PASSIVE BILATERAL CONTROL OF TELEOPERATORS UNDER CONSTANT TIME-DELAY[1]

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Abstract: We propose a novel control scheme for teleoperators consisting of a pair of multi-degree-of-freedom (DOF) nonlinear robotic systems under a constant communication time delay. By passifying the communication and control blocks together, the proposed control scheme guarantees energetic passivity of the closed-loop teleoperator in the presence of parametric uncertainty and a constant communication delay without relying on widely utilized scattering or wave formalisms. The proposed control scheme also achieves master-slave position coordination and bilateral static force reflection. The proposed control scheme is symmetric in the sense that the control and communication laws of the the master and slave are of the same form. Simulations are performed to validate properties of the proposed control scheme.

Keywords: nonlinear teleoperator, time-delay, passivity, Lyapunov-Krasovskii functionals, Parseval's identity

1. INTRODUCTION

Energetically, a closed-loop teleoperator is a two-port system (see figure 1). Thus, the foremost goal of the control (and communication) design is to ensure interaction safety and coupled stability (Colgate, 1994) when it is mechanically coupled with a broad class of slave environments and human operators. For this, energetic passivity (i.e. mechanical power as the supply rate (Willems, 1972)) of the closed-loop teleoperator has been widely utilized as the control objective (Anderson and Spong, 1989; Niemeyer and Slotine, 1991; Lawrence, 1993; Stramigioli *et al.*, 2002; Lee and Li, 2003*a*; Lee and Li, 2002). This is because 1) the feedback interconnection of the passive teleoperator with *any* passive environments/humans is necessarily stable (Colgate, 1994); and 2) in many cases, slave environments are passive (e.g. pushing a wall) and humans can be assumed as passive systems (Hogan, 1989). Also, passive teleoperator would be potentially safer to interact with, since the maximum extractable energy from it is bounded, thus, possible damages on environments/humans are also limited.

How to ensure passivity of the time-delayed bilateral teleoperation was a long standing problem. In (Anderson and Spong, 1989), scattering theory was proposed to passify the delayed-communication, and passivity of the teleoperator is ensured for arbitrary constant time-delay. In (Niemeyer and Slotine, 1991), this result is extended and the notion of wave variables was introduced. Since these two seminal works, scattering

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theory (or wave formalism) has been virtually the only way to enforce passivity of the delayed bilateral teleoperation (Stramigioli *et al.*, 2002; Chopra *et al.*, 2003; Yokokohji *et al.*, 1999; Niemeyer and Slotine, 2004).

In this paper, we propose a novel bilateral control scheme for teleoperators consisting of a pair of multi-DOF nonlinear robots under a constant communication time delay. By passifying the combination of the communication and control blocks together (see figure 1) rather than achieving their individual passivity as in scattering based schemes, the proposed control scheme guarantees energetic passivity of the closed-loop teleoperator in the presence of parametric uncertainty and constant time-delay of arbitrary magnitude. The proposed control scheme also ensures asymptotic convergence of the position coordination which is guaranteed only implicitly in the scattering based approaches. The proposed control scheme is also symmetric, i.e. the master and slave control and communication modules are of the same form.

The rest of this paper is organized as follows. The control problem is formulated in section 2. In section 3, control law is designed and its properties are detailed. Simulations are performed in section 4 and concluding remarks are given in section 5.

2. PROBLEM FORMULATION

2.1 Plant

Let us consider a nonlinear mechanical teleoperator consisting of a pair of n-DOF robotic systems:

$$M_1(q_1)\ddot{q}_1(t) + C_1(q_1,\dot{q}_1)\dot{q}_1 = T_1(t) + F_1(t), \quad (1)$$
$$M_2(q_2)\ddot{q}_2(t) + C_2(q_2,\dot{q}_2)\dot{q}_2 = T_2(t) + F_2(t), \quad (2)$$

where $q_i, F_i, T_i \in \Re^n$ are configurations, human/environmental force, and controls, and $M_i(q_i)$, $C_i(q_i,\dot{q}_i) \in \Re^{n\times n}$ are symmetric and positive-definite inertia matrices and Coriolis matrices s.t. $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ are skew-symmetric ($i = 1, 2$).

Considering a constant time-delay $\tau \geq 0$ (see figure 1), we will define the controls $T_1(t), T_2(t)$ in (1)-(2) to be functions of the current and stored local information and the remote information delayed by the constant time-delay $\tau \geq 0$ s.t. $T_i(t) := T_i(q_i(t), \dot{q}_i(t), q_i(t-\tau), \dot{q}_i(t-\tau), q_j(t-\tau), \dot{q}_j(t-\tau)) \in \Re^n$, where $i, j \in \{1, 2\}, i \neq j$.

2.2 Control Objectives

We would like to design the controls $T_1(t), T_2(t)$ to achieve the following *master-slave position coordination*: when $(F_1(t), F_2(t)) = 0$,

$$q_E(t) := q_1(t) - q_2(t) \to 0, \qquad t \to \infty. \quad (3)$$

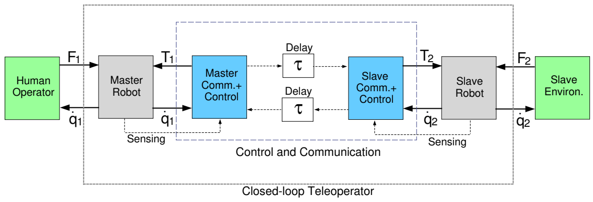

Fig. 1. Closed-loop teleoperator as a two-port system.

We would also like to achieve the *bilateral force reflection*: when $(\ddot{q}_1(t), \ddot{q}_2(t), \dot{q}_1(t), \dot{q}_1(t)) \to 0$,

$$F_1(t) \to -F_2(t). \quad (4)$$

For safe interaction and coupled stability, we would like to enforce the following *energetic passivity* condition for the two-port teleoperator (1)-(2): there exists a finite constant $d \in \Re$ s.t. $\forall t \geq 0$,

$$\int_0^t \left[F_1^T(\theta)\dot{q}_1(\theta) + F_2^T(\theta)\dot{q}_2(\theta)\right] d\theta \geq -d^2, \quad (5)$$

i.e. maximum extractable energy from the two-port closed-loop teleoperator is always bounded. Let us also define *controller passivity* condition (Lee, 2004; Lee and Li, 2003*b*) : there exists a finite constant $c \in \Re$ s.t. $\forall t \geq 0$,

$$\int_0^t \left[T_1^T(\theta)\dot{q}_1(\theta) + T_2^T(\theta)\dot{q}_2(\theta)\right] d\theta \leq c^2, \quad (6)$$

i.e. energy generated by the two-port controller (figure 1) is always limited.

Lemma 1. (Lee, 2004; Lee and Li, 2003*b*) For the mechanical teleoperator (1)-(2), controller passivity (6) implies energetic passivity (5).

Proof: Let us define the total kinetic energy

$$\kappa_f(t) := \tfrac{1}{2}\dot{q}_1^T M_1(q_1)\dot{q}_1 + \tfrac{1}{2}\dot{q}_2^T M_2(q_2)\dot{q}_2, \quad (7)$$

then, using the dynamics (1)-(2) and its skew-symmetric property, we have

$$\frac{d}{dt}\kappa_f(t) = T_1^T\dot{q}_1 + T_2^T\dot{q}_2 + F_1^T\dot{q}_1 + F_2^T\dot{q}_2. \quad (8)$$

Thus, by integrating above equality with the controller passivity condition (6) and the fact that $\kappa_f(t) \geq 0$, we have, $\forall t \geq 0$, $\int_0^t [F_1^T(\theta)\dot{q}_1(\theta) + F_2^T(\theta)\dot{q}_2(\theta)]d\theta \geq -\kappa_f(0) - c^2 =: -d^2$. ■

3. CONTROL DESIGN

To achieve coordination (3), force reflection (4), and passivity (5), we design the master and slave controls $T_1(t), T_2(t)$ to be

$$\begin{aligned} T_1(t) := &-(K_d + 2K_v + P_\epsilon)\dot{q}_1(t) \\ &+ K_v\dot{q}_1(t-\tau) + K_v\dot{q}_2(t-\tau) \\ &- K_p q_1(t-\tau) + K_p q_2(t-\tau), \end{aligned} \quad (9)$$

$$\begin{aligned} T_2(t) := &-(K_d + 2K_v + P_\epsilon)\dot{q}_2(t) \\ &+ K_v\dot{q}_2(t-\tau) + K_v\dot{q}_1(t-\tau) \\ &- K_p q_2(t-\tau) + K_p q_1(t-\tau), \end{aligned} \quad (10)$$

where $K_v, K_p \in \Re^{n\times n}$ are symmetric and positive-definite proportional-derivative (PD) control gains, $K_d \in \Re^{n\times n}$ is the the dissipation gain to enforce energetic passivity and designed to satisfy the following "gain-setting" condition s.t.

$$K_d - \frac{2sinw\tau}{w} K_p \succcurlyeq 0, \quad \forall w \in \Re, \tag{11}$$

(i.e. $K_d - \frac{2sinw\tau}{w} K_p$ is positive-semidefinite), and $P_\epsilon \in \Re^{n\times n}$ is an additional dissipation gain (e.g. inherent device damping) to be exploited for the position coordination proof in theorem 2. Since $\tau - \frac{sinw\tau}{w} \geq 0$, $\forall w \in \Re$, one possible solution for the condition (11) is given by

$$K_d = 2K_p\tau. \tag{12}$$

Theorem 2. Consider the mechanical teleoperator (1)-(2) under the controls (9)-(10) designed to satisfy the gain-setting condition (11).

1) The closed-loop teleoperator is passive (i.e. satisfies (5)) regardless of parametric uncertainty in (1)-(2) (i.e. robust passivity (Lee and Li, 2003*b*));

2) Suppose that the human operator and slave environment are energetically passive, i.e. $\exists$ finite constants $d_1, d_2 \in \Re$ s.t.

$$\int_0^t \underbrace{-F_\star^T(\theta)\dot{q}_\star(\theta)}_{\substack{\text{mechanical power inflow to}\\ \text{human/environment}}} d\theta \geq -d_\star^2, \tag{13}$$

$\forall t \geq 0$ ($\star \in \{1,2\}$), i.e. the maximum extractable energy from them are bounded. Then, $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_\infty$. Thus, if the human and slave environment are $\mathcal{L}_\infty$-stable input-output impedance maps, $F_1(t), F_2(t) \in \mathcal{L}_\infty$;

3) Suppose that the human and slave environment are passive in the sense of (13). Suppose further that $M_i^{jk}(q_i)$, $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ and $\frac{\partial^2 M_i^{jk}(q_i)}{\partial q_i^m \partial q_i^l}$ are all bounded, where $M_i^{jk}(q_i)$ and q_i^m are the jk-th and the m-th components of $M_i(q_i)$ and q_i. Then, $q_E(t) = q_1(t) - q_2(t)$ is bounded $\forall t \geq 0$. Moreover, if $(F_1(t), F_2(t)) = 0$, $(q_E(t), \dot{q}_E(t)) \to 0$ (i.e. (3) is achieved);

4) If $(\dot{q}_1(t), \dot{q}_2(t), \ddot{q}_1(t), \ddot{q}_2(t)) \to 0$, then, $F_1(t) \to -F_2(t) \to -K_p(q_1(t) - q_2(t))$ (i.e. (4) is achieved).

Proof: 1) Let us denote the mechanical power generated by the controls (9)-(10) by

$$\begin{aligned} s_c(t) :&= T_1^T(t)\dot{q}_1(t) + T_2^T(t)\dot{q}_2(t) \\ &= s_v(t) + s_p(t) - P(t), \end{aligned}$$

where $P(t)$ is the following quadratic form:

$$P(t) := \begin{pmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{pmatrix}^T \begin{bmatrix} P_\epsilon & 0 \\ 0 & P_\epsilon \end{bmatrix} \begin{pmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{pmatrix}, \tag{14}$$

and the terms $s_v(t)$ and $s_p(t)$ are defined by

$$\begin{aligned} s_v(t) :=& -2\dot{q}_1^T K_v \dot{q}_1 + \dot{q}_1^T K_v \bar{\dot{q}}_1 + \dot{q}_1^T K_v \bar{\dot{q}}_2 \\ & -2\dot{q}_2^T K_v \dot{q}_2 + \dot{q}_2^T K_v \bar{\dot{q}}_2 + \dot{q}_2^T K_v \bar{\dot{q}}_1, \end{aligned} \tag{15}$$

$$\begin{aligned} s_p(t) :=& -\dot{q}_1^T K_d \dot{q}_1 - \dot{q}_1^T K_p \bar{q}_1 + \dot{q}_1^T K_p \bar{q}_2 \\ & -\dot{q}_2^T K_d \dot{q}_2 - \dot{q}_2^T K_p \bar{q}_2 + \dot{q}_2^T K_p \bar{q}_1, \end{aligned} \tag{16}$$

where we denote $(\star(t), \star(t-\tau))$ by $(\star, \bar{\star})$.

Similar to the Lyapunov-Krasovskii functionals (Gu and Niculescu, 2003), we define

$$V_v(t) := \tfrac{1}{2} \int_{-\tau}^0 \dot{q}_L^T(t+\theta) K_v \dot{q}_L(t+\theta) d\theta \geq 0, \tag{17}$$

where $q_L(t) := q_1(t) + q_2(t) \in \Re^n$, and its time-derivative is given by

$$\frac{d}{dt}V_v(t) = \tfrac{1}{2}\left[\dot{q}_L^T(t) K_v \dot{q}_L(t) - \dot{q}_L^T(t-\tau) K_v \dot{q}_L(t-\tau)\right].$$

Then, using the above equality and the fact that

$$2\left[\dot{q}_1^T K_v \dot{q}_1 + \dot{q}_2^T K_v \dot{q}_2\right] \geq \dot{q}_L^T K_v \dot{q}_L, \tag{18}$$

the supply rate $s_v(t)$ in (15) can be written as

$$\begin{aligned} s_v(t) &\leq -\dot{q}_L^T K_v \dot{q}_L + \dot{q}_L^T K_v \bar{\dot{q}}_L \\ &= \underbrace{-\tfrac{1}{2}\dot{q}_L^T K_v \dot{q}_L + \dot{q}_L^T K_v \bar{\dot{q}}_L - \tfrac{1}{2}\bar{\dot{q}}_L^T K_v \bar{\dot{q}}_L}_{=-\frac{1}{2}[\dot{q}_L - \bar{\dot{q}}_L]^T K_v [\dot{q}_L - \bar{\dot{q}}_L]} \\ &\quad \underbrace{+\tfrac{1}{2}\bar{\dot{q}}_L^T K_v \bar{\dot{q}}_L - \tfrac{1}{2}\dot{q}_L^T K_v \dot{q}_L}_{=-\frac{d}{dt}V_v(t)} \leq -\frac{d}{dt}V_v(t), \end{aligned} \tag{19}$$

where we denote $(\star(t), \star(t-\tau))$ by $(\star, \bar{\star})$. Thus, by integrating the inequality (19), we have

$$\int_0^t s_v(\theta) d\theta \leq -V_v(t) + V_v(0). \tag{20}$$

Let us consider the supply rate $s_p(t)$ in (16). Then, using the definition of $q_E(t) = q_1(t) - q_2(t)$ in (3) and the following inequality similar to (18) s.t.

$$2\left[\dot{q}_1^T K_d \dot{q}_1 + \dot{q}_2^T K_d \dot{q}_2\right] \geq \dot{q}_E^T K_d \dot{q}_E, \tag{21}$$

we have

$$\begin{aligned} s_p(t) &\leq -\tfrac{1}{2}\dot{q}_E^T(t) K_d \dot{q}_E(t) - \dot{q}_E^T(t) K_p q_E(t-\tau) \\ &= -\tfrac{1}{2}\dot{q}_E^T(t) K_d \dot{q}_E(t) - \frac{d}{dt}V_p(t) \\ &+ \dot{q}_E^T(t) K_p \underbrace{\left[\int_0^t \dot{q}_E(\theta) d\theta - \int_0^{t-\tau} \dot{q}_E(\theta) d\theta\right]}_{=q_E(t) - q_E(t-\tau)}, \end{aligned} \tag{22}$$

where $V_p(t)$ is the spring potential energy, i.e.

$$V_p(t) := \tfrac{1}{2} q_E^T(t) K_p q_E(t). \tag{23}$$

Let us define truncated signal $\tilde{\dot{q}}_E^t(\theta)$ of $\dot{q}_E(t)$ s.t.

$$\tilde{\dot{q}}_E^t(\theta) := \begin{cases} \dot{q}_E(\theta) & \text{if } \theta \in [0,t] \\ 0 & \text{otherwise} \end{cases}, \tag{24}$$

with its Fourier transform given by $V_E^t(w) = \int_{-\infty}^{+\infty} \tilde{\dot{q}}_E^t(\theta) e^{-jw\theta} d\theta = \int_0^t \dot{q}_E(\theta) e^{-jw\theta} d\theta$ ($j = \sqrt{-1}$).

Then, using and Parseval's identity (Goldberg, 1961), the gain-setting condition (11), and the fact that $\tilde{V}_E^{t*}(w)K_d\tilde{V}_E^t(w)$ and $\tilde{V}_E^{t*}(w)K_p\tilde{V}_E^t(w)$ are even-functions w.r.t. w, we have: $\forall t \geq 0$,

$$\begin{aligned}
&\int_0^t s_p(\theta)d\theta \leq -V_p(t) + V_p(0) \\
&\quad - \tfrac{1}{2}\int_{-\infty}^{+\infty} \tilde{V}_E^{t*}(w)K_d\tilde{V}_E^t(w)dw \\
&\quad + \int_{-\infty}^{+\infty} \tilde{V}_E^{t*}(w)K_p\frac{1-e^{-jw\tau}}{jw}\tilde{V}_E^t(w)dw \\
&= -V_p(t) + V_p(0) \\
&\quad - \tfrac{1}{2}\int_{-\infty}^{+\infty} \tilde{V}_E^{t*}(w)\left(K_d - \frac{2sinw\tau}{w}K_p\right)\tilde{V}_E^t(w)dw \\
&\leq -V_p(t) + V_p(0),
\end{aligned} \tag{25}$$

where we denote the complex conjugate transpose of a complex vector $\star \in \mathbb{C}^n$ by $\star^*$ (i.e. $\star^* = \bar{\star}^T$).

Thus, by summing up (20) and (25) with the fact that $V_v(t) \geq 0$ and $V_p(t) \geq 0$, $\forall t \geq 0$, we can prove controller passivity (6) s.t. for all $\forall t \geq 0$,

$$\begin{aligned}
&\int_0^t T_1^T(\theta)\dot{q}_1(\theta) + T_2^T(\theta)\dot{q}_2(\theta)d\theta \\
&\quad = \int_0^t [s_v(\theta) + s_p(\theta)]\, d\theta - \int_0^t P(\theta)d\theta \\
&\quad \leq V_v(0) + V_p(0) =: c^2,
\end{aligned} \tag{26}$$

where $V_v(0)$ will be zero if $(\dot{q}_1(t), \dot{q}_2(t)) = 0$ $\forall t \in (-\infty, 0]$ and $V_p(0)$ is the initial spring potential energy. Thus, from lemma 1, energetic passivity (5) follows. Since controller passivity (6) doesn't depend on the parameters in (1)-(2), this achieved passivity is robust against parametric uncertainty.

2) By integrating the equality (8) with the controller passivity (26) and the human/slave environment passivity (13), we have, for all $t \geq 0$,

$$\begin{aligned}
&\kappa_f(t) + V_v(t) + V_p(t) \\
&\leq \kappa_f(0) + V_v(0) + V_p(0) - \int_0^t P(\theta)d\theta + d_1^2 + d_2^2.
\end{aligned} \tag{27}$$

where $P(t) \geq 0$ (14). Therefore, $\kappa_f(t)$ is bounded, thus, $\dot{q}_1(t), \dot{q}_2(t)$ are also bounded $\forall t \geq 0$ (i.e. $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_\infty$). Moreover, if the human and slave environment are $\mathcal{L}_\infty$-stable impedance maps, $F_1(t), F_2(t) \in \mathcal{L}_\infty$.

3) Boundedness of $q_E(t) = q_1(t) - q_2(t)$ is a direct consequence of the inequality (27) and the definition of $V_p(t)$ in (23).

First step of the convergence proof is to show that $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$. Suppose that $F_1(t), F_2(t) = 0$, $\forall t \geq 0$. Then, from (27) with $d_1 = d_2 = 0$ and the boundedness of $P_\epsilon, M_1(q_1), M_2(q_2)$, we have:

$$\begin{aligned}
\kappa_f(t) &\leq \kappa_f(0) + c^2 - \int_0^t P(\theta)d\theta \\
&\leq \kappa_f(0) + c^2 - \gamma\int_0^t \kappa_f(\theta)d\theta,
\end{aligned} \tag{28}$$

$\forall t \geq 0$, where $P(t)$ and c are defined in (14) and (26), and $\gamma > 0$ is a constant scalar. Here, since $\kappa_f(t) \geq 0$, the term $\int_0^t \kappa_f(\theta)d\theta$ is monotonically increasing and upper bounded, thus, it converges to a limit. Therefore, following Barbalat's lemma, if $\kappa_f(t)$ is uniformly continuous, $\kappa_f(t)$ will also converge to 0 (i.e. $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$). To show this, let us consider $\frac{d}{dt}\kappa_f(t)$. Then, from (8) with $F_1(t) = F_2(t) = 0$, we have $\frac{d}{dt}\kappa_f(t) = T_1^T(t)\dot{q}_1(t) + T_2^T(t)\dot{q}_2(t)$, where $\dot{q}_1(t), \dot{q}_2(t)$ are bounded (from item 2 of this theorem), and $T_1(t), T_2(t)$ are also bounded from their definitions (9)-(10) with bounded $q_E(t) = q_1(t) - q_2(t)$ (from item 2 of this theorem). Thus, $\kappa_f(t)$ is uniformly continuous, therefore, $\kappa_f(t) \to 0$ and $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$.

Second step is now to show $(\ddot{q}_1(t), \ddot{q}_2(t)) \to 0$ to establish $q_1(t) \to q_2(t)$. Let us consider the teleoperator dynamics (1)-(2) with $F_1(t) = F_2(t) = 0$, where, as shown in the above paragraph, the controls T_1, T_2 in (9)-(10) are bounded. Also, from the boundedness assumption of $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$, the Coriolis terms $C_i(q_i, \dot{q}_i)\dot{q}_i$ $(i = 1, 2)$ in (1)-(2) are bounded. Thus, the accelerations $\ddot{q}_1(t), \ddot{q}_2(t)$ are also bounded $\forall t \geq 0$. Now, let us consider the acceleration $\ddot{q}_i(t)$ in (1)-(2) (with $F_i(t) = 0$):

$$\ddot{q}_i = -M_i^{-1}(q_i)C_i(q_i, \dot{q}_i)\dot{q}_i + M_i^{-1}(q_i)T_i(t), \tag{29}$$

$i = 1, 2$. Then, the time-derivatives of the terms in the right hand side of (29) are all bounded due to the boundedness of $\ddot{q}_i(t), \dot{q}_i(t), q_E(t)$, $\frac{d}{dt}M_i^{-1}(q_i) = -M_i^{-1}(q_i)\frac{d}{dt}M_i(q_i)M_i^{-1}(q_i)$ (from the boundedness assumption of $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$) and $\frac{d}{dt}C_i(q_i, \dot{q}_i)$ (from the boundedness assumption on $\frac{\partial^2 M_i^{jk}(q_i)}{\partial q_i^m \partial q_i^l}$). This implies that the right hand side of (29) is uniformly continuous. Thus, $\ddot{q}_1(t), \ddot{q}_2(t)$ are also uniformly continuous. Therefore, following Barbalat's lemma, $(\ddot{q}_1(t), \ddot{q}_2(t)) \to 0$ as $(\dot{q}_1(t), \dot{q}_2(t)) \to 0$. Moreover, from the dynamics (1)-(2) with $(\ddot{q}_1(t), \ddot{q}_2(t), \dot{q}_1(t), \dot{q}_2(t)) \to 0$ and $F_1(t) = F_2(t) = 0$ $t \geq 0$, we have $K_p(q_1(t) - q_2(t)) \to 0$, i.e. $q_1(t) \to q_2(t)$, since K_p is positive-definite.

4) Suppose that $(\dot{q}_1(t), \dot{q}_2(t), \ddot{q}_1(t), \ddot{q}_2(t)) \to 0$. Then, from the dynamics (1)-(2) with the controls (9)-(10), we have:

$$\begin{aligned}
F_1(t) &\to -K_p(q_1(t) - q_2(t)), \\
F_2(t) &\to -K_p(q_2(t) - q_1(t)),
\end{aligned} \tag{30}$$

where we use $\dot{q}_i(t - \tau) \to 0$ and $q_i(t - \tau) \to q_i(t)$ as $(\dot{q}_1(t), \dot{q}_2(t), \ddot{q}_1(t), \ddot{q}_2(t)) \to 0$. ∎

In the human/environment passivity condition (13), the negative sign in the integration comes from the fact that the power inflows to those systems are given by $(-F_i(t))^T\dot{q}_i(t)$ where $-F_i(t)$ is the reaction force. Also, boundedness of $M_i^{jk}(q_i)$, $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ and $\frac{\partial^2 M_i^{jk}(q_i)}{\partial q_i^m \partial q_i^l}$ can be achieved if the master and slave configuration spaces are compact and their inertia matrices are smooth. This

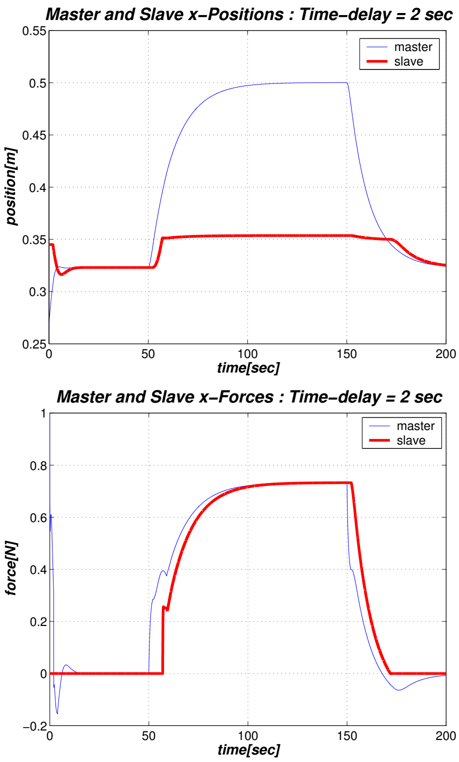

Fig. 2. Stable interaction with master-slave position coordination and force reflection under $2sec$ time-delay.

condition is satisfied in many practical robots (e.g. revolute joint robots).

The proposed control law (9)-(10) requires symmetric delays (i.e. forward and backward delays are the same) and their accurate estimates. To relax this requirement which might limit applicability of the proposed scheme, we recently extend it to the case where the delays are constant but asymmetric and their estimate are inaccurate. Due to the space limitation, we don't include it into this paper and leave it for future publication.

The key step in the proof of theorem 2 is the Parseval's identity in (25) which we assume to be true. A sufficient condition for this is that $\dot{q}_E(t) \in \mathcal{L}_2$ (Goldberg, 1961). As the following lemma shows, this condition can be guaranteed if the human and slave environment are passive in the sense of (13), and the Coriolis matrices in (1)-(2) are bounded w.r.t. (i.e. regardless of) q_1, q_2. The Parseval's identity was also used in (Colgate and Schenkel, 1997) to ensure the energetic passivity of haptic-interfaces under zero-order-hold.

Lemma 3. Suppose that the human and slave environment are passive in the sense of (13) and define $\mathcal{L}_\infty$-stable impedance maps. Suppose further that $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ are bounded, where $M_i^{jk}(q_i)$ and q_i^m are the jk-th and the m-th components of $M_i(q_i)$ and q_i $(i = 1, 2)$. Then, if $\dot{q}_1(0), \dot{q}_2(0), q_E(0)$ are bounded, $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_2$. Thus, $\dot{q}_E(t) \in \mathcal{L}_2$, and the Parseval's identity (25) is justified.

Proof: If $\dot{q}_1(t), \dot{q}_2(t), q_E(t)$ are bounded, $T_i(t)$ will be bounded (from (9)-(10)), and also $F_i(t)$ will be bounded, since the human and slave environment are assumed to be $\mathcal{L}_\infty$-stable impedance map $(i = 1, 2)$. Moreover, from (1)-(2) with the bounded $\frac{\partial M_i^{jk}(q_i)}{\partial q_i^m}$ (i.e. the Coriolis matrices are bounded functions w.r.t q_1, q_2), $\ddot{q}_1(t), \ddot{q}_2(t)$ will also be bounded. Therefore, if $\dot{q}_1(0), \dot{q}_2(0), q_E(0)$ are bounded, $\ddot{q}_1(0), \ddot{q}_2(0)$ are also bounded.

With these bounded $\dot{q}_i(0), \ddot{q}_i(0), q_E(0)$ $(i = 1, 2)$, we can find $\bar{t} > 0$ for a sufficiently large $\bar{M} > 0$ s.t. $\forall t \in \bar{I} := [0, \bar{t})$, $\int_0^t P(\theta)d\theta < \bar{M}$, where $P(t)$ is given in (14). Thus, on $\bar{I}$, the Parseval's identity (25) holds and, following the inequality (27), $\dot{q}_1(t), \dot{q}_2(t), q_E(t)$ are all bounded.

Suppose that $\dot{q}_1(t) \notin \mathcal{L}_2$ or $\dot{q}_2(t) \notin \mathcal{L}_2$. Then, because $\int_0^t P(\theta)d\theta$ is continuous on $\bar{I}$ (since $\dot{q}_1(t), \dot{q}_2(t)$ are bounded $\forall t \in \bar{I}$) and monotonically increasing $\forall t \geq 0$, there should exist t_o and a sufficiently large $\bar{M} > 0$ s.t. $0 < t_o < \bar{t}$ and $M < \int_0^{t_o} P(\theta)d\theta < \bar{M}$ where $M := \kappa_f(0) + V_v(0) + V_p(0) + d_1^2 + d_2^2$. However, this is not possible, because, on the interval $\bar{I}$, the Parseval's identity (25) holds, thus, from (27), we have: for all $t \in \bar{I}$,

$$
\begin{aligned}
0 \leq& \kappa_f(t) + V_v(t) + V_p(t) \\
\leq& \kappa_f(0) + V_v(0) + V_p(0) + d_1^2 + d_2^2 - \int_0^t P(\theta)d\theta.
\end{aligned}
$$

This implies that $\int_0^t P(\theta)d\theta \leq M$ $\forall t \in \bar{I}$ (i.e. $P(t)$ should be uniformly bounded by M on $\bar{I}$ and cannot blow up) and contradicts with the assumption that $M < \int_0^{t_o} P(\theta)d\theta$. Therefore, $\dot{q}_1(t), \dot{q}_2(t) \in \mathcal{L}_2$ and $\dot{q}_E(t) \in \mathcal{L}_2$. Thus, following (Goldberg, 1961), Parseval's identity (25) is valid $\forall t \geq 0$. ∎

4. SIMULATION

We consider a pair of 2-links planar robots. We also model the human as PD-type position tracking controller (spring+damper). To evaluate the contact stability, we implement a lightly damped wall in the slave environment at $x = 0.35m$ reacting only along the x-direction. We set the gains K_d and K_p in (9)-(10) following the condition (12), while the dissipation gain P_ϵ is set to be $0.01K_d$. We also impose time-delay $\tau = 2sec$. During $0 - 50sec$, the human operator stabilizes the slave before the wall. Then, $50 - 150sec$, s/he pushes the slave into the wall, and $150 - 200sec$, s/he retracts the slave from the wall.

As shown in figure 2, interaction with the wall is stable with $2sec$ time-delay. Master-slave position coordination is also achieved when the slave does not interact with the wall. When the slave interacts with the wall, slave force is reflected to the human through the deformation of the spring gain K_p in the control (9)-(10). A force peak in figure 2 around $58sec$ occurs when the human starts moving the master. This is because of the dissipation gain K_d in (9)-(10), which, according to the gain-setting condition (12), needs to be high when we want to achieve high spring gain K_p for better force reflection. If we decrease K_d to mitigate this force peak, we also need to decrease K_p to satisfy the gain-setting condition (12), then, fidelity of force reflection would be compromised. Thus, under the gain-setting condition (12) (i.e. passivity requirement), we have bandwidth trade-off between motion agility and force reflection.

5. CONCLUSIONS

We propose a novel passive bilateral control law for nonlinear mechanical teleoperators under a constant communication time-delay. In contrast to widely-utilized scattering theory (or wave formalism) based approaches where the control and communication blocks are passified individually and position coordination is ensured only implicitly, the proposed control scheme enforces energetic passivity of the closed-loop teleoperator by passifying the combination of the communication and control blocks, and also, by explicitly communicating position signals, ensures the master-slave position coordination when there is no mismatched environment and human forces. The proposed scheme also achieves force reflection in static manipulations. Thus, performance and transparency are enhanced substantially with guaranteed interaction stability. The proposed control scheme is symmetric in the sense that the communication and control structure for the master and slave systems have the same forms. Simulation is performed to validate the properties of the proposed control scheme.

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