ON-RAMP DECENTRALIZED NONLINEAR CONTROL WITH DISTURBANCE REJECTION

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Abstract: A method based on disturbance estimation and bounded controls is proposed to reduce congestion in highways where on-ramp metering can be implemented. An existing model is first extended to account for friction and other interactions between the mainstream and the on-ramp flows. A local variable structure observer is then proposed to estimate the lumped upstream and downstream flows that affect one section of the highway. Using the estimates of the disturbances, a local nonlinear controller stabilizes the system while maintaining the control effort bounded. The boundedness of the control effort is necessary to guarantee the feasibility of the control law. The controller is robust in the sense that the estimator accounts also for modeling errors and parameter variations.

Keywords: Road Traffic, Constraints, Disturbance Rejection, Nonlinear Control Systems, Variable Structure

1. INTRODUCTION

Ramp-metering control is by far the most effective way to maximize the use of traffic capacity in highways. Recognizing the importance of such a technique, several researchers have proposed different centralized and decentralized solutions based on different approaches. However, most of these proposals fail to consider at least one of the following important properties of the on-ramp metering control problem:

a) The available control input (the on-ramp flow) is positive, bounded, and limited by the main-stream flow.
b) The state of the system is bounded and positive.
c) On-ramp metering produces friction and affects both the conservation and the momentum equations of the plant.

This work analytically takes these facts into account by modifying an accurate model for highway pipelines (Karaaslan *et al.*, 1991). The resultant modified model includes the effect of metering on the momentum equation–Subsection 2.1–as well as the effect of the mainstream density on the flow entering from the on-ramp–Subsection 2.2. As an original contribution, a continuous approximation of the anticipation term in the momentum equation smooths out the model–Subsection 2.3–in order to simplify the control design. Subsection 2.4 introduces a change of coordinates and prepares the model for the estimation of the disturbances through variable structure methods–Section 3. Based on the estimates of the disturbances, an almost smooth control law stabilizes the system while enforcing boundedness of the control effort–Section 4. The theoretical results are supported by

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Table 1. Typical model parameters.

λ	1	ρ_{jam}	110(veh/Km)	δ	200(Km/h^2)
l	1.86	$q_{\mathrm{r}M}$	1800(veh/h)	L	0.5(Km)
α	0.95	χ	40(veh/Km)	τ	0.0057(h)
μ_f	0.001	v_{f}	93.1(Km/h)	μ_1	12(Km^2/h)
		ζ	120(veh/Km)	μ_2	6(Km^2/h)
		σ	35(veh/Km)	χ'	4(Km/h)

a set of simulations presented in Section 5. Finally, a brief discussion in Section 6 summarizes and evaluates the results. Because of space limitations, some details have not been included but can be found in (Becerril-Arreola and Aghdam, 2005).

2. AN IMPROVED TRAFFIC FLOW MODEL

The macroscopic traffic flow model presented next is one of the most accurate ones for highway pipelines (Karaaslan *et al.*, 1991). Let the subscripts "us" and "ds" denote that the variables they accompany correspond to the upstream and downstream neighboring sections, respectively. The state of the model is denoted by: $\rho(t)$, the average traffic density (veh/Km); $v(t)$, the mean speed of traffic (Km/h); and $q(t)$, the traffic flow (veh/h). The space-discretized equations will then be:

$$q(t) = \alpha\rho(t)v(t) + (1-\alpha)\rho_{\mathrm{ds}}(t)v_{\mathrm{ds}}(t) \tag{1a}$$

$$q_{\mathrm{us}}(t) = \alpha\rho_{\mathrm{us}}(t)v_{\mathrm{us}}(t) + (1-\alpha)\rho(t)v(t) \tag{1b}$$

$$\dot{\rho}(t) = \frac{1}{L}\left[q_{\mathrm{us}}(t) - q(t) + \frac{r(t)}{\lambda} - \frac{p(t)}{\lambda}\right] \tag{1c}$$

$$\dot{v}(t) = \frac{v_{\mathrm{e}}(\rho(t)) - v(t)}{\tau} - \frac{\mu(t)}{\tau L}\frac{\rho_{\mathrm{ds}}(t) - \rho(t)}{\rho(t)+\chi} + \frac{\rho_{\mathrm{us}}(t)v_{\mathrm{us}}(t)}{\rho(t)+\chi'}\frac{\sqrt{v_{\mathrm{us}}(t)v(t)} - v(t)}{L} + \delta, \tag{1d}$$

where δ is the modeling error,

$$\mu(t) := \begin{cases} \mu_1 \dfrac{\zeta}{\rho_{\mathrm{jam}} - \rho_{\mathrm{ds}}(t) + \sigma} & \text{if } \rho_{\mathrm{ds}}(t) \geq \rho(t) \\ \mu_2 & \text{otherwise,} \end{cases}$$

and

$$v_{\mathrm{e}}(\rho) = v_{\mathrm{f}}\left[1 - (\rho/\rho_{\mathrm{jam}})^{l}\right] \tag{2}$$

is one of the existing models for the equilibrium speed-density relationship (Castillo and Benitez, 1995). The variables $r(t)$ and $p(t)$ represent the flows transferring from the on-ramp into the mainstream and from the mainstream into the off-ramp, respectively. The constant parameters of the model are defined as follows: λ is the number of lanes in the mainstream section, τ is the relaxation time, v_{f} is the free speed, ρ_{jam} is the jam density, and L is the length of the section. The constants $\chi, \chi', \zeta, \sigma, \mu_1$, and μ_2 are measured parameters whose typical values appear in Table 1.

Certain modifications can further improve the accuracy of the model (1) by accounting for the additional phenomena addressed next.

2.1 Friction due to merge

One can take the friction effect (Liu *et al.*, 1996) into account by adding to the right side of the momentum equation (1d) the term:

$$G(t) = -\mu_{\mathrm{f}}|g_{\mathrm{t}}(t)|\rho(t)v(t),$$

where μ_{f} is a constant friction coefficient and $g_{\mathrm{t}}(t)$ is the so called generation term, which in the particular case analyzed here, equals the difference between the on-ramp and off-ramp flows. The first derivative of $G(t)$ with respect to $g_{\mathrm{t}}(t)$ is not continuous due to the presence of the absolute value operator. However, this limitation can be removed by assuming that the sensors measuring the speed and density are located before and after the on-ramp such that no off-ramp is present between them. As a result, the generation term $g_{\mathrm{t}}(t)$ will be equal to the on-ramp flow $r(t)$. Since this flow is always positive, $G(t)$ will then be:

$$G(t) = -\mu_{\mathrm{f}}r(t)\rho(t)v(t). \tag{3}$$

2.2 Bounds to the on-ramp flow

The present development considers the following model for the unsignalized merging of two traffic flows because it is given in terms of macroscopic variables (Liu *et al.*, 1996):

$$r(t) = (1 - \rho/\rho_{\mathrm{jam}})\, q_{\mathrm{r}}(t), \tag{4}$$

where $q_{\mathrm{r}}(t)$ is the metered on-ramp flow.

2.3 Discontinuity smoothing

For most values of μ_1, ζ, σ, and μ_2, the anticipation term in (1d) introduces a discontinuity into the vector field of the system dynamics. The solution to differential equations with a discontinuous vector field might not exist or might not be unique. Furthermore, some of the multiple solutions might lead to attractors other than the equilibria, such as accumulation points. Although collected data suggests that discontinuities are present in the traffic flow dynamics (Hall, 2002), no observation has yet detected the presence of attractors other than the equilibrium points described by (2) and the alternative speed-density relationships. The unnatural solutions to (1) can be avoided by means of the following C^{∞} (smooth) approximation to μ:

$$\hat{\mu} = \frac{\mu_1\zeta}{\rho_{\mathrm{jam}} - \rho_{\mathrm{ds}} + \sigma}\left(\tilde{\mu} + \frac{1}{2}\right) + \mu_2\left(\frac{1}{2} - \tilde{\mu}\right)$$
$$\tilde{\mu} := \frac{\arctan\left[\epsilon\left(\rho_{\mathrm{ds}} - \rho\right)\right]}{\pi}, \tag{5}$$

where ϵ is a design parameter that specifies how well $\hat{\mu}$ approximates μ. Fig. 1 presents a graphical comparison between μ–the mesh–and its continuous approximation $\hat{\mu}$–the solid surface–for the set of typical values listed in Table 1.

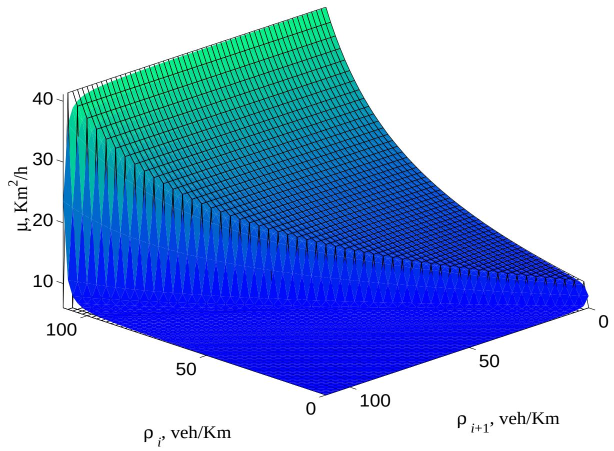

Fig. 1. Comparison between the original anticipation term and its smooth approximation.

2.4 Change of coordinates

The traffic control problem is normally formulated as the stabilization of the density, or both the speed and the density, around a given set-point ρ_d, or (ρ_d, v_d) with $v_d = v_{\mathrm{e}}(\rho_d)$. A standard procedure is to define the state transformation:

$$x_1 = v - v_d$$
$$x_2 = \rho - \rho_d.$$

Expressed in this set of coordinates and modified by (3), (4), and (5), the system equations read as:

$$\begin{aligned}\dot{x}_1 =& -\frac{1}{\tau}x_1 - \frac{v_{\mathrm{f}}}{\tau}\left(\frac{x_2+\rho_d}{\rho_{\mathrm{jam}}}\right)^l + \psi \\ & - \mu_{\mathrm{f}}\frac{(x_2+\rho_d)}{(x_1+v_d)^{-1}}\left(1-\frac{x_2+\rho_d}{\rho_{\mathrm{jam}}}\right)q_{\mathrm{r}}\end{aligned} \tag{6a}$$

$$\begin{aligned}\dot{x}_2 =& \frac{1-2\alpha}{L}\left(\rho_d x_1 + v_d x_2 + x_1 x_2\right) \\ & + \frac{1}{\lambda L}\left(1-\frac{x_2+\rho_d}{\rho_{\mathrm{jam}}}\right)q_{\mathrm{r}} + \phi,\end{aligned} \tag{6b}$$

where all constant terms and unknown variables have been lumped together into the disturbances $\psi(t,\delta)$ and $\phi(t)$ given by:

$$\begin{aligned}\psi =& \frac{1}{\tau}(v_{\mathrm{f}} - v_d) - \frac{\mu(t)}{\tau L}\frac{\rho_{\mathrm{ds}}(t)-\rho(t)}{\rho(t)+\chi} + \delta \\ & + \frac{\rho_{\mathrm{us}}(t)}{\rho(t)+\chi'}\frac{v_{\mathrm{us}}(t)}{L}\left[\sqrt{v_{\mathrm{us}}(t)v(t)} - v(t)\right]\end{aligned} \tag{7}$$

and:

$$\phi = \frac{1-2\alpha}{L}\rho_d v_d + \frac{\alpha}{L}q_{\mathrm{us}}(t) - \frac{1-\alpha}{L}q_{\mathrm{ds}}(t). \tag{8}$$

If δ, the additive modeling error in (1d), is bounded inside the operating range, so are the above disturbances because the state of each highway section is bounded inside the closed set:

$$\mathcal{X} = \left\{\rho, v \in \mathbb{R}^2 \middle| 0 \le \rho \le \rho_{\mathrm{jam}}, 0 \le v \le v_{\mathrm{f}}\right\}, \tag{9}$$

which constitutes the operating space.

The control input $q_{\mathrm{r}}(t)$ is bounded inside the interval $\mathcal{R} = [0, q_{\mathrm{r}M}]$, where $q_{\mathrm{r}M}$ is a constant value determined by the geometry of the on-ramp and depends on other conditions such as weather and incidents.

The system equations are now in the form $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})u$ with $\boldsymbol{f}(\boldsymbol{0}) \neq \boldsymbol{0}$ and an equilibrium point inside $\mathcal{X}$ that is unique for a given value of ρ_d and instantaneous values of $\psi(t,\delta)$ and $\phi(t)$. The disturbances $\psi(t,\delta)$ and $\phi(t)$ are non-matched non-vanishing perturbations (Khalil, 1996) and $\boldsymbol{g}(\boldsymbol{x})|_{\rho=\rho_{\mathrm{jam}}} = \boldsymbol{0}$, which implies that standard control design techniques cannot be directly applied without considering possible control singularities.

3. ESTIMATION OF DISTURBANCES

Variable structure observers can provide very precise estimates of the instantaneous values of lumped disturbances (Rundell *et al.*, 1996), (Becerril *et al.*, 2004). Consider and observer with the following equations:

$$\begin{aligned}\dot{\hat{x}}_1 =& -\frac{1}{\tau}x_1 - \frac{v_{\mathrm{f}}}{\tau}\left(\frac{x_2+\rho_d}{\rho_{\mathrm{jam}}}\right)^l + \bar{K}_1\mathrm{sgn}(s_1) \\ & - \mu_{\mathrm{f}}(x_2+\rho_d)(x_1+v_d)\left(1-\frac{x_2+\rho_d}{\rho_{\mathrm{jam}}}\right)q_{\mathrm{r}} \\ \dot{\hat{x}}_2 =& \frac{1-2\alpha}{L}\rho_d x_1 + \frac{1-2\alpha}{L}v_d x_2 + \frac{1-2\alpha}{L}x_1x_2 \\ & + \frac{1}{\lambda L}\left(1-\frac{x_2+\rho_d}{\rho_{\mathrm{jam}}}\right)q_{\mathrm{r}} + \bar{K}_2\mathrm{sgn}(s_2)\end{aligned}$$

and the sliding surfaces defined by $s_1 \triangleq x_1 - \hat{x}_1$ and $s_2 \triangleq x_2 - \hat{x}_2$. The associated error dynamics are given by:

$$\begin{aligned}\dot{s}_1 &= \psi(t,\delta) - \bar{K}_1\mathrm{sgn}(s_1) \\ \dot{s}_2 &= \phi(t) - \bar{K}_2\mathrm{sgn}(s_2).\end{aligned} \tag{10}$$

Since the traffic flow through the adjacent highway sections is limited, upper bounds to the disturbances exist and can be represented by:

$$|\psi(t,\delta)| < \psi_M, \; |\phi(t)| < \phi_M. \tag{11}$$

These values can be used to define the observer gains $\bar{K}_1 = \psi_M + K_1$ and $\bar{K}_2 = \phi_M + K_2$, which make the sliding surfaces s_1 and s_2 globally attractive for any positive constants K_1 and K_2. The proof of attractiveness results from the Lyapunov functions $V_{s1} = \frac{s_1^2}{2}$ and $V_{s2} = \frac{s_2^2}{2}$, with derivatives:

$$\begin{aligned}\dot{V}_{s1} &= s_1\psi(t,\delta) - (\psi_M + K_1)|s_1| \\ \dot{V}_{s2} &= s_2\phi(t) - (\phi_M + K_2)|s_2|.\end{aligned}$$

It follows from (11) that $s_1\psi(t,\delta) - |s_1|\psi_M \le 0$ and $s_2\phi(t) - |s_2|\phi_M \le 0$ for all $\psi(t,\delta)$, $\phi(t)$, s_1, and s_2. Consequently, $\dot{V}_{s1} \le -K_1|s_1|$, $\dot{V}_{s2} \le -K_2|s_2|$, and the sliding surfaces s_1 and s_2 are globally attractive for any positive K_1 and K_2. An equivalent mode is thus reached so that $\dot{s}_i = 0$ for $i = 1, 2$. Let the subscript "eq" denote equivalent mode values. The global attractiveness of the sliding surfaces and (10) imply that $\psi(t,\delta) = \bar{K}_1 [\mathrm{sgn}(s_1)]_{\mathrm{eq}}$ and $\phi(t) = \bar{K}_2 [\mathrm{sgn}(s_2)]_{\mathrm{eq}}$, and so the estimates of the disturbances can be found.

4. CONTROL DESIGN

Like previous work in the field, the proposed design focuses on controlling the variable x_2 because ρ determines whether the state of the system lies inside the stable or the unstable region of the traffic fundamental diagram.

Let $q_{\mathrm{r}} = \beta(\boldsymbol{x}, t)$ be a control signal that ensures $x_2 \to 0$ as $t \to 0$. Assuming that the disturbances vary slowly and that the stabilizing control $\beta(\boldsymbol{x}, t)$ exists, one can obtain an expression for the steady-state control signal $q_{\mathrm{r}} = q_{\mathrm{rss}}$ as well as for the steady-state velocity. From (6b), the value of the control q_{r} when the solution $x_2(t) = 0$ has been reached is given by:

$$q_{\mathrm{rss}} = \frac{\lambda L}{1 - \frac{\rho_d}{\rho_{\mathrm{jam}}}} \left(\frac{2\alpha - 1}{L} \rho_d x_1 - \phi(t) \right). \quad (12)$$

The existence of this control is necessary for regulation but the nature of the plant is such that this control might lie outside of the feasible range of values of q_{r}. This implies that, under certain conditions, the solution $x_2(t) = 0$ cannot be reached. From (12) and the bounds on the control effort $0 \le q_{\mathrm{rss}} \le q_{\mathrm{r}M}$, the conditions for the existence of q_{rss} are given by:

$$-\frac{q_{\mathrm{r}M}\left(1 - \frac{\rho_d}{\rho_{\mathrm{jam}}}\right)}{\lambda L} \le \phi(t) - \frac{2\alpha - 1}{L} \rho_d x_1 \le 0. \quad (13)$$

As stated before, the flow q_{r} is strictly positive and so is the term it multiplies in (6b). It follows that the effect of any non-zero q_{r} is to increase the deviation between the desired and the actual densities. More specifically, any non-zero value of the on-ramp flow contributes to increase the actual density, as suggested by (1c). q_{r} also affects x_2 through x_1 by increasing it because the friction reduces the speed and so reduces the flow exiting the section. Consequently, a properly designed q_{r} can drive $\rho(t)$ to ρ_d only when $x_2 < 0$, i.e, the mainstream flow can be increased when its density is below the desired level but it cannot be reduced when its density surpasses the prescribed limit. When $x_2 > 0$, q_{r} must be maintained as small as possible in order to avoid congestion in the mainstream–these facts can be easily inferred from (1c). On the other hand, the available demand is an upper bound to q_{r}. This implies that the controller might not be able to achieve a given ρ_d when the demand is too low. Given these limitations on the control effort, the control design must maintain the state ρ below and as close as possible to a predefined value ρ_d so as to maximize the utilization of the capacity of the highway section. Since an insufficient on-ramp demand would lead to a value of ρ below ρ_d, which is acceptable when the main objective is to prevent congestion, one can assume that the available control signal q_{r} is always at its maximum $q_{\mathrm{r}M}$.

The globally stabilizing universal formula proposed in (Lin and Sontag, 1994) provides an effective way to stabilize a class of nonlinear systems by means of a control signal whose magnitude is bounded by 1. Although this formula was conceived originally for autonomous systems, it can be applied to this problem because the availability of the disturbance estimates allows one to treat the plant (6) as an autonomous system as long as certain conditions are satisfied. To find these conditions, the plant must be rewritten in a different form so that $x_2 = 0$ is an unforced equilibrium point of the subsystem (6b).

Lemma 1. For a set of disturbances $\phi(t)$ satisfying (13), there exists a control signal $q_{\mathrm{rss}}(t)$ such that, when applied to (6), the closed-loop system has the form $\dot{x}_2 = f_2(\boldsymbol{x}) + g_2(\boldsymbol{x})u(t)$ and satisfies the condition $f_2(\mathbf{0}) = 0$.

PROOF. The existence of the feedback $q_{\mathrm{rss}}(t)$ that makes $x_2 = 0$ an unforced equilibrium point is subject to (13) because this condition guarantees that $q_{\mathrm{rss}}(t)$ lies inside the feasible range of $q_{\mathrm{r}}(t)$, i.e. that $q_{\mathrm{rss}} \in [0, q_{\mathrm{r}M}]$. Therefore, the preliminary feedback that transforms (6) into the form $\dot{x}_2 = f_2(\boldsymbol{x}) + g_2(\boldsymbol{x})u(t)$ with $f_2(\mathbf{0}) = 0$ is given by $q_{\mathrm{r}}(t) = q_{\mathrm{rss}}(t) + q_{\mathrm{rC}}(t)$, where $q_{\mathrm{rC}}(t)$ is the control that drives the system to the state $x_2(t) = 0$ and $q_{\mathrm{rss}}(t)$ has been defined in (12) as the control that keeps $x_2(t)$ at zero by enforcing $\dot{x}_2(t) = 0$. The closed-loop density subsystem is then defined by:

$$\begin{aligned} f_2 =& \frac{1 - 2\alpha}{L} \left(\rho_d x_1 + v_d x_2 + x_1 x_2 \right) \\ &+ g_2 q_{\mathrm{rC}M} q_{\mathrm{rss}} + \phi(t) \\ g_2 =& \frac{1}{q_{\mathrm{rC}M} \lambda L} \left(1 - \frac{x_2 + \rho_d}{\rho_{\mathrm{jam}}} \right). \end{aligned} \quad (14)$$

■

Since $q_{\mathrm{rss}}(t)$ is uniquely defined for given values of the disturbances, $q_{\mathrm{rC}}(t)$ must now be designed so that the solution $x_2(t) = 0$ is attractive and $q_{\mathrm{r}}(t) \in [0, q_{\mathrm{r}M}]$. The signal $q_{\mathrm{rC}}(t)$ relates to the control input $u(t)$, $|u(t)| \le 1$, through $q_{\mathrm{rC}}(t) = u(t)/q_{\mathrm{rC}M}$ where $q_{\mathrm{rC}M}$ is the maximum value of $q_{\mathrm{rC}}(t)$. Once the bound $q_{\mathrm{rC}M}$ is defined, the design procedure continues by proposing $V_2 = x_2^2/2$ as a Control Lyapunov function for $\dot{x}_2 = f_2(\boldsymbol{x}) + g_2(\boldsymbol{x})u(t)$ inside the interval $x_2 \in [-\rho_d, \rho_{\mathrm{jam}} - \rho_d)$.

Theorem 1. For a set of disturbances $\phi(t)$ satisfying (13), there is a control signal $u(t)$ given by:

$$u = \begin{cases} -\dfrac{f_2 + \sqrt{f_2^2 + x_2^2 g_2^4}}{g_2\left(1 + \sqrt{1 + x_2^2 g_2^2}\right)} & \text{if } b_2 \neq 0 \\ 0 & \text{if } b_2 = 0. \end{cases} \quad (15)$$

$$b_2 = \nabla V_2 g_2, \quad (16)$$

such that, when applied to (14), ρ_d is an asymptotically stable equilibrium point of the closed-loop density dynamics.

PROOF. It is trivial to verify that the disturbances are the only variables that could introduce discontinuities into the model. Since the control (12) completely compensates for them, the signal $u(t)$ can be continuous and therefore the Lyapunov function satisfies the small control property (Lin and Sontag, 1994). Moreover, Lemma 1 guarantees that, inside a time-varying set that depends on the choice of $q_{\mathrm{r}EM}$, the plant satisfies $f_2(\mathbf{0}) = 0$. Since these two main conditions for the existence of the bounded stabilizing feedback method presented in (Lin and Sontag, 1994) are satisfied, such a stabilizing feedback exists.

The universal stabilizing formula for bounded controls introduced in (Lin and Sontag, 1994) requires the definition of the functions $a_2(\boldsymbol{x}) = \nabla V_2 f_2(\boldsymbol{x})$ and $b_2(\boldsymbol{x})$ as in (16) that are to be substituted into the following formula:

$$u = \begin{cases} -\dfrac{a_2 + \sqrt{a_2^2 + b_2^4}}{b_2\left(1 + \sqrt{1 + b_2^2}\right)} & \text{if } b_2 \neq 0 \\ 0 & \text{if } b_2 = 0. \end{cases} \quad (17)$$

Since $a_2(\boldsymbol{x})$ and $b_2(\boldsymbol{x})$ are both scalars, the common factor $\nabla V_2 = x_2$ cancels out and (17) reduces to (15). ■

This control law is bounded by $|u| \leq 1$ and guarantees the asymptotic stability of the system (Lin and Sontag, 1994) as long as there exists a control q_{rE} such that $f_2(\mathbf{0}) = 0$. If q_{rE} does not lie inside the feasible range, then the intrinsic properties of the system prevent arbitrary set-point stabilization. A nice property of the control signal (15) that is hard to attain through other control techniques is its well-definiteness at $x_2 = \rho_{\mathrm{jam}} - \rho_d$, i.e., when $g_2(\boldsymbol{x}) = 0$.

Finally, to prevent the case when the on-ramp queue pervades the surface roads in the surroundings, one can set a positive lower bound q_{rm} to the on-ramp flow. If this option is to be implemented with the control law proposed above, the control signal q_{rE} must be maintained above the value of $q_{rm} + q_{\mathrm{r}CM}$ in order to guarantee that $q_{\mathrm{r}} \geq q_{rm}$.

Table 2. Observer and controller parameters.

$\bar{K}_1$	$\bar{K}_2$	$q_{\mathrm{r}CM}$	q_{rm}
10 000	10 000	1 (veh/h)	200 (veh/h)

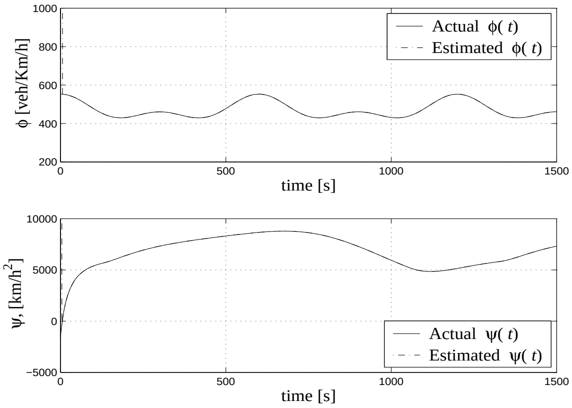

Fig. 2. Convergence of the disturbance estimator states.

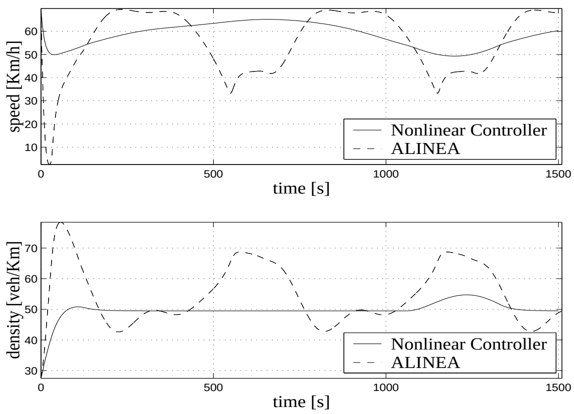

Fig. 3. Convergence comparison between ALINEA and the nonlinear controller.

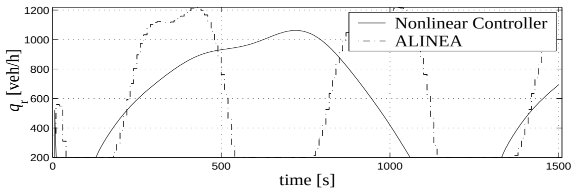

Fig. 4. Control effort comparison between ALINEA and the nonlinear controller.

5. SIMULATION RESULTS

Simulations were conducted to compare the performance of the proposed strategy with the popular ALINEA controller (Smaragdis *et al.*, 2003). The justification for using this linear controller as a reference is its wide use and copious available information. Furthermore, both controllers address the stabilization of the ρ dynamics only.

The discrete-time integral control law reads as:

$$q_{\mathrm{r}}[k] = \max\left\{q_{\mathrm{r}}[k-1] - K(\rho[k] - \rho_d), q_{rm}\right\},$$

where K is a constant frequently set to 16 and k is the time index. Although a sampling period equal to 40s is proposed for the integral control law in (Papageorgiou *et al.*, 1990), these simulations used a shorter period of 10s to increase the speed of response of the controller. In turn, the disturbance observer and the nonlinear controller were tuned with the parameters listed in Table 2. The plant employed the parameters listed in Table 1 and was affected by the disturbances:

$$v_{\text{us}} = \frac{1}{2}v_{\text{f}} + \frac{1}{10}v_{\text{f}}\sin\left(\frac{t}{150\pi}\right)$$
$$\rho_{\text{ds}} = \frac{1}{2}\rho_{\text{jam}} + \frac{3}{5}\rho_{\text{jam}}\cos\left(\frac{t}{300\pi}\right).$$

For initial conditions $\boldsymbol{x}(0) = [3/4v_{\text{f}} \quad 1/4\rho_{\text{jam}}]^T$ and desired density $\rho_d = 49.5$veh/Km, the controllers performed as shown in Figs. 2, 3, and 4.

Fig. 2 shows that the disturbance estimates approach the actual values very fast and, consequently, that the nonlinear controller becomes effective very quickly. Fig. 3 reveals that the nonlinear controller drastically reduces the amplitude of the density variations and clearly outperforms the linear control law. The figure also shows that the nonlinear controller produces a second overshoot, during which no controller can stabilize the system because a feasible stabilizing control input does not exist. Fig. 4 shows that the integral controller over-reacts to the fast-varying disturbances and this causes large state swings that do not occur when the nonlinear controller is used.

6. CONCLUSIONS

Taking into account the usually neglected effects of merging, this work proposes local nonlinear controllers to reduce congestion in highways. The control strategy is robust because it estimates and compensates for any difference between the mathematical model and the actual plant. Therefore, it is effective in the presence of incidents, changing weather conditions, and seasonal variations.

In contrast to previous approaches, the proposed control law addresses the fact that the on-ramp flow may be limited by the mainstream flow. This is an important advantage since it allows the controller to react faster. The simulations confirm this by showing that the new controller acts more promptly than a similar well-known control strategy, thus achieving better performance.

Besides being computationally inexpensive, this approach is also affordable because its localized nature eliminates the need for expensive communication systems. If necessary, it can also be combined with wide-area centralized methods–such as the one presented in (Kotsialos *et al.*, 2002)–to achieve optimal performance.

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