ROBUST STRING STABILITY CONTROLLER DESIGN OF A PLATOON OF VEHICLES BASED ON LMI APPROACH*

Lijuan Wu [1,2] **Yuanwei Jing** [1] **Xue-Bo Chen** [2] **Wei Wang** [3]

1:School of Information Science and Engineering, Northeast University, Shenyang, 110004, P.R.China, wuleejuan@yahoo.com.cn, ywjjing@peoplemail.com.cn
2:School of Electronic and Information Engineering, Anshan Science and Technology University, Anshan, 114044, P.R.China, xuebochen@sohu.com
3:Dalian University of Technology, Dalian 116023,P.R.China

Abstract: This paper is devoted to robust stable control for a platoon system with mismatched parametric uncertainty. A stable sufficient condition of the platoon of vehicles is given in terms of linear matrix inequalities (LMI), based on which the corresponding controller is also developed. The results are illustrated by an example.

Keywords: a platoon of vehicles, LMI, stable control, robust control.

1. INTRODUCTION

The problem of highway congestion has long been a nightmare of urban residents in many countries. Congestion comes from the fact that highway capacity is not able to meet traffic demand. To alleviate the problem, several policies could be applied (Varaiya, 1993):

1) Local policies such as unbalanced lane structure and ramp flow control,
2) Building more highway,
3) Raising tolls or other taxes,
4) Promoting mass transit or carpooling,
5) Developing other types of high-speed communication networks that can effectively replace the need for travel, e.g., virtual reality meet technology, and
6) The Intelligent Vehicle/Highway Systems (IVHS)

Some of the items listed above are now being employed in most countries to increase traffic flow, e.g., items 1), 2), 3) and 4). Due to the scarcity of land, the policy of building more highways will become infeasible. It is estimated, for example, that in the United States in the year 2010, total roadway travel will be more than double relative to 1992 (Bender, 1991), and correspondingly, there is an inability to build enough new roads to accommodate this increase. So other policies have been placed emphasis on improving highway congestion. Item 5) can solve some parts of the problem, but this solution is in the research stage.

Intelligent vehicle highway system (IVHS) have drawn much concern since they are assumed to be safe and effective methods to meet the increasing traffic demand and resultant problems such as traffic accidents, traffic congestion and air pollution, which integrate control, communication and computing technologies implemented on the highway and vehicles so that the driving will be partly or even taken over by the system. The principle motivation for an IVHS is to increase highway capacity in a safe manner by effective utilization of the existing infrastructure. There are several approaches that can be used to increase traffic flow. Among them, platooning is one approach that has attracted much attention in recent years (Huang, *et al.*, 1999).

A platoon is one or more vehicles traveling together as a group with relatively small spacing to improve

*) This research is supported by NSFC of China under grant No.60074002, and by the USRP of Liaoning Education Department under grant No.202192057

capacity and to reduce relative velocity in case of accidents. Inter-platoon spacing is large enough to allow emergency stopping. By using advanced control, communication, and computing technologies, dynamic lane changing, merging, and exiting problems can solved, and it appears that platooning is most feasible (Horowitz, 2000).

The platoon control system must be designed to take into account the string stability of a vehicle stream, in addition to good vehicle following performance. A platoon of vehicles is said to be string stability if, under no other excitation, the range errors decreased as they propagate along the vehicle stream. The ‘string stability ’ of a vehicle platoon under automatic control has been an active topic of research in recent decades (Swaroop, 1996).

A platoon of vehicles is said to be string stable if the spacing errors do not amplify upstream from one vehicle to another in the string. Mathematically, if the transfer function from the range error of a vehicle to that of its following vehicle has a magnitude of less than or equal to 1, then it is string stability. This work was developed by Stankovic (2000), in his paper he designed a decentralized controller using LQ approach and analyzed the platoon string stability by its transfer function. Swaroop (1996) studied the string stability of a countably infinite interconnected of a class of nonlinear systems. Lee (2002) adopted fuzzy-sliding mode algorithm to study the string stability of platoon system.

In this paper, the string stability of a platoon system is studied based on LMI approach. The sufficient condition of stability was obtained first, then based on which the corresponding controller is also developed, and at last the unstable bounds can be obtained from it. The paper is organized as follows: The following section presents a description of the string stability analysis method. Based on this method, we make a simulation in section 4. We present conclusion in Section 5.

2. MODEL STRUCTURE OF SYSTEMS

In this paper, it will be supposed that the *i*-th vehicle in a close platoon consisting of *N* vehicles can be represented by the following nonlinear third-order model (Stankovic, 2000):

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= f_i(v_i, a_i) + g_i(v_i)\eta_i \end{aligned} \qquad i = 1, 2, \ldots, N \tag{1}$$

where $d_i = x_i - x_{i-1}$ is the distance between two consecutive vehicles, x_{i-1} and x_i being their position, v_i and a_i are the velocity and acceleration respectively, while η_i is the engine input. Functions $f_i(.,.)$ and $g_i(.,.)$ are given by

$$f_i(v_i, a_i) = -\frac{2k_{di}}{m_i} v_i a_i - \frac{1}{\tau_i(v_i)}[a_i + \frac{K_{di}}{m_i} v_i^2 + \frac{d_{mi}}{m_i}],$$
$$g_i(v_i) = \frac{1}{m_i \tau_i(v_i)} \tag{2}$$

where m_i represent the vehicle mass, τ_i is the time-constant of the engine, K_{di} the aerodynamic drag coefficient and d_{mi} the mechanical drag. Assuming that the parameters in (2) are a priori known, we shall adopt the following control law structure:

$$\eta_i = m_i u_i + k_{di} v_i^2 + d_{mi} + 2\tau_i k_{di} v_i a_i \tag{3}$$

where u_i is the input signal. After introducing (3), the third equation in (1) becomes

$$\dot{a}_i = -\tau_i^{-1} a_i + \tau_i^{-1} u_i \tag{4}$$

Assuming that every vehicle have the same τ_i, so τ_i can be replaced by τ. Then (1) will be

$$\begin{aligned} \dot{d}_i &= v_{i-1} - v_i \\ \dot{v}_i &= a_i \\ \dot{a}_i &= -\tau^{-1} a_i + \tau^{-1} u_i \end{aligned} \tag{5}$$

The resulting linearized vehicle model is the basis of realizing the platoon control strategies.

We change the model by the following expressions

$$\Delta d_i = d_i - d_r, \Delta v_i = v_i - v_r, \Delta a_i = a_i - a_r \tag{6}$$

where d_r is the reference value of distance between two consecutive vehicles, v_r, a_r are the reference values of velocity and acceleration, respectively. When the vehicle is stable, we demand d_r, v_r are constant values, so a_r =0. Obviously $\Delta d_i, \Delta v_i, \Delta a_i$ are the deviation values of the corresponding values.

From (5) and (6) we get the deviation state equation of vehicle

$$\begin{aligned} \Delta\dot{d}_i &= \Delta v_{i-1} - \Delta v_i \\ \Delta\dot{v}_i &= \Delta a_i \\ \Delta\dot{a}_i &= -\tau^{-1}\Delta a_i + \tau^{-1} u_i \end{aligned} \tag{7}$$

or

$$\dot{x}_i = A_v x_i + B_v u_i$$

The following state model of the entire platoon can be formulated

$$S: \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \cdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A}_v & 0 & \cdots & & 0 \\ \mathbf{A}_d & \mathbf{A}_v & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & \mathbf{A}_d & \mathbf{A}_v \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} + \begin{bmatrix} \mathbf{B}_v & 0 & \cdots & 0 \\ 0 & \mathbf{B}_v & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \mathbf{B}_v \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \cdots \\ u_n \end{bmatrix}. \quad (8)$$

Where

$$x_i^T = [\Delta d_i \quad \Delta v_i \quad \Delta a_i],$$
$$x_1^T = [\Delta v_1 \quad \Delta a_1],$$
$$\mathbf{A}_v = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\tau^{-1} \end{bmatrix};$$
$$\boldsymbol{A}_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_v = \begin{bmatrix} 0 \\ 0 \\ \tau^{-1} \end{bmatrix}.$$

A_d is the interconnection between the vehicles.

3. STRING STABILITY CONTROLLER DESIGN

The platoon control system is modeled from the outset of stability analysis as an interconnection of a number of subsystems. The problem of structural perturbations arises in a natural way.

Let's consider the interconnected system (Siljak and.Stipanovic, 2000)

$$\dot{x} = A_i x_i + B_i u_i + h_i(t, x), \quad i \in N \quad (9)$$

which is composed of N linear time-invariant subsystems, which model is the same as equation (8), we list it again in the following:

$$\dot{x}_i = A_i x_i + B_i u_i, \qquad i \in N$$

where $x_i \in R^{ni}$ are the states, $u_i \in R^{mi}$ are the inputs, $h_i : R^{n+1} \to R^{ni}$ are the interconnection, and $N = \{1, 2, \ldots, N\}$. The state of the overall system $x = (x_1^T, x_2^T, \cdots, x_N^T)^T$, $\sum_{i=1}^{N} n_i = n$, $A_i = A_v, B_i = B_v$, the subsystems are disjoint.

For the linear part of the system we require that all pairs $\{A_i, B_i\}$ be stablizable. To the nonlinear interconnection, we require that they all satisfy the quadratic constraints

$$h_i^T(t,x) h_i(t,x) \le \alpha_i^2 x^T H_i^T H_i x \quad (10)$$

where $\alpha_i > 0$ are interconnection bounds. The bounding matrices H_i are constant and are a necessary ingredient in formulating the connective stabilization problem of the overall system.

The constraints (10) can further be interpreted as

$$\|h_i(t,x)\| \le \alpha_i \|H_i x\| \quad (11)$$

where $\|\bullet\|$ is the Euclidean norm. If we define the constant matrix H_i as a block matrix

$$H_i = [H_{i1}, \quad H_{i2}, \quad \cdots, \quad H_{iN}] \quad (12)$$

with the blocks H_{ij} compatible with the subsystems state vectors x_i, the constraints (11) can be rewritten as

$$\|h_i(t,x)\| \le \alpha_i \left\| \sum_{j=1}^{N} H_{ij} x_j \right\| \le \alpha_i \sum_{j=1}^{N} \|H_{ij}\| \|x_j\| \quad (13)$$

and arrive at the inequality

$$\|h_i(t,x)\| \le \alpha_i \sum_{j=1}^{N} \xi_{ij} \|x_j\| \quad (14)$$

which is the standard interconnection constraint with

$$\xi_{ij} = \|H_{ij}\|$$

The overall interconnected system can be rewritten in a compact form

$$\dot{x} = A_D x + B_D u + h(t, x) \quad (15)$$

where $x \in R^n$ is the state, $u \in R^m$ is the input, and

$$A_D = diag\{A_1, \quad A_2, \quad \cdots, \quad A_N\},$$
$$B_D = diag\{B_1, \quad B_2, \quad \cdots, \quad B_N\}$$

are constant matrices of appropriate dimensions. We assume that the subsystems are disjoint, that is,

$$x = \left(x_1^T, \quad x_2^T, \quad \cdots, \quad x_N^T\right)^T$$
$$u = \left(u_1^T, \quad u_2^T, \quad \cdots, \quad u_N^T\right)^T.$$

In the compact notation (15), the interconnection function

$$h: R^{n+1} \to R^n,$$
$$h = \left(h_1^{\,T}, \quad h_2^{\,T}, \quad \cdots, \quad h_N^{\,T}\right)^T,$$

is constrained as

$$h^T(t,x)h(t,x) \le x^T \left(\sum_{i=1}^{N} \alpha_i^{\,2} H_i^{\,T} H_i \right) x \qquad (16)$$

Assume each subsystem is controlled by only its locally available state. The assumption implies that the *i*-th subsystem is controlled by the local control law:

$$u_i(x_i) = K_i x_i, \qquad i \in N \qquad (17)$$

where K_i is a $m_i \times n_i$ constant matrix. The control law for the overall system has the familiar block-diagonal form

$$u(x) = K_D x \qquad (18)$$

where $K_D = diag\{K_1, K_2, \cdots, K_N\}$ is a $m \times n$ constant matrix with diagonal blocks compatible with those of A_D and B_D.

To compute the gain matrix K_D, so that the closed-loop system

$$\dot{x} = (A_D + B_D K_D)x + h(t,x). \qquad (19)$$

is robustly string stable in the large under the constraint (16) on the interconnection function $h(t,x)$, we use the change of variables

$$K_D Y_D = L_D, \qquad (20)$$

and express K_D as

$$K_D = L_D Y_D^{-1} \qquad (21)$$

Then, using the results of (Siljak, 2000), we formulate the following optimization problem:

minimize $\sum_{i=1}^{N} \gamma_i$,

subject to $Y_D > 0$,

$$\begin{bmatrix} A_D Y_D + Y_D A_D^{\,T} + B_D L_D + L_D^{\,T} B_D^{\,T} & I & Y_D H_1^{\,T} & \cdots & Y_D H_N^{\,T} \\ I & -I & 0 & \cdots & 0 \\ H_1 Y_D & 0 & -\gamma_1 I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ H_N Y_D & 0 & 0 & \cdots & -\gamma_N I \end{bmatrix} < 0 \qquad (22)$$

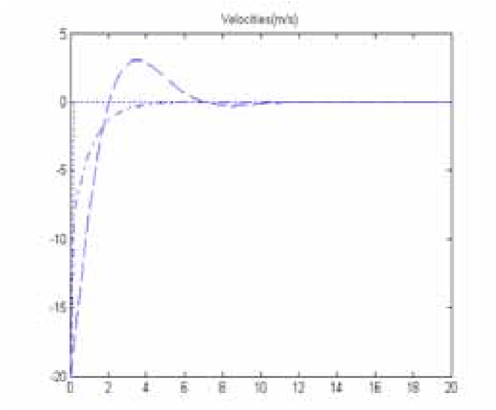
where $\gamma_i = 1/\alpha_i^{\,2}$.

We have the following:

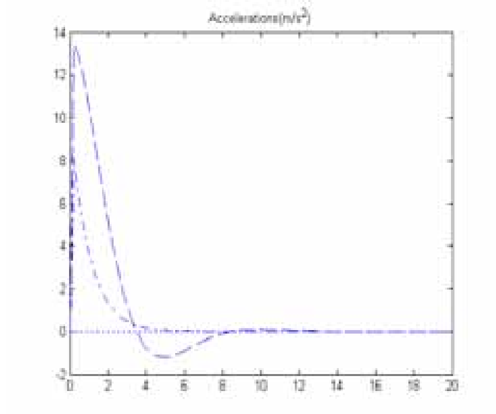
Theorem 1. The interconnected closed-loop system (15) is robustly string stable with degree vector $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_N)$ by control law (18), if the problem (22) is feasible.

4. SIMULATION

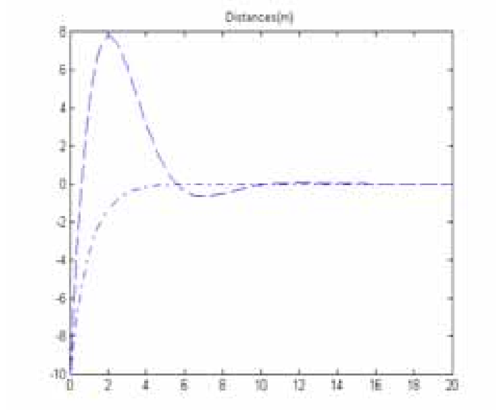
In the simulation, we set *N*=3 and we consider the uncertainty of the vehicle model parameters. The model of every subsystem (every vehicle) can represent as



(a)



(b)



(c)

Fig.1.Three-vehicle platoon with $\tau = 0.1$

$$\dot{x}_i = A_v x_i + A_d e(t,x) x_{i-1} + B_v u_i \qquad (23)$$

The interconnection between the *i*-th vehicle and (*i*-1)th vehicle is

$$\begin{aligned} h_i(t,x) &= A_d e(t,x), \quad 2 \le i \le N \\ h_1(t,x) &= 0 \end{aligned} \qquad (24)$$

According the method described in section 3, and set the reference values of velocity and acceleration are $v_r = 20km/h, a_r = 0m/s^2$ respectively, the reference value of distance between two consecutive vehicles $d_r = 10m$. At $t = 0$, all the vehicle start with the maximum acceleration and they are positioned with zero initial headway spacing. So the beginning distance deviation is 10*m*.

We can solve problem (22) to get the feedback gains of the subsystems as follow

$$\begin{aligned} K_1 &= [-1.9070 \quad -0.1172] \\ K_2 &= [2.8687 \quad -3.6291 \quad -0.2420] \\ K_3 &= [0.0000 \quad -0.0149 \quad -0.0015] \end{aligned}$$

Thus we get the decentralized feedback control law

$$u_i(x_i) = K_i x_i \qquad (25)$$

At the same time, the robust stability bounds are obtained.

$$\alpha_1 = 17.8735, \alpha_2 = 9.4978, \alpha_3 = 0.3221$$

Finally, the result of the simulation is shown in Fig. 1. From the result it can be seen that the string stability is achieved.

The three lines in Fig.1 (a) and (b) represent the velocities and accelerations deviation of three vehicles, respectively. In Fig.1(c) the two lines represent the distances between the consecutive vehicles. Good velocity tracking and very small transient distance errors are achieved for each vehicle.

5. CONCLUSION

In this paper, a new methodology for control design of platoon of vehicles is proposed. It is based on LMI approach. The decentralized controllers can make the platoon system with string stability; At the same time also can get the unstable bounds. A simulation has been carried out and the result has been given. The result show the proposed controller achieve good velocity tracking performance and small distance error while assuring platoon safety.

REFERENCES

An-Chyau Huang, Gee-Shen Chuang and Yen John Chen (1999). Longitudinal Control of a Vehicle Platoon with Bounded Parametric Uncertainties, *Journal of Chinese Institute of Engineers*, **Vol.22, No.3**, pp.365-374.

Bender. J. G (1991). An Overview of Systems Studies of Automated Highway Systems, *IEEE Trans on Vehicular Technology,* **Vol. 40, No. 1**. pp.82-89.

D. D. Siljak and D.M.Stipanovic (2000). Robust Stabilization of Nonlinear Systems: The LMI Approach, *Mathematical Problems in Engineering,* **Vol.6**, pp.461-493.

D. D. Siljak and D.M.Stipanovic (2001). Organically-Structured Control, *Proceedings of the American Control Conference, Arlington*, VA June 25-27.

D. Swaroop and J.K.Hedrick (1996). String Stability of Interconnected Systems, *IEEE Transaction on Automatic Control,* **Vol. 41, No. 3**.

G. D. Lee and S. W. Kim (2002), A Longitudinal Control System for a Platoon of Vehicles using a Fuzzy-sliding Mode Algorithm, *Mechatronics* 12, pp.97-118.

R. Horowitz and P. Varaiya (2000). Control Design of an Automated Highway System. *IEEE Proceedings of the IEEE,* **Vol.88, No.7**.

S. S. Stankovic and M. J. Stanojevic (2000). Decentralized Overlapping Control of a Platoon of Vehicles. *IEEE Trans on Control Technology,* **Vol.8**, pp.816-832.

Varaiya.P. (1993). Smart Cars on Smart Roads: Problems of Control. *IEEE Transaction on Automatic Control*, **Vol. 38,** pp.195-207.