NONCOOPERATIVE GAME ON SCHEDULING: THE SINGLE MACHINE CASE

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Abstract: Considering the diversified requirements of modern manufacturing, the paper studies a group of scheduling problems where jobs and machine have independent performance objectives. Based on noncooperative game, it is modeled as a two-layer optimization problem. In job-layer, the jobs' strategies resulted from competition for machine resource achieve Nash Equilibrium (NE), while in machine-layer, machine induces the NE to some global optimum by indirectly influencing jobs' selfish behaviors. Referring to Lagrangian relaxation, an iterative algorithm is developed to solve the problem. Numerical example is also given for illustration.

Keywords: Manufacturing system. Production costs, Scheduling algorithms, Nash games, Multiobjective optimizations.

1. INTRODUCTION

Production scheduling is an important issue in modern manufacture. Traditionally, the optimization objective of scheduling problem only reflects the production cost of manufacturing enterprise. In line with current trends towards competitive market, where both supplier and demanders of production have the diversified requirements, the manufacturers should not only consider their own costs, but also satisfy the independent requirements of individual demanders. It implies that the scheduling concept is changed from enterprise-based to demander-based, and more attention should be paid on demanders' independent objectives in optimization. For this reason, the scheduling problem here is formulated as a multiperson multiobjective problem where both jobs and machine have their own objectives. Conventional scheduling research only considers the global objective combined by the jobs' homogeneous objectives. The diversity among jobs' production requirements can be roughly handled by the methodology of weighted combination where weighted values reflect the priorities of the orders. However, this simple weighted combination is imprecise and fails to exactly accommodate the flexibility and variety of different demanders' requirements.

Recently, some researchers follow a new way where scheduling problem is regarded as jobs' competition for machine resource, and the individuality of jobs is emphasized. Lin and Solberg (1992) organized a schedule by the negotiatory behaviors of jobs and machines in a market-like environment. Price of resource is introduced in negotiation to reflect the collision, and then, the problem of bottleneck is preferably solved. However, necessary mathematical description and performance analysis are lacking in this work. Auction was adopted in Walsh, *et al.* (1998) and Erhan and Wu (1999) to investigate single and multi-machine scheduling problem respectively. The scheduling problem is viewed as the economic problem where jobs bid for machine resource. Especially, Erhan and Wu (1999) introduced the general equilibrium to build the auction model, discussed the relationship between the model and Lagrangian relaxation, and gave the solving algorithm. The above work assumes the job has independent decision ability, thus the difficulty of global decision is decentralized to individual problem and the calculation is simplified. However, the heterogeneous objectives of different demanders are not considered.

Teredesai and Ramesh (1998) as well as David Ben-Arieh and Manoj Chopra (1998) investigated the above multiperson multiobjective problem from the viewpoint of game theory which provides a good mathematic tool for individual competition. The former refers to the idea of "coopetition" to study multi-machine scheduling problem, and establishes a theoretic game model, in which each job and machine has its own objective. While, the latter is based on evolutionary game theory and Nash Equilibrium (NE) concept, where operations of jobs are modeled as decision makers and the machines as strategies. NE schedule can be found through searching in payoff matrix. However, with the increasing of problem scale, the construction of payoff matrix becomes much more difficult, which results in the complexity in theoretic analysis and solving. Although the above works follow the new research idea, it is still lacked in strict mathematical description, necessary theoretic analysis and efficient algorithm.

In our previous work Wang and Xi (2004), Noncooperative Game (NG) model for single machine scheduling problem was given, where the jobs' heterogeneous objectives were considered. The existence of NE schedule is proved in general case and the performance bound of NE schedules measured by the global objective of minimizing total completion time is given. In this paper, a key issue how to choose the NE schedule by the guidance of some global performance will be further studied. The scheduling problem is modeled as the resource competition among individual jobs with independent objectives. A two-layer optimization model which reflects the objectives of jobs and machine respectively is established. An algorithm to generate the NE schedule which satisfies the individual objectives of both jobs and the machine is developed. Simulation is given to illustrate the effectiveness of the new scheduling method.

2. PROBLEM FORMULATION

2.1 Variables and Model

For a single machine scheduling instance with n jobs, H is the length of the scheduling horizon. Job J_i arrives at r_i, then needs w_i time slots to wait and p_i time slots to process, and is completed at C_i with due date d_i. All these variables are integer. Without preemption, we have $r_i+w_i+p_i=C_i$, see Fig. 1.

For given r_i, p_i, choose w_i as the job's decision variable. Then, the feasible strategy of J_i is:

$$w_i^{valid}=\{w_i|0\le w_i\le H-r_i-p_i,\ w_i\in Z\},\ i=1,\dots,n \quad (1)$$

The feasible strategies of n jobs form a n dimensional space:

$$W=w_1^{valid}\times w_2^{valid}\times\dots\times w_n^{valid} \quad (2)$$

Given each job's strategy w_i, a schedule is formed by:

$$w=(w_1,w_2,\dots,w_n),\ w\in W \quad (3)$$

An arbitrary J_i has its own objective to minimize $f_i(w_i)$ by choosing appropriate waiting time w_i:

$$\min_{w_i\in W_i^{valid}} f_i(w_i),\ i=1,\dots,n \quad (4)$$

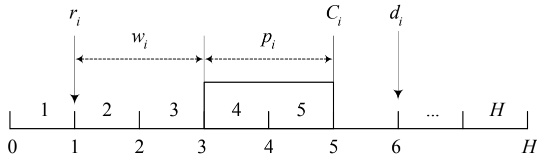


Fig.1. Illustration of job's basic variables

However, constrained by scarce machine capacity, jobs' choices may conflict. In Wang and Xi (2004), variable $\Delta w_{ij}=|(w_j+r_j+p_j/2)-(w_i+r_i+p_i/2)|$ was introduced to describe the collision between J_i and J_j. When $\Delta w_{ij}<(p_j+p_i)/2$, J_i overlaps with J_j; otherwise, no collision. For investigating detailed jobs' competition degree on each machine time slot, 0-1 variable δ_{ik} is introduced as Erhan and Wu (1999) and Luh P.B., *et al*. (1990) to describe the jobs process on each time slot. If job J_i is active at time slot k, δ_{ik} equals 1; otherwise, 0. For example, in Fig. 1, $\delta_{i4}=\delta_{i5}=1$. Obviously, δ_{ik} is decided by w_i, i.e.:

$$\delta_{ik}(w_i)=\begin{cases}1, & r_i+w_i+1\le k\le r_i+w_i+p_i\\ 0, k<r_i+w_i+1, or & k>r_i+w_i+p_i\end{cases} \quad (5)$$

Formula (5) implies the constraints of release date and nonpreemption. Further, the capacity constraint of machine can be described by:

$$\sum_{i=1}^{n}\delta_{ik}(w_i)\le 1,\ k=1,\dots,H \quad (6)$$

Formula (4-6) forms the job-layer model P which is a multiperson multiobjective optimization problem essentially. The job's individual objective (4) reflects the demander's own production requirement. However, the schedule generated from P may be poor efficient and even irrational from the viewpoint of manufacturer. For the schedule's global efficiency, machine's objective $M(w_1, w_2,\dots,w_n)$ which reflects enterprise's interior requirement should be introduced:

$$\min M(w)=\min\sum_{i=1}^{n}\omega_i\overline{f_i}(w) \quad (7)$$

Denote the schedule optimizing M as w^{opt} with value M^*. The function of machine-layer is to choose the NE schedules from P which have better global performance. Similar to conventional research, the global objective in M is combined by the jobs' homogenous objectives $\overline{f_i}(w)$. Further, machine-layer model is also subject to constraints (5) and (6).

Due to the complex constraints and jobs' heterogeneous property, it's difficult to handle the jobs' and machine's requirements simultaneously. Hence, complicating constraint is relaxed firstly. Variable $P_{ik}(w)$ is introduced to describe J_i's payment of using time slot k. Integrating it into the job's objective, relaxation version of job-layer model is as follows:

$$R_i:\ \min_{0\le w_i\le H-r_i-p_i} K_i(r,p,d,w)=f_i(w_i)+\sum_{r_i+w_i+1\le k\le r_i+w_i+p_i}P_{ik}\ ,\quad i=1,\dots,n \quad (8)$$

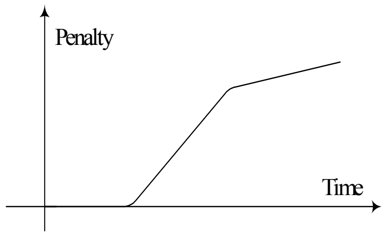


Fig.2. Pinedo type objective

Constrained by (5) and (6), the jobs' strategic choices influence mutually in original problem P. While in relaxation model, mutual restrictions among jobs are included in penalty term $P_{ik}(w)$. For the multiperson multiobjective optimization model R combined by R_i, NE solution concept in NG is used to define the solution of relaxed model R. Find a set of strategies $w^*=(w_1^*, w_2^*, \ldots, w_n^*)$, so that the following holds for each J_i and $w_i^* \in w_i^{valid}$:

$$K_i(w_1^*, \ldots, w_{i-1}^*, w_i^*, w_{i+1}^*, \ldots, w_n^*) \le K_i(w_1^*, \ldots, w_{i-1}^*, w_i, w_{i+1}^*, \ldots, w_n^*) \quad (9)$$

w^* is called the NE schedule of R. Wang and Xi (2004) proved the existence of NE schedule, and indicated that the NE schedules usually were not unique. Denote the set of all w^* by W^*.

Since constraint (5) and (6) have been included in job-layer, the function of machine-layer is only to choose the appropriate NE schedule from the job-layer. Hence, the problem in machine-layer can be redefined as:

$$\min M(w^*) = \min \sum_{i=1}^{n} \omega_i \overline{f_i}(w^*), \quad s.t.\ (9) \quad (10)$$

Denote the optimal schedule of (10) by w^{opt} with value $M(w^{opt})$.

2.2 Performance Objective

Pinedo (1995) showed that job in practical production usually had the penalty function as in Fig. 2. If job is completed before its due date, its penalty equals zero. Then, its penalty increases over the completion time at a given rate. While the job reaches a point, penalty increases at a much slower rate. Furthermore, completion time, lateness and their square are also common used objectives to describe the job's performance. These objectives are all nondecreasing with the waiting time of the job. This paper mainly focuses on such job's objective. Obviously, it is difficult for simple weighted combination of conventional scheduling to accurately consider the jobs' heterogeneous objectives.

Formula (10) gives machine-layer's objective which is combined by homogeneous jobs' objective $\overline{f_i}(w^*)$. For example, machine-layer optimization problem can be $1|r_j|\sum\omega_j C_j$ or $1|r_j|\sum\omega_j L_j$ when $\overline{f_i}(w^*)$ is taken as C_i or L_i (=max{0,$C_i - d_i$}) respectively.

3. ALGORITHM DESIGN

In the above section, the complicating constraint is relaxed by the adding of penalty term which reflects the job's extra payment for violating capacity constraint. Here, the detailed form of penalty term will be further discussed, then an algorithm to solve NE schedule through iterative coordination between job-layer and machine-layer is presented.

P_{ik} is used to relax constraints and reduces resource collision. Hard penalty was used in Wang and Xi (2004), i.e. if a job conflicted with other jobs, its penalty was $+\infty$; otherwise, 0. This type of penalty guarantees the feasibility of the result, but fails to quantitatively reflect the competition degree and mutual influence among jobs. Instead, formula (5) is adopted here to model collision and the soft penalty is given as below:

$$P_{ik}(w) = \pi_k \left(\sum_{i=1}^{n} \delta_{ik} \right) \delta_{ik} \quad (11)$$

(11) can be intuitively explained that job J_i must pay the fee P_{ik} for the using of machine time slot k according to resource price π_k. This kind of penalty was widely used in conventional scheduling research such as Erhan and Wu (1999) and Luh P.B., e*t al.* (1990). Jobs would compete for appropriate continuous machine resource interval to accomplish its process according to their relaxed objectives (8) where π is fixed by machine-layer according to its own performance objective (10).

Thus, an iterative algorithm to solve the two-layer relaxation model is given. Given the price π_k of each time slot, the task of job-layer is to choose the best combination of resource slots according to job's own objective. Jobs' strategies will feedback to the machine-layer. The machine-layer then updates the resource price as the foundation of next iteration. The process is repeated until a solution which balances the interests of each participator in production is obtained. Obviously, jobs choose strategies according to (8) in job-layer, which achieves a NE solution of R. While, the updating of π in machine-layer not only reduces the collision, but also induces the jobs' selfish behaviors to some global objective. In the whole procedure, the incentive mechanism, i.e. the mechanism of adjusting π plays a key role.

Here gives Lagrangian relaxation model of the machine-layer model by referring Erhan and Wu (1999) and Luh P.B., e*t al.* (1990) at first:

$$R_M = \min_w \left[\sum_{i=1}^{n} \omega_i \overline{f_i} + \sum_{k=1}^{H} \pi_k \left(\sum_{i=1}^{n} \delta_{ik} - 1 \right) \right], \quad s.t.\ (5) \quad (12)$$

The lower bound of $M(w^{opt})$ can be achieved by solving (12) to optimality. In order to get closer lower bound, construct its dual problem:

$$D_M = \max_\pi \left[-\sum_{k=1}^{H} \pi_k + \min_w \sum_{i=1}^{n} \left(\omega_i \overline{f_i} + \sum_{k=1}^{H} \pi_k \delta_{ik} \right) \right], \quad s.t.\ (5), \pi_k \ge 0, k=1, \ldots, H \quad (13)$$

In order to solve (13), subgradient algorithm is used in adjusting π:

$$\pi_k^{r+1} = \pi_k^r + \Delta^r\left(\sum_{i=1}^{n}\delta_{ik}^r - 1\right),\ k=1, \ldots, H \qquad (14)$$

where, $\left(\sum_{i=1}^{n}\delta_{ik}^r - 1\right)$ is the subgradient of D_M.

The step size is given by:

$$\Delta^r = \lambda \frac{\overline{M} - D_M^r}{\sum_{k=1}^{H}\left(\sum_{i=1}^{n}\delta_{ik}^r - 1\right)^2},\ 1 \le \lambda \le 2 \qquad (15)$$

where, $\overline{M}$ is an estimate of the optimal global solution, D^r_M is the value of (13) at the rth iteration. Here, machine-layer adjusts π according to (14) and (15) in order to induce jobs' strategic behaviors, consequently make D_M close to $\overline{M}$.

Since R is the relaxed model of original problem, the NE schedule generated from R perhaps is infeasible. Refer to "List scheduling" algorithm used in Luh P.B., e*t al.* (1990), the feasible NE schedule can be formed as follows: find the processing sequence of NE schedule of R at first, then schedule the jobs according to this sequence as machine becomes available. Since job's objective is nondecreasing with its waiting time, this feasible schedule must be the NE solution of P.

Thus, the procedure of solving the original constrained, multi-person multi-objective, two-layer optimization problem can be obtained. H is assigned as total processing time plus the maximum of all the jobs' arrive times, so that there is enough resource to process all the jobs. If there is no change in jobs' strategies and resource price, satisfactory NE schedule is achieved and the procedure terminates. The entire algorithm is given as follows:

Step0 r=0, λ=1, $H=\max_j\{r_j\}+\sum_j(p_j)$. Initialize $\pi_k^r=0$.
Step1 Given π_k^r of each time slot, each job chooses its optimal waiting time w^r according to (8).
Step2 Calculate D^r_M according to (10). Construct the feasible schedule, then calculate the global objective $\overline{M}$ of feasible schedule.
Step3 Update π_k^r according to (14) and (15). Stop calculation when w^r and π_k^r remain unchanged in last three iterations or the upper bound of iterative time (3000 times) reaches.
Step4 $r=r+1$, goto *Step 1*.

Because the global objective and jobs' objectives are not compatible, i.e. the global objective isn't combined by the jobs' objectives, the competitive results of individual jobs don't always compose the optimal schedule of relaxed problem (12). In other words, theory of lower bound in conventional Lagrangian relaxation algorithm doesn't always hold and D_M in iterative procedure isn't the lower bound of $M(w^{opt})$. The experiments in the following Section will validate it.

There is large space for the choice of optimization objective: global objective can be arbitrary and jobs' objectives are nondecreasing. The coming simulation focuses on the global objective as total completion time with some popular job's objectives.

4. EXAMPLE

In the experiments here, jobs with independent objectives cited in Section 2.2 is considered. As showed in Fig. 3, type Ⅰ and Ⅱ objective correspond to completion time and lateness. Kethleya R. B. and Bahram Alidaeeb (2002) defined type Ⅲ objective as:

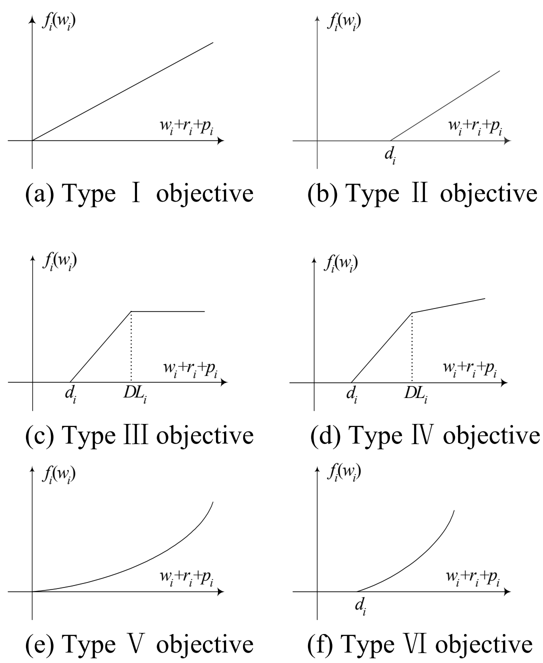
$$f_i(w_i) = \begin{cases} 0, & C_i \le d_i \\ \omega_i(C_i - d_i), & d_i < C_i < DL_i \\ \omega_i(DL_i - d_i), & DL_i \le C_i \end{cases} \qquad (16)$$

where, DL_i is a constant larger than d_i. Here, further modify it to get type Ⅳ objective which is very similar to Pinedo type objective:

$$f_i(w_i) = \begin{cases} 0, & C_i \le d_i \\ \omega_{i1}(C_i - d_i), & d_i < C_i < DL_i \\ \omega_{i1}(DL_i - d_i) + \omega_{i2}(C_i - DL_i), & DL_i \le C_I \end{cases} \qquad (17)$$

Moreover, two quadratic objectives, square of completion time (see Townsend, 1978; Cheng, and Liu, 2004) and lateness (see Sun *et al*., 1999), are considered. Obviously, the above objectives are all nondecreasing with the job's waiting time.

A single machine scheduling problem with parameters in Tab. 1 is studied. Each job has its own objective: $f_1(w_1)=\min C_1$. $f_2(w_2)=\min(\max\{0,C_2-d_2\})$. $f_3(w_3)$ is type Ⅲ as (16) where DL_3=25, ω_3=1. $f_4(w_4)$ is type Ⅳ as (17) where DL_4=25, ω_{41}=1, ω_{42}=0.5. $f_5(w_5)=\min C_5^2$. $f_6(w_6)$ equals to 0 if C_6 is smaller than d_6; otherwise, $(C_6-d_6)^2$. Furthermore, the objective of machine is to minimize the total completion time, i.e. $\min\sum C_i$. Tab. 2 gives the optimal schedule only minimizing the machine's objective, where the value of machine's objective is 137; the value of total jobs' independent objective is 399.



(a) Type Ⅰ objective (b) Type Ⅱ objective
(c) Type Ⅲ objective (d) Type Ⅳ objective
(e) Type Ⅴ objective (f) Type Ⅵ objective

Fig.3. Job's Performance Objective

Tab.1 Single machine example

Job	J_1	J_2	J_3	J_4	J_5	J_6
r_i	0	3	2	5	4	10
p_i	10	4	8	6	3	12
d_i	$+\infty$	15	15	20	$+\infty$	30
$f_i(w_i)$	Ⅰ	Ⅱ	Ⅲ	Ⅳ	Ⅴ	Ⅵ

Tab.2. The global optimal schedule

Job	J_1	J_2	J_3	J_4	J_5	J_6	
Sequence	5	1	4	3	2	6	
C_i	34	7	24	16	10	46	$\sum C_i$=137
$f_i(w_i)$	34	0	9	0	100	256	$\sum f_i$=399

Tab.3. The schedule of our method

Job	J_1	J_2	J_3	J_4	J_5	J_6	
Sequence	1	3	6	4	2	5	
C_i	10	17	43	23	13	35	$\sum C_i$=141
$f_i(w_i)$	10	2	10	3	169	25	$\sum f_i$=219

Using the algorithm presented above, 16 iterations are required to solve the problem. Tab. 3 gives the result of our method where the values of machine's objective and total jobs' independent objectives are 141 and 219 respectively. It has been shown that the jobs' heterogeneous objectives can be greatly satisfied by giving up a little portion of machine's performance in our method.

5. CONCLUSION

In this paper, the new scheduling idea of serving demanders is introduced. Production scheduling problem is modeled as a constrained, multi-person multi-objective, two-layer optimization problem. In the job-layer, each job optimizes its own objective by selfish competition, which composes the NE schedule of the model. While in the machine-layer, machine induces the NE schedule to satisfy some global objective. A relaxed and iterative algorithm is designed to solve the NE schedule by referring to Lagrangian relaxation. Heterogeneous jobs' objectives can be considered in calculation and computational experiment indicates that the new method satisfies diversified objectives in production.

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