E_{beam} + \beta \rho A \int_0^l x w_x(w_t + v_s w_x)dx. \qquad (8)$$

where $\beta < \alpha/\beta_1$ and $\beta_1 = l \cdot \max\{1, \rho A/T_{s,\min}\}$. []

Hence, with Lemma 1 and assuming that the disturbance $|d(t)|$ is uniformly bounded by μ_d, i.e., $\mu_d \geq |d(t)|$, where μ_d is an unknown positive constant, a positive definite functional $V(t)$, as the total energy of the moving beam system including the actuator, is defined as follows:

$$V(t) = V_{beam} + V_{act} + V_{dist},\qquad(9)$$

where

$$V_{act} = \frac{m_c}{\xi_1}\{\xi_1 w_t(l) + \xi_2 w_x(l)\}^2,\ \xi_1 = \frac{\alpha}{2},\ \xi_2 = \frac{(\alpha v_s + \beta l)}{2},\qquad(10)$$

$$V_{dist} = \frac{1}{2\gamma_d}\widetilde{\mu}^2,\ \text{and}\ \widetilde{\mu}_d = \hat{\mu}_d - \mu_d,\qquad(11)$$

where $\hat{\mu}_d$ is the adaptive estimate of μ_d, which will be specified in Theorem 1 following.

In this paper, the functional $V(t)$ in (9) is considered as a Lyapunov function candidate. For sufficiently high tension as mentioned in Remark 2, the positive values α and β can be chosen to satisfy

$$\left\{\beta\left(T_{s,\min} - \rho A v_s^2\right) - \alpha(T_s)_{t,\max} - (\beta l + \alpha v_s)(T_s)_{x,\max}\right\} > 0.$$

Finally, the main theorem of this paper is stated as follows:

Theorem 1: Consider system (1)-(3). Let the control force $f_c(t)$ be given by

$$f_c(t) = -g_1 w_{xt}(l) + (d_c - g_2)w_t(l) - m_c\dot{v}_s w_x(l) + f_d(t),\qquad(12)$$

where $g_1 = \frac{m_c(\alpha v_s + \beta l)}{\alpha}$, $g_2 = \frac{\beta\rho A l v_s}{(\alpha v_s + \beta l)}$, $\alpha > 0$,

$$\text{and}\ 0 < \beta < \min\{\alpha v_s/l,\ \alpha/\beta_1\},\qquad(13)$$

and the additional term $f_d(t)$ is regarded as a new input signal determined based on robust control strategy and is given by

$$f_d(t) = -\frac{\hat{\mu}_d^2(t)}{\hat{\mu}_d(t)|\overline{w}(l)| + \varepsilon_d}\overline{w}(l),\qquad(14)$$

where $\overline{w}(l) = \{\alpha w_t(l) + (\alpha v_s + \beta l)w_x(l)\}$ and $\varepsilon_d > 0$. The adaptation law $\hat{\mu}_d$ in (14) is proposed as follows:

$$\dot{\hat{\mu}}_d(t) = -\delta_d\hat{\mu}_d(t) + \gamma_d|\overline{w}(l)|,\qquad(15)$$

where $\delta_d > 0$ and $\gamma_d > 0$. Suppose that $T_s(l) > \rho A v_s^2$ and $\left\{\beta\left(T_{s,\min} - \rho A v_s^2\right) - \alpha(T_s)_{t,\max} - (\beta l + \alpha v_s)(T_s)_{x,\max}\right\} > 0$. Then, the dynamics of the closed-loop system is uniformly ultimately bounded, that is,

$$\dot{V}(t) \leq -\lambda V(t) + \upsilon,\qquad(16)$$

where $\upsilon = \varepsilon_d + \frac{\delta_d}{2\gamma_d}\mu_d$, $\lambda > 0$, and

$$\lambda = \min\left[\begin{array}{cc} \dfrac{3\beta}{\alpha(\alpha+\beta\beta_1)}, & \dfrac{\beta\left(T_{s,\min}-\rho A v_s^2\right)-\alpha(T_s)_{t,\max}-(\beta l+\alpha v_s)(T_s)_{x,\max}}{2\alpha T_{s,\max}(\alpha+\beta\beta_1)}, \\[3mm] \dfrac{\beta}{2\alpha(\alpha+\beta\beta_1)}, & \dfrac{\beta\left(T_{s,\min}-\rho A v_s^2\right)-\alpha(T_s)_{t,\max}-(\beta l+\alpha v_s)(T_s)_{x,\max}}{4\alpha\rho A v_s^2(\alpha+\beta\beta_1)}, \\[3mm] \dfrac{\alpha\beta\left(T_s(l)-\rho A v_s^2\right)}{2m_c\left(\alpha v_s+\beta l\right)^2}, & \dfrac{\beta\rho A l}{m_c\alpha}\left(\dfrac{\alpha v_s}{\alpha v_s+\beta l}-\dfrac{1}{2}\right),\ 2\delta_d \end{array}\right]$$

$$(17)$$

Since υ in (16) is a uniformly bounded positive constant as $\varepsilon_d + \frac{\delta_d}{2\gamma_d}\mu_d$, the uniform ultimate boundedness region of $V(t)$ can be made arbitrarily small near to zero by making sufficiently small ε_d, δ_d and relatively large γ_d (see Corless and Leitmann, 1981).

Remark 3: From (12)-(16), it is seen that the closed-loop system with the control law f_c without f_d in (12) can be exponential stable if the effect of the disturbance $d(t)$ from the uncontrolled span is ignored, i.e., $V(t) \leq V(0)e^{-\lambda t}$ from (16). Also, in the case, the open-loop system with only the boundary damping can be stable if the damping coefficient is sufficiently large from (16). However, the damping value never gives any effect to the slope terms as shown in (16) and which means how large damping value cannot improve the performance of the open-loop system. Further, the effect of $d(t)$ cannot be such simply disregarded in actual systems and then the closed-loop system with only f_c as well as the open-loop system in the case would not guarantee any stability, without mentioning the exponential stability.

Remark 4: Note that Theorem 1 is a sufficient condition but not a necessary condition since the value of $(T_s)_t$ is impossible to maintain as a positive value for all t in an actual system, given that it is close to a periodic pattern. Hence, for a system with high-frequency variation of the tension, that is, $\beta T_{s,\min} < \alpha(T_s)_{t,\max}$, the stability analysis of the closed-loop system should be performed via complex mathematic schemes and/or numerical simulations by considering the system parameters and control gains. However, in the case of non-viscoelastic materials (see Remark 2), for almost all translating beams, such as a high-tensioned steel strip in actual processing lines, the proper control gains satisfying the conditions in Theorem 1 can be assured.

Remark 5: Robust adaptive control laws (12)-(15) are given for velocity $w_t(l)$, slope $w_x(l)$, and slope rate $w_{xt}(l)$ on the controlled side of the actuator at $x = l$. By using an encoder (or photodiode) on the actuator and two laser sensors, the actuator displacement $w(l)$ and the slope $w_x(l)$ on the actuator, respectively, can be measured. The actuator velocity $w_t(l)$ and the slope rate $w_{xt}(l)$ can then be implemented by the backward differencing of the signals (see Li *et al.*, 2002; Li and Rahn, 2000).

Remark 6: Consider V_{beam} in (8) for $l < x < l_T$ which is equivalent to the mechanical energy of the uncontrolled span. Since time-varying conditions at the left and right boundaries of the uncontrolled span part are really bounded, it can be easily concluded by using Lemma 1, Lemma 3, and Theorem 1 that the mechanical energy of the uncontrolled span is

uniform ultimate bounded. However, the uniform ultimate boundedness region could not be made arbitrarily small despite of the control action at $x = l$ due to the periodic excitation of the support roller at $x = l_T$.

Remark 7: Note that a dynamic model of a time-varying translating string can be easily obtained by setting $E = 0$ in the beam model given as (1)-(3). Hence, the robust adaptive controller proposed can be directly applied to the axially moving string system without any modifications for ensuring the vibration reduction.

4. NUMERICAL SIMULATIONS

In this section, the effectiveness of the robust adaptive controller proposed is illustrated by numerical simulations using a finite difference scheme. The parameter values used for the beam are as follows: $\rho = 700$ kg/m^3 , $A = 35 \times 10^{-5}$ m^2 , $I = (0.07 \times 0.005^3)/12$ m^4 , $l = 25$ m , $l_T - l = 25$ m , and $E = 2 \times 10^8$ N/m^2. Also, the parameters and control gains of the control laws in (12)-(15) are given as $m_c = 2$ kg , $d_c = 0.5$ N/m/sec , $\alpha = 1$, $\beta = 0.015$, $\gamma_d = 10^{-3}$, $\delta_d = 10^{-5}$, and $\varepsilon_d = 10^{-6}$. Let the initial conditions of the beam satisfying (2) be

$w(x,0) = (0.3 \times 10^{-8})x^2(l-x)^3(l_T - x)^2$ m, $w_t(x,0) = 0$ m/sec,

and the initial conditions of the proposed controller be zero.

To show the influence of the time-varying speed and control gains on the system's stability, only the controlled span is first considered, i.e., the uncontrolled span in Fig. 1 is disregarded. Hence, in the case, the exponential stability can then be guaranteed for the closed-loop system using only f_c without f_d in (12) as mentioned in Remark 4. The simulation results of this system are presented in Figs. 2-5: Fig. 2 depicts the vibration energy $E_{beam}(t)$ in (6) of the time-varying beam system with $v_s(t) = 2 + \sin 10t$ m/sec and the time-invariant beam system with $v_s = 2$ m/sec , respectively, under the proposed controller, for which the initial tension was given as $T_0 = 115$ N. As shown in Fig. 2, the vibration energy of the time-varying beam system diverges despite the boundary controller at $x = l$, whereas that of the time-invariant beam system converges. This instability result of the time-varying beam system is due to $\beta T_{s,\min} \cong 0.8 << \alpha (T_s)_{t,\max} \cong 612$, as analyzed in Section 2 and Section 3.

Fig. 3 shows the influence of the control gains α and β in (12), in which the control gain β was given as 0.001 and 0.038 , respectively, instead of $\beta = 0.015$. For this simulation, the varying traveling speed and the initial tension of the moving beam were given as $v_s(t) = 2 + 0.01 \sin 3t$ m/sec and

$T_0 = 115$ N , respectively. As analyzed in Theorem 1 and Remark 3, it is seen from Fig. 3 that the exponential index λ in (17) depends on the values of α and β ; accordingly, $E_{beam}(t)$ of the system with higher β decays more quickly. However, the value of β cannot be set too high due to the limit mentioned in Theorem 1; here the limit is $\beta < \alpha/l = 0.04$. Even though a local increase in $E_{beam}(t)$ of the closed-loop system is shown in Fig. 3, it is verified from Fig. 4 that the Lyapunov energy $V(t)$ in (27) still exponentially decays without any local increases, as analyzed in Remark 3.

In Fig. 5, the closed-loop system given as Fig. 4 was compared with the open-loop system with only the boundary damping set by $d_c = 5 \times 10^4$ N/m/sec. From the result, it is seen that $E_{beam}(t)$ of the open-loop system controlled by the boundary damper converges more slowly than that of the closed-loop system with the feedback boundary controller proposed despite such a high damping value as analyzed in Remark 4. Further, the present damping value $d_c = 5 \times 10^4$ N/m/sec is too large to be implemented in an actual system.

Now, consider the total span containing the uncontrolled span part as shown in Fig. 1. The simulation results of the time-varying translating beam and string are demonstrated through Figs. 6-7 and 8-9, respectively: For the beam, the varying translating speed, the initial tension, and the periodic excitation at $x = l_T$ were given as $v_s(t) = 1 + 0.2 \sin 3t$ m/sec , $w(l_T) = 0.5 \sin 30t$ m , and $T_0 = 5000$ N , respectively. Fig. 6 shows the vibration energy $E_{beam}(t)$ of the controlled and uncontrolled spans using only f_c without f_d , in which it is clearly seen that $E_{beam}(t)$ of the closed-loop system doesn't converge to zero any more and remains at a level due to the vibration effect of the uncontrolled span as mentioned in Remark 3. Hence, to overcome the unknown undesired vibration effect from the uncontrolled span, robust adaptive laws (14)-(15) are also needed. Fig. 7 depicts the vibration energy $E_{beam}(t)$ of the controlled and uncontrolled spans using f_c with f_d. As analyzed in Theorem 1 and Remark 6, it is seen from the results that the initial vibration energy of the controlled span dissipates asymptotically with the control action at $x = l$ while the vibration energy of the uncontrolled span still remains almost at the same level. Also, a local increase in the mechanical energy is shown due to the boundary disturbance on the controlled span part

The same results as the above can be obtained for the string as mentioned in Remark 8. Figs. 7 and 8 show the vibration energy $E_{beam}(t)$ in (13) of the controlled and uncontrolled spans using only f_c without f_d and using f_c with f_d , respectively, where the varying translating speed, the initial

tension, actuator mass, and the periodic excitation at $x = l_T$ were given as $v_s(t) = 1 + 0.2\sin 3t$ m/sec, $T_0 = 2600$N, $m_c = 0.5$kg, and $w(l_T) = 0.25\sin 30t$ m, respectively, and the initial conditions were given as $w(x,0) = 0.5\sin 6\pi x$ m and $w_t(x,0) = 0$ m/sec.

As shown in Figs. 7 and 9, $E_{beam}(t)$ of the closed-loop system controlled by the robust adaptive control laws in (12)-(15) converges in a stable manner, and, also, the control gains given by $\alpha = 1$ and $\beta = 0.015$ would never be an implementation problem. The transverse displacement $w(l)$ at the middle point and the control input force $f_c(t)$ in (12), used in obtaining Figs. 6-9, are predicted to converge near to zero from the results. Also, it is easily predicted that the value $\hat{\mu}_d$ to estimate the unknown disturbance d converges to a constant value because $w_t(l)$ and $w_x(l)$ approach near to zero, however, it is noted that the convergence of the estimated parameter to the exact value is not essential in this control scheme.

5. CONCLUSIONS

This paper has investigated a robust adaptive control scheme to suppress the transverse vibration of an axially moving beam with a varying traveling speed and a nonlinear spatiotemporally varying tension. Since the feedback terms in the control laws are the velocity, slope, and the slope rate on the controlled span side of the actuator, the vibration suppression of the controlled span can be successfully implemented while ensuring the bounded vibration of the uncontrolled span. Because the beam was modeled as an Euler-Bernoulli beam equation with a time-varying speed, the control method developed can be applied to any system including string and belt systems in a similar form. In our future research, the proposed control strategy will be extended to applications, such as axially moving continua with arbitrarily varying lengths including elevators, cranes, and robotic manipulators with axially moving flexible arms, in which the traveling velocity and tension terms are truly described as time-varying and spatiotemporally varying functions, respectively.

ACKNOWLEDGMENT

REFERENCES

Romero, I. and R. Vignjevic (2002). Deployable space manipulator closed-loop control, ideas and possibilities of using GPS as a sensor, *Advanced in Space Research*, **30**, pp. 419-425.

Chen, L.Q., N.H. Zhang and J.W. Zu (2003). The regular and chaotic vibrations of an axially moving viscoelastic string based on 4-order Galerkin truncation, *Journal of Sound and Vibration*, **261**, pp. 764-773.

Choi, J.Y., K.S. Hong and K.J. Yang (2004). Exponential stabilization of an axially moving tensioned strip by passive damping and boundary control, *Journal of Vibration and Control*, **10**, pp. 661-682.

Corless, M.J. and G. Leitmann (1981). Continuous state feedback guaranteeing uniform boundedness for uncertain dynamic systems, *IEEE Transactions on Automatic and Control*, **26**, pp. 1139-1144.

Lee, S.Y. and C.D. Mote (1999). Wave characteristics and vibration control of translating beams by optimal boundary damping, *Journal of Vibration and Acoustics*, **121**, pp. 18-25.

Li, Y., D. Aron and C.D. Rahn (2002). Adaptive vibration isolation for axially moving strings: theory and experiment, *Automatica*, **38**, pp. 379-390.

Li, Y. and C.D. Rahn (2000). Adaptive vibration isolation for axially moving beams, *IEEE/ASME Transactions on Mechatronics*, **5**, pp. 419-428.

Wickert, J.A. and C.D. Mote (1989). On the energetics of axially moving continua, *Journal of Acoustic Society of America*, **85**, pp. 1365-1368.

Yang, K.J., K.S. Hong and F. Matsuno (2005). The rate of change of energy functional for axially moving continua, submitted to *IEEE Transactions on Automatic and Control* for review.

Zhu, W.D. and J. Ni (2000). Energetics and stability of translating media with an arbitrarily varying length, *ASME Journal of Vibration and Acoustics*, **122**, pp. 295-304.
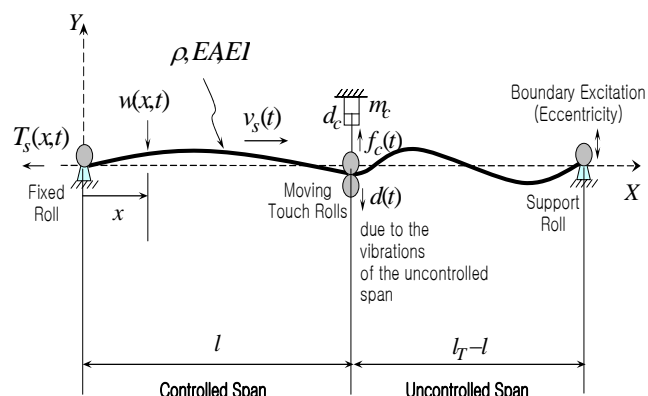
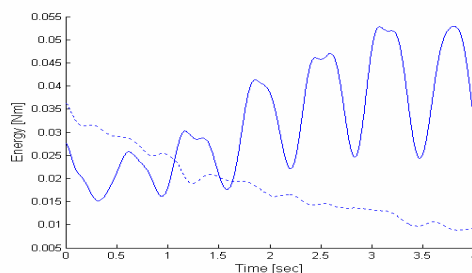Fig. 1 A control-oriented model of the axially moving beam with a hydraulic actuator.

Fig. 2 Instability of the time-varying case: $E_{beam}(t)/2$ with $v_s = 2 + \sin 10t$ m/sec (solid line) and $E_{beam}(t)$ with $v_s = 2$ m/sec (dotted line).
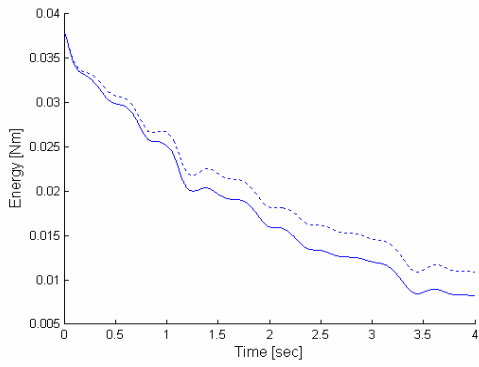
Fig. 3 The effect of control gain β :
$E_{beam}(t)$ with $\beta = 0.038$ (solid line) and that with
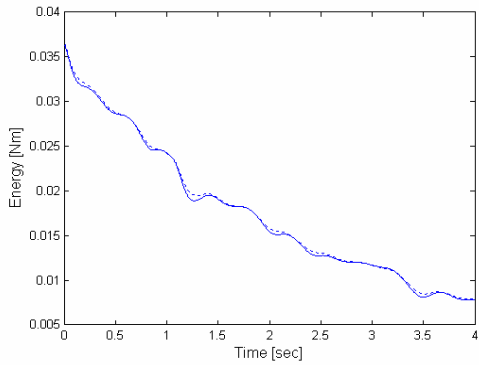$\beta = 0.001$ (dotted line).

Fig. 4 Comparison of $E_{beam}(t)$ (solid line) and $V(t)$
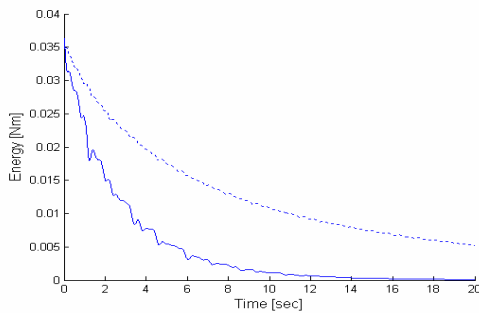(dotted line): $\beta = 0.038$.

Fig. 5 Comparison of $E_{beam}(t)$ of the closed-loop system
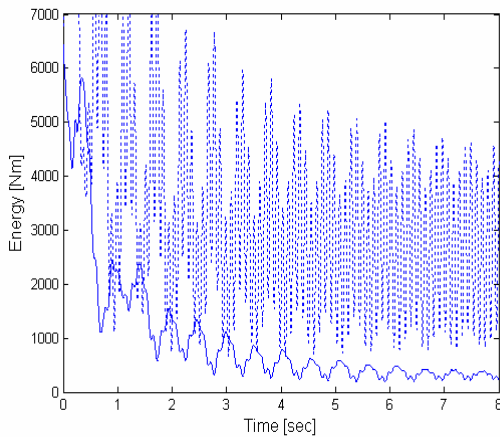(solid line) and that with $f_c = 0$ (open-loop, dotted line).

Fig. 6 Responses of the closed-loop system without
disturbance compensation, that is, f_d in (12) is zero:
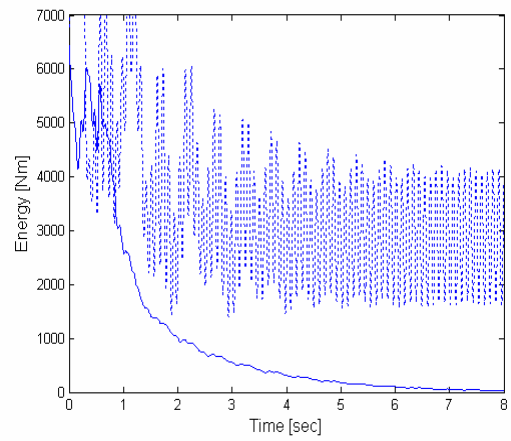$E_{beam}(t)$ of controlled span (solid line) and uncontrolled
span (dotted line)

Fig. 7 Responses of the closed-loop system (see Fig. 6)
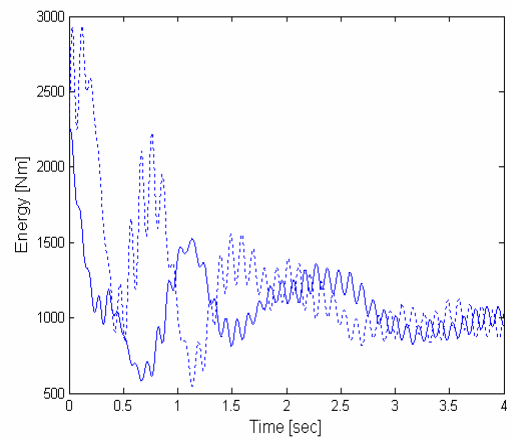with a f_d -term: improvement is shown.

Fig. 8 Responses of the axially moving string (i.e., (1)-(3)
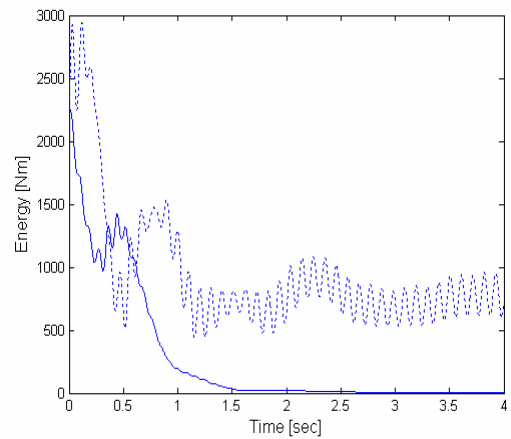with $E = 0$) when $f_d = 0$ in (12).

Fig. 9 Responses of the axially moving string (see Fig. 8)
with a f_d -term.