STATE ESTIMATE SCHEMES FOR DESCRIPTOR SYSTEMS WITH MULTI-TIME DELAYED MEASUREMENTS

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Abstract: This paper deals with discrete-time stochastic descriptor (singular) systems with instantaneous and multi-time delayed measurements. The estimability condition of the descriptor systems involving delayed measurements is given. Using the measurements reorganization approach, the optimal Kalman filter and corresponding estimate error covariance are derived. Furthermore, an algorithm for the linear unbiased minimum variance state estimation is given. A numerical example is presented to illustrate the given algorithm.

Keywords: State estimation, Descriptor systems, Measurement reorganization, Multi-time delayed measurements, Discrete-time systems.

1. INTRODUCTION

The descriptor system is receiving more and more interests in past years. The motivation of this activity partly comes from applications in robotic, electric, economic, and chemical systems, see Rossenbrock (1974) and Luenberger (1977). In past twenty years, there have been a considerable amount of researches on the problem of state estimation for linear stochastic descriptor systems, for example Darouach and Boutayeb (1992); Zasadzinski et al. (1990); Lewis and Mertzios (1989); Zhang et al. (1999); Dai (1987); Deng et al. (2000); Fridman and Shaked (2002), and references there in. Under assumption of regularity such as in Dai (1987); Darouach and Boutayeb (1992), the descriptor system is first transformed into a normal form via state augmentation and a so-called Weierstrass normal form is introduced, the filtering problem is solved by utilizing the results of nonsingular systems. Darouach and Boutayeb (1992) gives an online state and parameter estimation recursive algorithm for a linear time-invariant single input single output discrete-time descriptor system. Nikoukhah et al. (1992) considers a recursive descriptor Kalman filter and presents a general formulation of a discrete-time linear estimation without involving any transformation. Darouach et al. (1993b) considers a time-invariant descriptor system, a recursive estimate algorithm is obtained, and the convergence and stability of this estimation is studied later (Darouach et al. (1993a)).

In this paper, we are concerned with the optimal filtering for the descriptor systems with current and delayed measurements. This problem can find important application in the multiple

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sensor fusions and communication (Klein (1999); Leury and Hu (1999)). Traditionally, the delayed-measurements problem can be solved through state augmentation and Kalman filtering, which, however, leads to very complicated computation.

In the case for the normal systems (nonsingular systems), the estimation problem with current and delayed measurements has been treated, such as Zhang et al. (2001a); Zhang et al. (2001b); Bramanian and Sayed (2004); Zhang et al. (2003). In this paper, we shall investigate the state estimation problem for the descriptor systems with current and multiple delayed-measurements. The key to our discussion is to reorganize the instantaneous and the multi-time delayed measurements for the descriptor systems.

This note is organized as follows. First, the estimation problem to be considered in this paper is formulated in Section 2. Then in Section 3, the concept and condition of estimability for the descriptor systems with both current measurements and delayed measurements are introduced, and the optimal Kalman filter and corresponding estimate error covariance matrix are obtained. A numerical example is used to illustrate the effective of the given algorithm in Section 4. At last, some concluding remarks are drawn in Section 5.

The following symbols are used throughout this paper.
$\varepsilon(.)$: Mathematical expectation.
δ_{ij}: Kronecker delta function.
T (superscript): Vector or matrix transposition.

2. PROBLEM FORMULATION

We consider the discrete-time stochastic descriptor systems with both instantaneous and multi-time delayed measurements as

$$Ex(k+1) = \Phi x(k) + \Gamma w(k), \tag{1}$$

$$y_i(k) = H_i x(k_i) + v_i(k),\ i = 0, 1, \ldots, l, \tag{2}$$

where $x(k) \in \mathbb{R}^n, w(k) \in \mathbb{R}^r$ are its state, disturbance input, and matrices E and Φ are $p \times n$ constant matrices, H_i are $m_i \times n$ matrices and Γ is known constant matrix of approapriate dimension. Equation (1) need not be square and even if it is, we do not require $\{E, \Phi\}$ to define a regular pencil. The state $x(k)$ is observed by different systems with current and delayed measurements as (2), where

$$k_i = k - d_i, i = 0, 1, \ldots, l \tag{3}$$

with $d_0 = 0$. $y_0(k) \in \mathbb{R}^{m_0}$ is instantaneous measurement and suppose that

$$0 \le d_i < d_{i+1}, i = 0, 1, \ldots, l. \tag{4}$$

For $i > 0, y_i(k) \in \mathbb{R}^{m_0}, v_i(k)$ are delayed measurements and delayed measurement noises respectively, d_i is an integer. The initial state $x(0)$ and the noise $w(k), v_i(k)$ are zero mean mutually uncorrelated white noise sequences with known covariance matrices as

$$\varepsilon\{x(0)x^T(0)\} = P_0, \tag{5}$$

$$\varepsilon\{w(k)w^T(j)\} = Q_w \delta_{kj}, \tag{6}$$

$$\varepsilon\{v_i(k)v_i^T(j)\} = Q_{v_i} \delta_{kj}. \tag{7}$$

According to (2) and (4), let $y_s(k)$ denotes the observation of the system (1)–(2) at time k, and $v_s(k)$ is the related observation noise at time k, then for $i = 0, 1, \ldots, l$,

$$y_s(k) = \begin{cases} \begin{bmatrix} y_0^T(k) & \cdots & y_{i-1}^T(k) \end{bmatrix}^T, d_{i-1} \le k < d_i \\ \\ \begin{bmatrix} y_0^T(k) & \cdots & y_l^T(k) \end{bmatrix}^T, k \ge d_l \end{cases} \tag{8}$$

$$v_s(k) = \begin{cases} \begin{bmatrix} v_0^T(k) & \cdots & v_{i-1}^T(k) \end{bmatrix}^T, d_{i-1} \le k < d_i \\ \\ \begin{bmatrix} v_0^T(k) & \cdots & v_l^T(k) \end{bmatrix}^T, k \ge d_l \end{cases} \tag{9}$$

Now we state the H_2 filtering problem for the descriptor systems with multi-time delayed measurements.

Problem 1. Given observation $\{y_s(i)\}_{i=0}^k$, find the linear minimum variance estimation $\hat{x}(k \mid k)$ of the state $x(k)$ for the system (1)–(2).

3. ESTIMATE FOR DESCRIPTOR SYSTEMS WITH MULTI-TIME DELAYED MEASUREMENTS

3.1 Measurements Reorganization

As is well known, the optimal estimation stated in Problem 1 can be dealt with by state augmentation and Kalman filtering. However, the augmentation approach may be computationally expensive, especially when the dimension of the system is high and (or) the measurement delays are large. In this subsection, our aim is to present measurements reorganization method to give a new and cost saving design estimator without resorting to system augmentation.

Observe from (2) that for $0 \le k < d_1$,

$$y_0(k) = H_0 x(k) + v_0(k), \tag{10}$$

and the estimator $\hat{x}(k \mid k)$ is a standard H_2 estimator associated with system (1)–(2). As for the case of $k < d_l$, without losing universality, supposed that $d_{l-1} \le k < d_l$, then (1)–(2) can be seen as the case of $(l-1)$ time delays system. So for the simplicity of discussion, we shall assume that $k \ge d_l$ in this paper.

It is easy to know that the linear space $\mathcal{L}\{y_s(i)\}_{i=0}^k$ is equivalent to

$$\mathcal{L}\{z_i(\tau) \mid_{k_{i+1} < \tau \le k_i}, i = 0, 1, \ldots, l\}, \tag{11}$$

where $k_{l+1} \triangleq 0$, k_i is defined in (3), and

$$z_i(\tau) = \begin{bmatrix} y_0(\tau) \\ y_1(\tau + d_1) \\ \vdots \\ y_i(\tau + d_i) \end{bmatrix}. \tag{12}$$

From (2) and (12), we have the following relationship:

$$z_i(\tau) = \bar{H}_i x(\tau) + V_i(\tau), i = 0, 1, \ldots, l \tag{13}$$

with

$$\bar{H}_i = \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_i \end{bmatrix}, V_i = \begin{bmatrix} v_0(\tau) \\ v_1(\tau + d_1) \\ \vdots \\ v_i(\tau + d_i) \end{bmatrix}. \tag{14}$$

Note that $z_i(\tau)$ contains measurements of the state $x(\tau)$ at time instants $\tau + d_i, i = 0, 1, \ldots, l$, and $V_i(\tau)$ are white noises with zero mean and covariance matrix

$$Q_{V_i} = \mathrm{diag}(Q_{V_0}, Q_{V_1}, \ldots, Q_{V_i}), i = 0, 1, \ldots, l. \tag{15}$$

Then the reorganized systems are given as

$$Ex(\tau + 1) = \Phi x(\tau) + \Gamma w(\tau), \tag{16}$$
$$z_i(\tau) = \bar{H}_i x(\tau) + V_i(\tau), \tag{17}$$

where $k_{i+1} < \tau \leq k_i, i = 0, 1, \ldots, l$, $k_{l+1} = 0$.

It is obvious that state estimation based on the observations $\{y_s(i)\}_{i=0}^k$ is equivalent to the one with the reorganized observations $\{z_i(\tau) \mid_{k_{i+1} < \tau \leq k_i}, i = 0, 1, \ldots, l\}$. Thus the estimation problem is to find the projection of $x(k)$ onto the linear space spanned by the observations sequence (11).

3.2 Condition of Estimability for the Descriptor Systems with Delayed Measurements

Definition 2. The given system (1)–(2) is said to be estimable if for each of k, let $w(\cdot) = 0, v_i(\cdot) = 0, i = 0, 1, \ldots, l$, the knowledge of observations $\{y_s(i)\}_{i=0}^k$ and model (1)–(2) is sufficient to determine uniquely $x(k)$ (Darouach et al. (1993a); Darouach et al. (1993b)).

The following theorem gives the condition of the system estimability for discrete-time descriptor systems with instantaneous and multi-time delayed measurements.

Theorem 3. The system (1)–(2) is estimable if and only if the matrix $\begin{bmatrix} E^T & H_0^T \end{bmatrix}^T$ is of full column rank.

PROOF. Let $w(\cdot) = 0, v_i(\cdot) = 0, i = 0, 1, \ldots, l$, then for $k \geq d_l$, the systems (16)–(17) can be rewritten as follows, attention to $k_{l+1} = 0$.

$$T_i \begin{bmatrix} x(k_{i+1} + 1) \\ \vdots \\ x(k_i) \end{bmatrix} = \begin{bmatrix} \Phi x(k_{i+1}) \\ 0 \\ \vdots \\ 0 \\ z_i(k_{i+1} + 1) \\ \vdots \\ z_i(k_i) \end{bmatrix} \tag{18}$$

where

$$T_i = \begin{bmatrix} E & 0 & 0 & \cdots & 0 \\ -\Phi & E & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & -\Phi & E \\ \bar{H}_i & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \bar{H}_i \end{bmatrix}, i = 0, 1, \ldots l \tag{19}$$

Rewrite (19), we obtain

$$T_i = U_i \begin{bmatrix} E & 0 & \cdots & 0 & 0 \\ \bar{H}_i & 0 & \cdots & 0 & 0 \\ -\Phi & E & \cdots & 0 & 0 \\ 0 & \bar{H}_i & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & -\Phi & E \\ 0 & 0 & \cdots & 0 & \bar{H}_i \end{bmatrix}, i = 0, 1, \ldots l \tag{20}$$

where U_i is a row permutation matrix and $\bar{H}_i$ is defined in (14). From (19), we can easily deduce that the matrix $\begin{bmatrix} E^T & \bar{H}_i^T \end{bmatrix}^T$ is of full column rank which is equivalent to T_i of full column rank. In view of (18) and note definition 2, the systems (16)–(17) are estimable if and only if $\begin{bmatrix} E^T & \bar{H}_i^T \end{bmatrix}^T$ is of full column rank. Furthermore, Note that

$$\bar{H}_i = \begin{bmatrix} \bar{H}_{i-1} \\ H_i \end{bmatrix}, i = 1, 2, \ldots l \tag{21}$$

and

$$\bar{H}_0 = H_0 \tag{22}$$

Then systems (16) are estimable if and only if $\begin{bmatrix} E^T & H_0^T \end{bmatrix}^T$ is of full column rank. Note that system (1)–(2) is equivalent to (16)–(17). Thus the theorem is proved.

3.3 Problem Solution

Definition 4. Given time instant k, the estimator $\hat{x}(s, j, i), (\ j \leq k_i)$ is the optimal estimation of $x(s)$, given the observation $\{z_l(\tau) \mid_{0 < \tau \leq k_l}, \ldots, z_i(\tau) \mid_{k_{i+1} < \tau \leq k_i}\}$, where k_i is defined in (3).

Definition 5. The filtering error covariance matrix of state $x(\tau)$ can be presented as the following equations respectively.

$$P_i^{\tau,\tau} = \varepsilon\{[x(\tau) - \hat{x}(\tau,\tau,i)][x(\tau) - \hat{x}(\tau,\tau,i)]^T\} \tag{23}$$

where for $i = 0, 1, \ldots, l, k_{i+1} < \tau \leq k_i, k_{l+1} = 0$.

Theorem 6. If $\begin{bmatrix} E^T & H_0^T \end{bmatrix}^T$ is of full column rank, then the optimal estimate $\hat{x}(k \mid k) = \hat{x}(k,k,0)$ can be calculated by the following steps.
Step 1: Calculate $\hat{x}(k_l,k_l,l)$ and $P_l^{k_l,k_l}$ by

$$\begin{aligned}&\hat{x}(k_l,k_l,l)\\ &= P_l^{k_l,k_l} E^T(\Phi P_l^{k_l-1,k_l-1}\Phi^T + \Gamma Q_w \Gamma^T)^{-1}\times\\ &\Phi\hat{x}(k_l-1,k_l-1,l) + P_l^{k_l-1,k_l-1}\bar{H}_l^T Q_{V_l}^{-1} z_l(k_l),\end{aligned} \tag{24}$$

where $P_l^{k_l,k_l}$ is the filtering error covariance matrix satisfies the generalized Riccati equation

$$\begin{aligned}P_l^{k_l,k_l} =& [E^T(\Phi P_l^{k_l-1,k_l-1}\Phi^T + \Gamma Q_w \Gamma^T)^{-1}E\\ &+ \bar{H}_l^T Q_{V_l}^{-1}\bar{H}_l]^{-1}\end{aligned} \tag{25}$$

with initial values $\hat{x}(0,0,l) = 0, P_l^{0,0} = P_0$.

Step 2: Calculate $\hat{x}(k_i,k_i,i)$ and $P_i^{k_i,k_i}$ for $i = l-1, l-2, \ldots, 0$,

$$\begin{aligned}&\hat{x}(k_i,k_i,i)\\ &= P_i^{k_i,k_i} E^T(\Phi P_i^{k_i-1,k_i-1}\Phi^T + \Gamma Q_w \Gamma^T)^{-1}\times\\ &\Phi\hat{x}(k_i-1,k_i-1,i) + P_i^{k_i-1,k_i-1}\bar{H}_i^T Q_{V_i}^{-1} z_i(k_i),\end{aligned} \tag{26}$$

and

$$\begin{aligned}P_i^{k_i,k_i} =& [E^T(\Phi P_i^{k_i-1,k_i-1}\Phi^T + \Gamma Q_w \Gamma^T)^{-1}E\\ &+ \bar{H}_i^T Q_{V_i}^{-1}\bar{H}_i]^{-1}\end{aligned} \tag{27}$$

with $\hat{x}(k_{i+1},k_{i+1},i) = \hat{x}(k_{i+1},k_{i+1},i+1), P_i^{k_{i+1},k_{i+1}} = P_{i+1}^{k_{i+1},k_{i+1}}$, where $z_i(\tau)$, $\bar{H}_i$, and Q_{V_i} is as in (14).

PROOF. Based on Theorem 3, we know that the system (16)–(17) (equivalent to system (1)–(2)) is estimable. Furthermore, measurements equation (17) can be divided into two segments, i.e.,

$$Ex(\tau+1) = \Phi x(\tau) + \Gamma w(\tau), \tag{28}$$
$$z_l(\tau) = \bar{H}_l x(\tau) + V_l(\tau), \tag{29}$$

where $0 \leq \tau \leq k$ and for $i = l-1, l-2, \ldots, 0$,

$$Ex(\tau+1) = \Phi x(\tau) + \Gamma w(\tau), \tag{30}$$
$$z_i(\tau) = \bar{H}_i x(\tau) + V_i(\tau), \tag{31}$$

where $k_{i+1} < \tau \leq k_i$. By using the generalized Kalman filter (Darouach and Boutayeb; 1992), From (28)–(29) we have (24) and (25) directly with $\hat{x}(0,0,l) = 0, P_l^{0,0} = P_0$, where $P_l^{k_l,k_l}$ is the error covariance defined as the form of (23). Similarly, From (30)–(31) and generalized Kalman filter, (26)–(27) can be obtained, where $i = l-1, l-2, \ldots, 0$. Take the definitions aforementioned into consideration, it can be verified that the initial values of each segments are formulated as $\hat{x}(k_{i+1},k_{i+1},i) = \hat{x}(k_{i+1},k_{i+1},i+1), P_i^{k_{i+1},k_{i+1}} = P_{i+1}^{k_{i+1},k_{i+1}}$. At last, the optimal estimator can readily be obtained by $i = 0$, i.e., $\hat{x}(k \mid k) = \hat{x}(k,k,0)$.

From (24) and (26), we can deduce the following innovation forms:

$$\begin{cases} E\hat{x}(\tau+1,\tau,i) = A\hat{x}(\tau,\tau,i),\\ E\hat{x}(\tau+1,\tau+1,i) = A\hat{x}(\tau,\tau,i)\\ \qquad\qquad + K(\tau+1,i)e(\tau+1,i),\\ e(\tau+1,i) = z_i(\tau+1) - \bar{H}_i\hat{x}(\tau+1,\tau,i),\end{cases}$$

where $k_{i+1} < \tau \leq k_i, i = 0, 1, \ldots, l$, and $k_{l+1} = 0$. Here $\{e(\tau,i), i = 0, 1, \ldots, l\}$ is an innovation sequence of the filter. $K(\cdot,\cdot)$ is given by

$$K(\tau+1,i) = EP_i^{\tau,\tau}\bar{H}_i^T Q_{V_i}^{-1}.$$

There still has an aspect which should be noted for this descriptor system filtering problem, that is, because of non-causal phenomena, we can not solve the predictor $\hat{x}(k+1,k,0)$ directly through the Kalman filtering as above mentioned.

In summary, we have the following procedure for calculating optimal estimator.

(1) Examine the system estimability condition;
(2) Reorganize the observation from $\{y_s(i)\}_{i=0}^k$ to $\{z_i(\tau) \mid_{k_{i+1} \leq \tau \leq k_i}, i = 0, 1, \ldots, l, k_{l+1} = 0\}$;
(3) Compute $\hat{x}(k_l,k_l,l)$ and $P_l^{k_l,k_l}$ through (24) and (25);
(4) Compute $\hat{x}(k_i,k_i,i)$ and $P_i^{k_i,k_i}$, $i = l-1, l-2, \ldots, 0$ through (26) and (27), the initial values for each i are

$$\hat{x}(k_{i+1},k_{i+1},i) = \hat{x}(k_{i+1},k_{i+1},i+1),$$
$$P_i^{k_{i+1},k_{i+1}} = P_{i+1}^{k_{i+1},k_{i+1}};$$

(5) The above steps repeat at each k.

4. NUMERICAL EXAMPLE

As an example, consider system (1)–(2) with delays $d_1 = 10, d_2 = 20$, $k = 100$, and

$$E = \begin{bmatrix} 1 & 0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}, \Phi = \begin{bmatrix} 0.1 & 0 & 0\\ 0 & 1 & 0\\ -0.5 & 0 & 0.5\end{bmatrix}, \Gamma = \begin{bmatrix} 1\\ 0\\ 1\end{bmatrix},$$
$$H_0 = \begin{bmatrix} 0 & 1 & 0\end{bmatrix}, H_1 = \begin{bmatrix} 1 & 0 & 0\end{bmatrix}, H_2 = \begin{bmatrix} 1 & 0 & 1\end{bmatrix}.$$

The covariance matrices of $x(0)$ and the noises $w(k)$ and $v_i(k)$ are $P_0 = \mathrm{diag}(0.6, 0.5, 0.7)$, $Q_w = 1, Q_{v_0} = Q_{v_1} = Q_{v_2} = 1$, respectively.

It is not difficulty to know that

$$\bar{H}_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \bar{H}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \bar{H}_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

$$Q_{V_i} = \mathrm{diag}(Q_{v_0}, Q_{v_1}, \ldots, Q_{v_i}), i = 0, 1, 2,$$

and $\begin{bmatrix} E^T & H_0^T \end{bmatrix}^T$ is of full column rank which implies that the system is estimable. The simulation results are drawn in Fig.1-Fig.3, where the dashed line is the estimate value while the solid line is the true value of the signal. It is clearly shown that the optimal estimator achieves very good tracking performance.

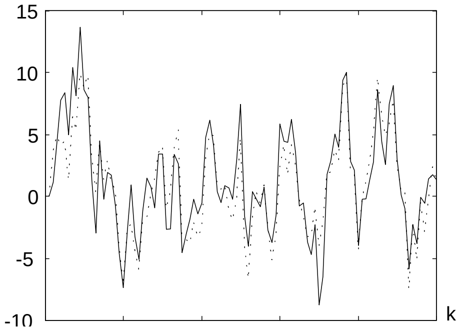

Fig. 1. True and optimal estimation of $x_1(k)$

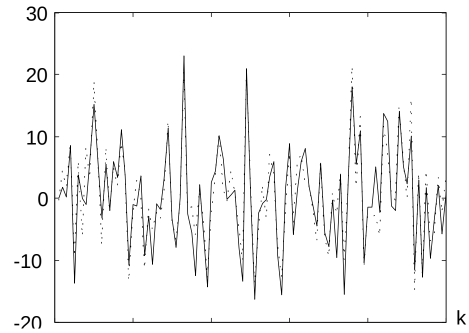

Fig. 2. True and optimal estimation of $x_2(k)$

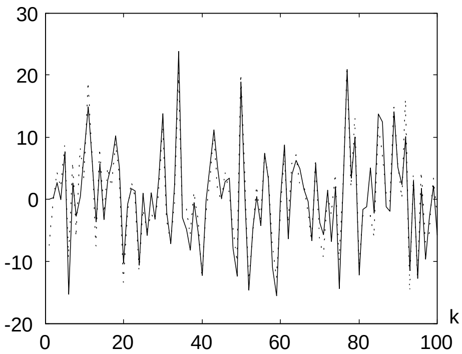

Fig. 3. True and optimal estimation $x_3(k)$

5. CONCLUSION

By using the notion of estimability of the general discrete-time descriptor systems and applying the method of measurements reorganization, we have established a simple state estimation algorithm for linear stochastic descriptor systems with instantaneous and multi-time delayed measurements. We solve the estimation problem without resorting to any system augmentation which greatly lessens the computational demand. A numerical example has been presented to illustrate the validity of the given algorithm.

(Chapter head:)*

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