A METHODOLOGY FOR IDENTIFICATION OF NARMAX MODELS APPLIED TO DIESEL ENGINES [1]

Gianluca Zito *,[2] **Ioan Doré Landau** *

* *Laboratoire d'Automatique de Grenoble ENSIEG, BP 46*
38402 Saint Martin d'Heres, France
{*gianluca.zito,landau*}*@inpg.fr*

Abstract: In this paper a nonlinear system identification methodology based on a polynomial NARMAX model representation is considered. Algorithms for structure selection and parameter estimation are presented and evaluated. The goal of the procedure is to provide a nonlinear model characterized by a low complexity and that can be efficiently used in industrial applications. The methodology is illustrated by means of an automotive case study, namely a variable geometry turbocharged diesel engine. The nonlinear model representing the relation between the variable geometry turbine command and the intake manifold air pressure is identified from data and validated.

Keywords: nonlinear models, system identification, identification algorithms, automotive.

1. INTRODUCTION

Industrial applications present challenging problems to face when dynamic models are required for the control of nonlinear systems. In model based control input-output nonlinear models can be either developed from physics principles or obtained from a system identification procedure. The first approach is the most adequate but, in practice, it often involves some main problems :

- it is difficult to set the correct values for the physical parameters, in order to get a relevant model for a specific application ;
- the identification of the physical parameters from data is not trivial, due to the structure of the nonlinear equations ;
- the models based on theoretical fundamentals can be very complicated and their use for control purposes is not straightforward.

An alternative solution, as in the linear case, is to use system identification algorithms : input-output nonlinear models, which have not necessarily a physical counterpart, are identified from data in order to obtain a control model. Several classes of nonlinear models are available for nonlinear system identification. A first classification can be done with respect to prior knowledge : "black-box models" are commonly defined as those models whose structure is chosen with no physical insight about the system. These models can be seen as nonlinear mappings from observed data to the output space (a direct mapping or a concatenation of mappings). See (Sjöoberg *et al.*, 1995) for an exhaustive discussion.

The polynomial NARMAX model representation is a black-box nonlinear model set that can be applied to a wide class of nonlinear systems and that can be easily integrated in a simple parameter estimation and structure selection procedure. In this paper a methodology to identify a polyno-

---

[1] This paper is submitted as regular paper to IFAC05.
[2] Corresponding author.

mial NARMAX model of a nonlinear system from data is presented, based on recursive parameter estimation and model structure selection.

The paper is organized as follows : in section 2 the NARMAX system identification procedure is illustrated. In section 3 a diesel engine system, used to test the procedure, is briefly described. The results obtained in this application are presented in section 4 and commented in section 5.

2. NARMAX SYSTEM IDENTIFICATION

2.1 NARMAX representation

The NARMAX model formulation was introduced in (Leontaris and Billings, 1987) as an extension for nonlinear systems of the linear ARMAX model, and is defined as

$$\begin{aligned} y(t) = F(&y(t-1),\dots,y(t-n_y),\\ &u(t-1),\dots,u(t-n_u),\\ &e(t-1),\dots,e(t-n_e)) + e(t) \end{aligned} \tag{1}$$

where $y(t)$, $u(t)$ and $e(t)$ represent the output, the input and the system noise signals respectively; n_y, n_u and n_e are the associate maximum lags and $F(\cdot)$ is a nonlinear function.

The NARMAX representation is a well-known tool for nonlinear modeling which includes several other nonlinear representations such as block-structured models and Volterra series. This class of models has the appealing feature to be linear-in-the-parameters, so that a straight implementation of least-squares techniques can be applied.

Expanding $F(\cdot)$ in (1) as a polynomial of degree L (where L is the degree of the nonlinearity) the expression of a polynomial NARMAX model is obtained as follows

$$y(t) = \sum_{i=1}^{n} \theta_i x_i(t) + e(t) \tag{2}$$

where

$$\begin{aligned} n &= \sum_{i=0}^{L} n_i,\ n_0 = 1\\ n_i &= n_{i-1}\frac{(n_y+n_u+n_e+i-1)}{i}, i = 1\dots L \end{aligned} \tag{3}$$

and

$\theta_i = ith$ model parameter
$x_1(t) = 1$

$$x_i(t) = \prod_{j=1}^{p} y(t-n_{yj}) \prod_{k=1}^{q} u(t-n_{uk}) \prod_{m=1}^{r} e(t-n_{em}) \tag{4}$$

$$i = 2,\dots,n, \quad p,q,r \geq 0, \quad 1 \leq p+q+r \leq L \tag{5}$$

$$1 \leq n_{yj} \leq n_y,\ 1 \leq n_{uk} \leq n_u, \quad 1 \leq n_{em} \leq n_e \tag{6}$$

The choice of a polynomial expression for the regressor is based on the possibility to derive nonlinear control algorithms for a nonlinear polynomial model as a direct extension of classic linear pole-placement control problem.

2.2 Input signal design

Input signal design is a very important step for nonlinear system identification. As for the linear case, the input signal should be persistently exciting. All the frequencies of interest for the system should be excited, and the input signal should cover the whole range of operation. A simple and effective implementation is realized by means of a concatenated set of small-signal tests. Small amplitude perturbing signals may be superposed to the different operating levels, exciting all dynamic modes of the system. Increasing and decreasing level amplitudes have to be considered in order to take into account direction dependent dynamics.

Different classes of signals can be employed for the identification process as multi-sine signals, maximum length binary sequences (MLBS) and classic pseudo-random signals. Documentation about identification signal design can be found in (Schroeder, 1970; Godfrey, 1993).

2.3 Structure selection

Structure selection is a key problem in a black-box system identification. A survey of the structure identification methods is in (Haber and Unbehauen, 1990), and an overview on the different approaches to nonlinear black-box modeling is in (Sjöoberg *et al.*, 1995). When the system to identify is nonlinear a direct estimation based on (2) generally leads to an over-parameterized model. If the values of n_y, n_u, n_e and L are increased to obtain a good accuracy, an excessively complex model will result together with a numerical ill-conditioning. A procedure is needed to select terms from the large set of candidates to provide a parsimonious model. A simple and effective procedure is based on error reduction ratio (ERR) defined in (Billings *et al.*, 1989) as

$$ERR_i = \frac{g_i^2 \sum_{k=1}^{N} w_i^2(t)}{\sum_{k=1}^{N} y_i^2(t)} \tag{7}$$

where $g_i(k)$ are the parameters and $w_i(k)$ the regressors of an auxiliary model constructed to be orthogonal over the data records:

$$y(t) = \sum_{i=1}^{n} g_i w_i(t) + e(t) \tag{8}$$

A model is found selecting the relevant terms from the full model set following a forward-regression algorithm (for more details see (Billings and Chen, 1989)): at each step the parameter with the highest ERR_i is added to the current model, following the principle that a parameter which reduces the variance more than the others is more important. An information criterion, could be used to stop the procedure, as the Akaike Information Criterion (Akaike, 1974), defined as

$$AIC = Nlog_e(\sigma_\epsilon^2(\theta_p)) + kp \tag{9}$$

where σ_ϵ^2 is the variance associated to the p–terms model and k is a penalizing factor. Several techniques have been proposed in the literature for selecting the best model structure, some of these are enhancements of the ERR algorithm or are used in conjunction with it as in (Aguirre and BIllings, 1995; Piroddi and Spinelli, 2003).

2.4 Parameter estimation

At the end of the selection process, a recursive identification is run with the selected parameters. An output error predictor is used, expressed in the form :

$$\hat{y}(t+1) = \hat{\theta}^T x(t) \tag{10}$$

where θ is defined in (2) and x is the same as in (4) but it now depends on the current and previous predicted outputs. At each instant t the parameter vector is updated with the adaptation algorithm :

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t+1)x(t)\varepsilon^0(t+1) \tag{11}$$

where F is an adaptive matrix gain and ε^0 is the *a-priori* prediction error :

$$\epsilon^0(t+1) = y(t+1) - \hat{y}^0(t+1) = y(t+1) - \hat{\theta}^T(t)x(t) \tag{12}$$

More details can be found in (Landau *et al.*, 1998).

2.5 Model validation

A statistical validation of the identified NARMAX model is performed with high order correlation functions defined in (Billings and Voon, 1986; Billings and Zhu, 1994) to detect the presence of unmodelled terms in the residuals of the nonlinear model. If the identified model is adequate, the following conditions should be satisfied by the prediction errors

$$\begin{aligned} \Phi_{\epsilon\epsilon}(k) &= \delta(k) && \text{(i.e. an impulse)} \\ \Phi_{u\epsilon}(k) &= 0 && \forall k \\ \Phi_{\epsilon(\epsilon u)}(k) &= 0 && k \geq 0 \\ \Phi_{u^{2'}\epsilon}(k) &= 0 && \forall k \\ \Phi_{u^{2'}\epsilon^{2'}}(k) &= 0 && \forall k \end{aligned} \tag{13}$$

where $\Phi_{xy(k)}$ indicates the cross–correlation function between $x(t)$ and $y(t)$, $\delta(k)$ is the Kronecker delta, $\overline{u^2(t)}$ is the the mean value of $u^2(t)$ and $u^{2'}(t) = u^2(t) - \overline{u^2(t)}$. If at least one of the correlation functions is well outside the confidence limits, a new model has to be identified. It is necessary, in order to check the ability of the model to represent system dynamics, to validate the estimated model on a new set of data (*validation data*) different from the set used for the identification (*learning data*).

Model prediction ability has to be assessed, together with statistical tests, with signals that may catch system nonlinearities. Triangular or step signals of different amplitude levels are ideal input signals used for time-domain model validation.

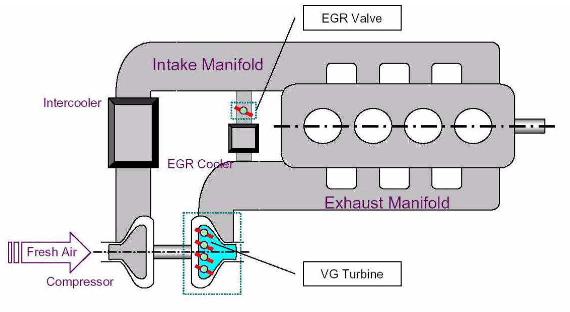


Fig. 1. The VGT/EGR diesel engine.

3. THE VGT TURBOCHARGED DIESEL ENGINE

A turbocharger is often used to enhance acceleration performances in diesel engines. Variable geometry turbochargers (VGT) are employed to achieve good boost at all speed conditions, with no lose in terms of efficiency and transient performances. A turbine is driven by the exhaust gas from the engine and drives the compressor which supplies the airflow into the engine as in Fig.1.

A Variable Geometry Turbocharger (VGT) is used to obtain high transient responses at low engine speeds and to avoid excessive airflow at high engine speeds. A pressure surge in exhaust manifold, in fact, has a detrimental effect for the engine acceleration performances.

A VGT is composed with adjustable vanes that can vary the effective flow area of the turbine,

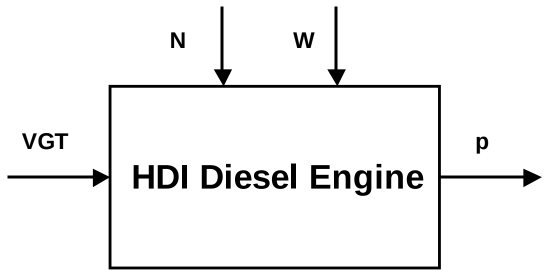


Fig. 2. Equivalent HDI diesel engine scheme for identification.

thereby affecting the compressor mass airflow in the exhaust manifold. VGT can also act as an emission control mechanism: it affects the pressure drop across the exhaust gas recirculation (EGR) vane, increasing the exhaust gas recirculation rate. The gas recirculated back into the engine through the EGR vane lowers the flame temperature and avoids the NOx (oxides of nitrogen) formation.

Examples of diesel engine models were presented in (Guzzella and Amstutz, 1998; Jankovic *et al.*, 2000; Kao and Moskwa, 1995) to be used in the control design phase. In this paper a procedure is presented to provide the nonlinear (discrete time) model of the dynamics between the VGT actuator command and the boost pressure in a turbocharged diesel engine from raw data. A polynomial NARMAX model is used in the identification algorithm, together with techniques for structure selection which preserve from over-parametrization.

4. SIMULATION RESULTS

4.1 Simulation setup

The identification algorithm presented in the previous sections is applied to a high pressure direct injection (HDI) engine model simulated with The MathWorks Simulink environment. The mechanical and thermodynamic interactions between the variables describing the engine operation are modelled with algebraic and differential equations, and with lookup tables recovered by real time experiments. Thus, the model is a low level description of the system showed in Fig.1 and, providing a close approximation of the real system, the nonlinear relation between the VGT signal command and the intake manifold air pressure (MAP) can be investigated in a large set of operative conditions.

For identification purposes the system could be seen as a SISO nonlinear black-box, as shown in Fig.2. The input (VGT) to the system is the command of the actuator that adjusts the angle of guide vanes placed to vary the incoming exhaust gas flow at the entrance of the turbine. The output (p) is the air pressure measured at the intake manifold (boost pressure). N and W are the speed engine and the air mass flow, respectively: a model is identified around an operating point defined by the pair (N, W).

Table 1. Diesel engine operating points: full acceleration.

Speed engine (rpm)	Air mass flow (mm^3/cp)
1000	45
1250	58.2
1500	64.75
1750	68.3
2000	72.31
2250	66.92
2500	66.37
2750	67.3
3000	66.7
3250	63.11
3500	62.11
3750	61.14
4000	60.95
4250	56.53
4500	52

Table 2. Diesel engine operating points: 50% acceleration.

Speed engine (rpm)	Air mass flow (mm^3/cp)
1000	23.68
1250	30.63
1500	34.3
1750	35.94
2000	37.7
2250	35.22
2500	35.8
2750	35.42
3000	35.1
3250	33.21
3500	32.69
3750	32.18
4000	32.08
4250	29.75
4500	27.37

The identification algorithm is feeded with input-output data sets generated from several simulations in order to find a polynomial NARMAX model of the VGT*–boost pressure* nonlinear relation for different pairs (N, W), that specify the operative conditions of interest for the engine. Tables 1 and 2 resume all the different operating points for a full and 50% driver acceleration.

4.2 Excitation signal design

The signal used for the identification is, for all the operating points, a concatenated data set of small signals. A sequence of increasing and decreasing steps describes the different regions of the VGT command, and small amplitude signals are superposed as excitation signals. The data set for the operating point defined by the pair $(N, W) = (3000\ rpm,\ 66.7\ mm^3/cp)$, and a full driver acceleration are considered. The VGT command is in the range 20%–65%, covered by a sequence of steps with an increasing/decreasing variation $\triangle = 5\%$ and superposed multi-sine signals.

4.3 VGT–boost pressure Model identification

The forward-regression estimation algorithm is applied to the data related to the pair $(N, W) = (3000\ rpm,\ 64\ mm^3/cp)$. The first choice for the parameters n_y, n_u and L is based on step responses analysis to estimate dynamics and nonlinearity orders. Tests for nonlinearity detection are presented in (Haber, 1985).

A general inspection reveals that a linear second order system is a good representation for small variations of the input and of the output. This means that the global nonlinear discrete time model, after a linearization, should provide a second order discrete time system. Thus, a model with $n_y = 2$, $n_u = 3$ and $L = 2$ is identified, and details about the parameters are given in table 3.

This procedure, iterated for all the pairs (N_i, W_i), where i is the generic operating point, leads to a set of nonlinear models that describes the diesel engine boost pressure as a nonlinear discrete time difference equation of the variables VGT, N and W. Thereby, (2) can be parameterized as

$$y(t) = \sum_{i=1}^{n} \theta_i(N, W) x_i(t) + e(t) \qquad (14)$$

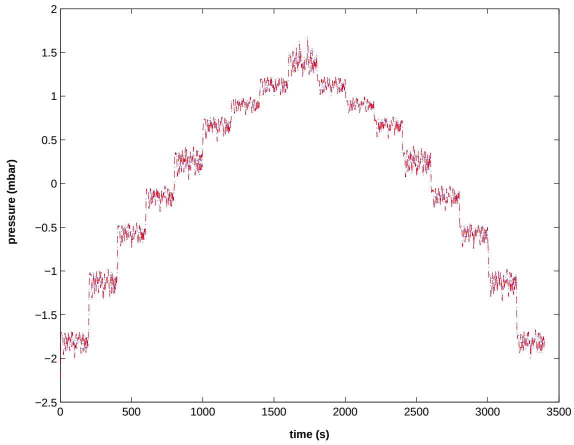
Each operating point has an associated nonlinear model of low complexity: for example, model in table 3 contains 10 parameters of the 21-terms full model. On the basis of this model efficient but still robust nonlinear control algorithms can be directly applied.

4.4 VGT–boost pressure Model validation

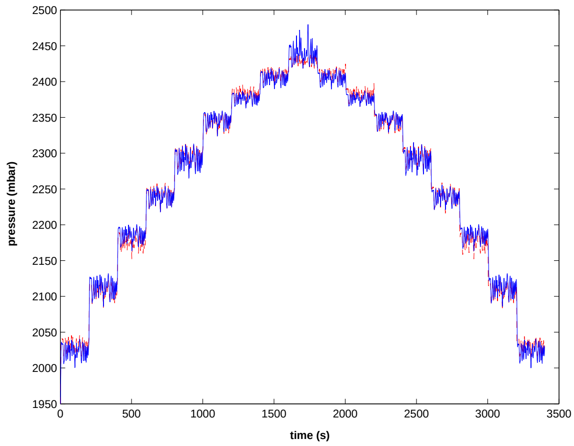
Statistical and time-domain validations are employed to assess the model quality. Fig.3 and Fig.4 show respectively model long-term prediction with *validation data* and step model validation with small and high amplitude data. In these

Table 3. NARMAX parameters.

Index selected	Parameter value	Model term
1	1902.2	constant
2	-0.52096	$y(t-1)$
3	0.013717	$y(t-2)$
4	6.2607	$u(t-1)$
5	1.6462	$u(t-2)$
6	9.7052	$u(t-3)$
7	0.00019272	$y^2(t-1)$
10	0.14749	$u^2(t-1)$
12	-0.40762	$u(t-1)u_(t-3)$
15	0.1361	$u^2(t-3)$



a)



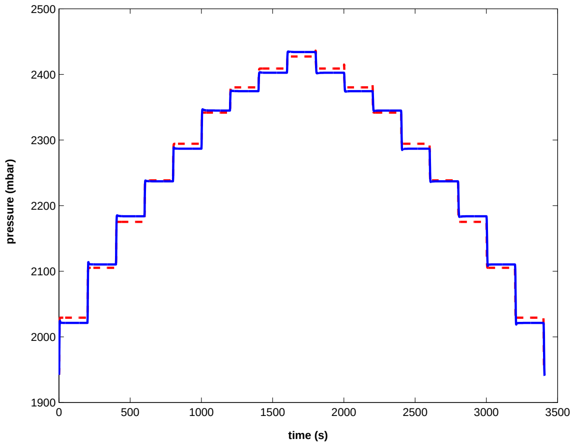
b)

Fig. 3. Model validation for (N,W)=(3000 rpm, 64 mm^3/cp): model prediction (dashed line), system output (solid line); a) 1-step-ahead predictor output (standardized data); b) long-term predictor output.

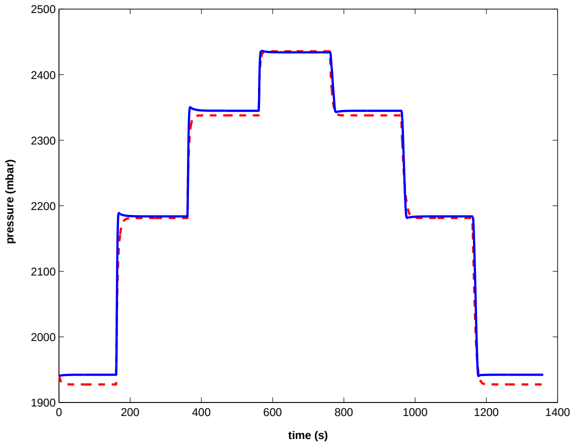
last two cases a step-sequence is applied to the identified model to verify that, for small and large variations in the input signal, the system output is matched from the nonlinear NARMAX model output. The first step sequence is the same used to sweep input amplitude range in the identification data acquisition ($\triangle = 5\%$), in the second one a larger amplitude variation is applied ($\triangle = 15\%$). This typical engine test confirm that the model is suitable to represent system dynamics in both input direction.

5. CONCLUSIONS

Model-based control design is a powerful tool in control of diesel engines. The availability of simple and control-oriented models is a key element in the phase of engine development and tuning. An efficient solution to the modeling problem is represented by a black-box nonlinear identification via polynomial NARMAX models. In this paper a practical identification procedure based on polynomial NARMAX modeling has been developed



a)



b)

Fig. 4. Model and real system step responses for (N,W)=(3000 rpm, 64 mm^3/cp): a) small amplitude; b) high amplitude; model output (dashed line), system output (solid line).

and applied to a HDI diesel engine. Parsimonious nonlinear models have been derived in view of an efficient nonlinear control algorithms implementation.

6. ACKNOWLEDGMENTS

The work of Gianluca Zito is supported by a Marie Curie Industry host Fellowship of the European Community.

REFERENCES

Aguirre, Luis A. and S. A. BIllings (1995). Improved structure selection for nonlinear models based on term clustering. *International Journal of Control* **62**(3), 569–587.

Akaike, H. (1974). A new look at the statistical model identification. *IEEE Transactions on Automatic Control* **19**(6), 716–723.

Billings, S. A. and Q. M. Zhu (1994). Nonlinear model validation using correlation tests. *International Journal of Control* **60**(6), 1107–1120.

Billings, S. A. and S. Chen (1989). Extended model set, global data and threshold mdeol identification for severely nonlinear systems. *International Journal of Control* **50**(6), 1897–1923.

Billings, S. A. and W. S. F. Voon (1986). Correlation based model validity tests for non–linear models. *International Journal of Control* **44**(1), 235–244.

Billings, S. A., S. Chen and M. J. Korenberg (1989). Identification of MIMO nonlinear systems using a forward-regression orthogonal estimator. *International Journal of Control* **49**(6), 2157–2189.

Godfrey, K.R. (1993). *Perturbation Signals for System Identification*. Prentice Hall. New York, NY.

Guzzella, L. and A. Amstutz (1998). Control of diesel engines. *IEEE Control System Magazine* **18**, 53–71.

Haber, R. (1985). Nonlinearity tests for dynamic processes. In: *IFAC Identification and System Parameter Estimation*. York, UK. pp. 409–413.

Haber, R. and H. Unbehauen (1990). Structure Identification of Nonlinear Dynamic Systems–A survey on Input/Output Approaches. *Automatica* **26**(4), 651–677.

Jankovic, M., M. Jankovic and I. Kolmanovsky (2000). Robust nonlinear controller for turbocharged diesel engines. *IEEE Transactions on Control System Technology*.

Kao, M. and J. J. Moskwa (1995). Turbocharged diesel engine modelling for nonlinear engine control and estimation. *ASME Journal of Dynamic Systems, Measurement, and Control*.

Landau, I.D., R. Lozano and M. M'Saad (1998). *Adaptive control*. Springer. London, GB.

Leontaris, I. J. and S. A. Billings (1987). Input-Output parametric models for non-linear systems–Part 1:Deterministic non-linear systems; Part 2: Stochastic non–linear systems. *International Journal of Control* **41**, 303–344.

Piroddi, L. and W. Spinelli (2003). An identification algorithm for polynomial NARX model based on simulation error minimization. *International Journal of Control* **76**(17), 1767–1781.

Schroeder, M. R. (1970). Synthesis of low peak-factor signals and binary sequences of low auto-correlation. *IEEE Transactions on Information Theory* **16**(6), 85–89.

Sjöoberg, J., Q. Zhang, L. Ljung, A. Benveniste, P. Glorennec B. Delyon, H. Hjalmarsson and A. Juditsky (1995). Nonlinear Black–box Modeling in System Identification: a Unified Overview. *Automatica* **31**(12), 1691–1724.