

## ROBUST DECENTRALIZED POLE ASSIGNMENT

Alireza Esna Ashari<sup>1</sup>  
Batool Labibi<sup>2</sup>

*1 Control and Intelligent Processing Center of Excellence  
University of Tehran, Tehran, Iran*

*2 K. N. Toosi University of Technology, Tehran, Iran*

**Abstract:** This paper considers the problem of assigning the closed-loop eigenvalues of linear large-scale systems in a desired stable region using decentralized control. A new method for robust decentralized control of the system via eigenstructure assignment of the subsystems is given. Based on the matrix perturbation theory, sufficient conditions for eigenstructure of each isolated subsystem are derived, when these conditions are satisfied, the closed-loop poles of the overall system will be assigned in the desirable region. Also, a new robustness measure is proposed, which does not need to calculate the eigenvectors and condition numbers. So the proposed measure has less computational burden than many of similar robustness measures. Based on the presented results, a new algorithm for decentralized robust pole assignment is suggested to design output feedback or state feedback. *Copyright © 2005 IFAC*

**Keywords:** Decentralized control, eigenstructure assignment, robust stability, large-scale systems, eigenvalue, and eigenvectors.

### 1. INTRODUCTION

Decentralized stabilization has been an active field of research for large-scale systems. Since 1960's, many authors have considered this problem (Bailey, 1966; Jamshidi, 1997; Siljak, 1991). The early works were based on Lyapunov methods (Araki and Kondo, 1972; Siljak, 1991). However, the stability results of the composite system are much dependent on the choice of the Lyapunov functions of the subsystems. Also, Lyapunov methods provide only sufficient conditions for stability of interconnected systems, and one may search in vain for stabilizing control.

In (Grosdidier and Morari, 1986) structured singular value interaction measures are used as a tool for the design of decentralized control. The interaction measures consider the stability and performance of the closed-loop system. This method provides a sufficient condition (in terms of the subsystem design constraints) under which aggregation of the stable subsystem design yields an overall stable

design. Also, Nett and Uthgenannt (1988) derived an explicit stability condition for 2-block systems. However, in both of these methods, it is assumed that the initial system is square, stable and minimum phase and for systems with high dimensions, it requires very complicated computations.

Recently, a novel sufficient condition for decentralized stabilization of large-scale systems is proposed (Labibi, et al., 2000). In this method, the stability sufficient condition is stated as maximum eigenvalues of the hermitian parts of the state matrix of each isolated subsystem and the interaction matrix. On the other hand, another sufficient condition for robust stabilization of large-scale systems is proposed (Labibi, et al., 2003). The authors have shown that by appropriately assigning the eigenstructure of each isolated subsystem, the robust stability of the overall closed loop system will be guaranteed.

However, in the mentioned approaches, exponential stability of the closed-loop system is considered and the poles can not be assigned in the desirable region. To overcome the mentioned problems in the previous methods, this paper presents a novel methodology for the design of decentralized controllers in which, the approach can be applied to non-square, non-minimum phase and open-loop unstable systems as well. The main contribution of the paper is that the new approach is capable of regional pole assignment using decentralized control. The proposed methodology guarantees the closed-loop robust stability.

In many practical design situations, there often exist perturbations or parameter variations in a system. In order to achieve closed-loop system with low eigenvalue sensitivity, some suitable measures of eigenvalue sensitivity has been introduced (Liu and Patton, 1998). In this paper, a new robustness index is introduced, which leads to computationally effective methods for robust controller design via normalizing the closed-loop state matrix. It allows the closed-loop poles to be assigned with minimum sensitivity to parameter perturbations within the specified region.

The paper is organized as follows: In section 2, the problem of finding suitable decentralized dynamical controllers for the subsystems of a linear large-scale system is presented. Section 3 provides the necessary mathematical background for the next section. In sections 4, new sufficient conditions are achieved, when satisfied the overall closed loop poles are assigned in a specified region. In sections 5 an illustrative example is presented to show the effectiveness of the proposed method.

## 2. PROBLEM FORMULATION

Consider a large-scale system  $G(s)$ , with the following state-space equations:

$$G: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where  $x \in R^n$ ,  $u \in R^m$ ,  $y \in R^p$ ,  $A \in R^{n \times n}$ ,  $B \in R^{n \times m}$ , and  $C \in R^{p \times n}$ , composed of  $L$  linear time-invariant subsystems  $G_i(s)$ , described by:

$$G_i: \begin{cases} \dot{x}_i = A_{ii}x_i + B_{ii}u_i + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ij}x_j \\ y_i = C_{ii}x_i \end{cases} \quad (2)$$

where  $A_{ii} \in R^{n_i \times n_i}$ ,  $B_{ii} \in R^{n_i \times m_i}$ ,  $C_{ii} \in R^{p_i \times n_i}$ ,  $x_i \in R^{n_i}$ ,  $u_i \in R^{m_i}$ ,  $y_i \in R^{p_i}$ ,  $\sum_{i=1}^L n_i = n$ ,  $\sum_{i=1}^L m_i = m$ , and

$\sum_{i=1}^L p_i = p$ . It is assumed that all  $(A_{ii}, B_{ii})$  are controllable and  $(A_{ii}, C_{ii})$  are observable, also all  $B_{ii}$  and  $C_{ii}$  are of full rank. In (2), the term  $\sum A_{ij}x_j$  is due to the interactions of the other subsystems. The objective in this paper is to design a local dynamical output feedback:

$$U_i(s) = K_i(s)(R_i(s) + Y_i(s)) \quad (3)$$

for each isolated subsystem

$$G_{di} = \begin{cases} \dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) \\ y_i(t) = C_{ii}x_i(t) \end{cases} \quad (4)$$

such that the closed loop poles are assigned appropriately. Note that  $R_i$  is the  $i$ -th reference input vector. Finally, the decentralized controller

$$K(s) = \text{diag}\{K_i(s)\} \quad (5)$$

assigns the closed loop poles of the overall system in the desirable region such that they have minimum sensitivity to parameter changes, if some sufficient conditions are satisfied.

## 3. PRELIMINARY MATHEMATICAL NOTES

In this section several basic Theorems which are necessary to prove the Theorem of the next section are presented. In the following Theorems, the matrix  $A$  which satisfies the property

$$A^H A = A A^H \quad (6)$$

is called normal where,  $H$  denotes the conjugate transpose of the matrix. Also Schur decomposition will be used (Golub and Van Loan, 1989) to give some results on matrix perturbation theory.

Theorem 3.1: Let

$$Q^H A Q = D + N \quad (7)$$

be a Schur decomposition of  $A \in C^{n \times n}$  where  $D = \text{diag}(\lambda_1, \dots, \lambda_n)$ ,  $N \in C^{n \times n}$  is a strictly upper triangular matrix and  $Q$  is an appropriate unitary matrix. Suppose that  $E \in C^{n \times n}$  is an arbitrary matrix. If  $\mu \in \lambda(A + E)$  and  $p$  is the smallest positive integer such that  $|N|^p = 0$ , then

$$\min_{\lambda \in \lambda(A)} |\mu - \lambda| \leq \max(\theta, \theta^{1/p}) \quad (8)$$

where:

$$\theta = \|E\|_2 \sum_{k=0}^{p-1} \|N\|_2^k \quad (9)$$

$\lambda(\cdot)$  means the eigenvalue of  $(\cdot)$  and  $|N| = \lfloor n_{ij} \rfloor$ ,  $\lambda$  is the eigenvalue of the matrix  $A$ .  
Proof: see (Golub and Van Loan, 1989).

Lemma 3.1: Matrix  $A \in C^{n \times n}$  is normal if and only if in the Schur decomposition of matrix  $A$ , the matrix  $N$  is equal to zero.

Proof: see (Golub and Van Loan, 1989).

Theorem 3.2: Matrix  $A \in C^{n \times n}$  is normal if and only if:

$$\|A\|_F^2 = \sum_{i=1}^n |\lambda_i(A)|^2 \quad (10)$$

where  $\|A\|_F$  is the Frobenius norm of  $A$ .

Proof: It is shown that unitary similarity transformations do not affect the Frobenius norm of a matrix. So, it can be concluded that:

$$\|Q^H A Q\|_F = \|A\|_F \quad (11)$$

If the matrix  $A$  is normal, then based on lemma 3.1, in equation (7)  $N=0$ , and

$$\|A\|_F^2 = \|D\|_F^2 = \sum_{i=1}^n |\lambda_i(A)|^2 \quad (12)$$

and the sufficient condition is proved. In order to show the necessary part of the Theorem, since the Schur decomposition of the matrix  $A$ , is given as follows

$$\|A\|_F = \|D + N\|_F = \sqrt{\sum_{i=1}^n |\lambda_i(A)|^2 + \sum_{j=1}^n \sum_{i=1}^n |n_{ij}|^2} \quad (13)$$

and

$$\|A\|_F^2 = \sum_{i=1}^n |\lambda_i(A)|^2 \quad (14)$$

it can be concluded that  $n_{ij} = 0$  and  $N=0$ , therefore the matrix is normal, and the proof is complete.  $\square$

#### 4. ROBUST DECENTRALIZED POLE ASSIGNMENT

Consider a large-scale system  $G(s)$ , with state space equations (1). The state space equations for each subsystem are given by (2). In general case, the designed controller for each subsystem is a dynamical controller. Assume that  $i$ -th controller  $K_i(s)$  in (3) has the following state-space equations

$$\begin{aligned} \dot{x}_{Ci}(t) &= A_{Ci}x_{Ci}(t) + B_{Ci}(r_i + y_i) \\ u_i(t) &= C_{Ci}x_{Ci}(t) + D_{Ci}(r_i + y_i) \end{aligned} \quad (15)$$

where  $x_{Ci} \in R^{n_{ci}}$  is the state vector of the controller and also  $\sum_{i=1}^N n_{ci} = n_c$ . It is simple to show that designing the dynamical output feedback for the subsystem can be converted to design of a static controller,  $\tilde{K}_i$  for the augmented subsystem with the following state space equation (Jamshidi, 1997; Liu and Patton, 1998):

$$\dot{\tilde{x}}_i(t) = (\tilde{A}_{ii} + \tilde{B}_{ii}\tilde{K}_i\tilde{C}_{ii})\tilde{x}_i(t) + \tilde{M}_i r_i \quad (16)$$

where:

$$\begin{aligned} \tilde{A}_{ii} &= \begin{bmatrix} A_{ii} & 0 \\ 0 & 0 \end{bmatrix}, \tilde{B}_{ii} = \begin{bmatrix} B_{ii} & 0 \\ 0 & I \end{bmatrix} \\ \tilde{C}_{ii} &= \begin{bmatrix} C_{ii} & 0 \\ 0 & I \end{bmatrix}, \tilde{M}_i = \begin{bmatrix} B_{ii}D_{Ci} \\ B_{Ci} \end{bmatrix} \\ \tilde{x}_i(t) &= \begin{bmatrix} x_i(t) \\ x_{Ci}(t) \end{bmatrix}, \tilde{K}_i = \begin{bmatrix} D_{Ci} & C_{Ci} \\ B_{Ci} & A_{Ci} \end{bmatrix} \end{aligned} \quad (17)$$

Applying the decentralized controller  $K(s) = \text{diag}\{K_i(s)\}$ , the next theorem on the overall stability can be proved.

Theorem 4.1: Assuming the decentralized controller  $K(s) = \text{diag}\{K_i(s)\}$  stabilizes the diagonal system  $G_d(s)$ , where  $G_d(s) = \text{diag}\{G_{di}(s)\}$  and  $G_{di}(s)$  is the transfer function of the  $i$ -th isolated subsystem, given by equation (4), then  $K(s)$  stabilizes  $G(s)$  if

$$\begin{aligned} \max_{j=1, \dots, n_i + n_{ci}} \text{Re}(\lambda_j(\tilde{A}_{ii} + \tilde{B}_{ii}\tilde{K}_i\tilde{C}_{ii})) &< -\max\{\theta_m, \theta_m^{1/p}\} \\ i &= 1, \dots, L \end{aligned} \quad (18)$$

where

$$\theta_m = \max_{i=1, \dots, L} \|H\|_2 \sum_{k=0}^{p-1} \|N_i\|_2^k \quad (19)$$

$H = A - \text{diag}\{A_{ii}\}$ ,  $\text{Re}(\cdot)$  is the real part of  $(\cdot)$  and  $N_i$  is obtained from the Schur decomposition of  $\tilde{A}_{ii} + \tilde{B}_{ii}\tilde{K}_i\tilde{C}_{ii}$  according to (7). Also  $p$  is the smallest positive integer such that  $|N_i|^p = 0$ .

In addition if:

$$\begin{aligned} \max_{j=1, \dots, n_i + n_{ci}} \text{Re}(\lambda_j(\tilde{A}_{ii} + \tilde{B}_{ii}\tilde{K}_i\tilde{C}_{ii})) &< -\alpha - \max\{\theta_m, \theta_m^{1/p}\} \\ i &= 1, \dots, L \end{aligned} \quad (20)$$

and:

$$\min_{j=1, \dots, n_i + n_{ci}} \operatorname{Re}(\lambda_j(\tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii})) > -\beta + \max\{\theta, \theta^{1/p}\} \quad (21)$$

the closed-loop poles of the overall system will be placed in the following region:

$$-\beta < \operatorname{Re}(\mu_k) < -\alpha \quad (22)$$

where  $\mu_k$  is the  $k$ -th eigenvalue of the overall closed loop system.

Proof: Without loss of generality and for the sake of simplicity, assume that there is no external reference input. Considering equations (16), the overall closed-loop system under the decentralized controller has the following state space equations:

$$\dot{\tilde{x}}(t) = (\tilde{A}_d + \tilde{B} \tilde{K} \tilde{C}) \tilde{x}(t) + \tilde{H} \tilde{x}(t) \quad (23)$$

where

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_2 \end{bmatrix} \quad \tilde{A}_d = \operatorname{diag}\{\tilde{A}_{ii}\} \quad \tilde{B} = \operatorname{diag}\{\tilde{B}_{ii}\} \\ \tilde{C} = \operatorname{diag}\{\tilde{C}_{ii}\} \quad \tilde{K} = \operatorname{diag}\{\tilde{K}_i\} \quad (24)$$

and:

$$\tilde{H} = \begin{bmatrix} 0 & 0 & A_{12} & 0 & \cdots & A_{1L} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ A_{21} & 0 & 0 & 0 & \cdots & A_{2L} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{L1} & 0 & A_{L2} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (25)$$

In Theorem 3.2

$$E = \tilde{H} \quad (26)$$

therefore the  $k$ -th eigenvalue of the overall closed-loop system satisfies the following relation:

$$\min_{j=1, \dots, n+n_{ci}} |\mu_k - \lambda_j| \leq \max(\theta, \theta^{1/p}) \quad (27)$$

where  $\lambda$  is an eigenvalue of closed-loop diagonal state matrix  $(\tilde{A}_d + \tilde{B} \tilde{K} \tilde{C})$ ,  $\mu_k$  is an eigenvalue of the overall closed-loop state matrix  $(\tilde{A}_d + \tilde{H} + \tilde{B} \tilde{K} \tilde{C})$ ,

$$\theta = \|\tilde{H}\|_2 \sum_{k=0}^{p-1} \|N_d\|_2^k \quad (28)$$

and  $N_d$  is the related matrix, obtained from Schur decomposition of the matrix  $(\tilde{A}_d + \tilde{B} \tilde{K} \tilde{C})$ . Since  $\|\tilde{H}\|_2 = \|H\|_2$ , equation (28) can be written as:

$$\theta = \|H\|_2 \sum_{k=0}^{p-1} \|N_d\|_2^k \quad (29)$$

Considering  $|\operatorname{Re}(\mu_k - \lambda_j)| \leq |\mu_k - \lambda_j|$ , condition (27) can be written as:

$$\operatorname{Re}(\lambda_j) - \max\{\theta, \theta^{1/p}\} < \operatorname{Re}(\mu_k) < \operatorname{Re}(\lambda_j) + \max\{\theta, \theta^{1/p}\} \\ j = 1, \dots, n + n_c \quad (30)$$

for the specified  $\lambda_j$ . In order to stabilize the overall closed loop system, the following conditions must be satisfied:

$$\operatorname{Re}(\mu_k) < 0 \quad k = 1 \dots n + n_c \quad (31)$$

Therefore sufficient conditions for overall stability can be given as:

$$\operatorname{Re}(\lambda_j) + \max\{\theta, \theta^{1/p}\} < 0 \quad j = 1, \dots, n + n_c \quad (32)$$

Since  $(\tilde{A}_d + \tilde{B} \tilde{K} \tilde{C})$  is block diagonal, (32) can be satisfied if the following conditions are satisfied for the  $i$ -th isolated subsystem:

$$\max_{j=1, \dots, n_i + n_{ci}} \operatorname{Re}(\lambda_j(\tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii})) < -\max\{\theta, \theta^{1/p}\} \quad (33)$$

In a similar way, in order to place the overall closed-loop poles in the following region

$$-\beta < \operatorname{Re}(\mu_k) < -\alpha \quad (34)$$

it suffices to have:

$$\max_{j=1, \dots, n_i + n_{ci}} \operatorname{Re}(\lambda_j(\tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii})) < -\alpha - \max\{\theta, \theta^{1/p}\} \quad (35)$$

and

$$\min_{j=1, \dots, n_i + n_{ci}} \operatorname{Re}(\lambda_j(\tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii})) > -\beta + \max\{\theta, \theta^{1/p}\} \quad (36)$$

Considering the Schur decomposition for the  $i$ -th isolated subsystem as given by

$$Q_i^H (\tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii}) Q_i = D_i + N_i \quad (37)$$

it is simple to show that the Schur decomposition for the diagonal system may have the following form:

$$Q_d^H (\tilde{A}_d + \tilde{B} \tilde{K} \tilde{C}) Q_d = N_d + D_d \quad (38)$$

where:

$$Q_d = \operatorname{diag}(Q_1, Q_2, \dots, Q_L), \\ N_d = \operatorname{diag}(N_1, N_2, \dots, N_L)$$

$$D_d = \text{diag}(D_1, D_2, \dots, D_L) \quad (39)$$

$$\|N_d\|_2 = \max_{i=1, \dots, L} \|N_i\|_2$$

From the above relations it is simple to show that

$$\theta = \max_{i=1, \dots, L} \|H\|_2 \sum_{k=0}^{p-1} \|N_i\|_2^k \quad (40)$$

and therefore  $\theta$  is equal to  $\theta_m$ , which is defined by (19). So, from (33) it can be concluded that if condition (18) is satisfied the overall stability is guaranteed. In a similar way, conditions (20) and (21) can be derived.  $\square$

Remark: It is also possible to drive similar conditions for the imaginary part of the closed-loop poles.

The final purpose is to provide a computational procedure for robust regional pole assignment via decentralized control. In order to have robustness of closed-loop poles against perturbations, several eigenvalue sensitivity indexes are introduced (Wilkinson, 1965; Kautsky, *et al.*, 1985; Liu and Patton, 1998). They are usually based on the spectral condition number of the closed-loop modal matrix. Due to the fact that it is difficult to handle the modal matrix, a new robustness index is proposed.

It is shown that if the set of eigenvectors of the system are assigned to be orthogonal, the closed-loop system will be well conditioned and the sensitivity of the closed-loop eigenvalues against perturbations and parameter variations of system will be minimized (Liu and Patton, 1998). Normal matrices have complete set of orthogonal eigenvectors (Strang, 1986). Therefore, based on theorem 3.2, the following function can be used as an eigenvalue sensitivity index for matrix A:

$$J = \left| \sum_{j=1}^{n_i+n_{ci}} |\lambda_j(A)|^2 - \|A\|_F^2 \right| \quad (42)$$

The suggested methodology for robust decentralized pole assignment for large scale systems is explained in the following algorithm.

Algorithm 4-1:

1) Select parameters  $\alpha$  and  $\beta$  such that:

$$2\|H\|_2 < \beta - \alpha \quad (43)$$

Note that, it is only a necessary condition for the problem to have a solution. If (43) is not satisfied, (20) and (21) will not be consistent

2) For each isolated subsystem the eigenvalues should be selected such that

$$\|H\|_2 - \beta < \text{Re}(\lambda_j) < -\|H\|_2 - \alpha \quad j = 1, \dots, n_i + n_{ci} \quad (44)$$

The above condition is another necessary condition for the problem.

3) for each isolated subsystem, minimize the following cost function

$$J_i = \left| \sum_{j=1}^{n_i+n_{ci}} |\lambda_j|^2 - \left\| \tilde{A}_{ii} + \tilde{B}_{ii} \tilde{K}_i \tilde{C}_{ii} \right\|_F^2 \right| \quad i = 1, \dots, L \quad (45)$$

subject to the conditions (20) and (21).

4) if the optimization problem does not have a solution, change  $\alpha$  and  $\beta$ , and go to step 1.

Since the algorithm tries to normalize the closed-loop matrix of each isolated subsystem (based on theorem 3.2), the eigenvectors of the closed-loop subsystems are assigned to be orthogonal. Therefore the closed-loop eigenvalues of the diagonal system has minimum sensitivity to perturbations of the parameters of the diagonal system. In order to perform robustness to perturbation of the interactions (off diagonal elements), the upper bound of the interactions is considered and relation (9) is modified such that:

$$\theta_m = \max_{i=1, \dots, L} \|H + \Delta H\|_2 \sum_{k=0}^{p-1} \|N_i\|_2^k \quad (46)$$

where  $\Delta H$  is the upper bound of perturbation for the matrix  $H$ .

Since this procedure does not need to calculate the eigenvectors, in comparison with many other similar methods, the proposed method has less computational burden.

## 5. ILLUSTRATIVE EXAMPLE

Consider the system whose dynamics are described by

$$A = \begin{bmatrix} 0.5 & -0.5 & 2 & 0.1 \\ 2 & -3.5 & 1 & -2.1 \\ 1 & 0.5 & 0.5 & 1 \\ -2 & 1.7 & 1 & -3.5 \end{bmatrix}, \quad B = \begin{bmatrix} -3 & 0 \\ -1.5 & 0 \\ 0 & 1 \\ 0 & 0.2 \end{bmatrix},$$

$$C = I_{4 \times 4}$$

The system is unstable. The objective is to design a decentralized controller such that the real parts of the closed-loop eigenvalues are less than  $\alpha = -0.2$  and more than  $\beta = -7$ . Since  $\|H\|_2 = 2.6675$ , condition (42) is satisfied. Using the method proposed in section 6, the decentralized controller:

$$K = \begin{bmatrix} -1.3187 & 0.11067 & 0 & 0 \\ 0 & 0 & 4.0778 & 0.95421 \end{bmatrix}$$

assigns the overall closed-loop poles at  $\{-5.2783, -3.4372 \pm 2.0211i, -1.9059\}$  and the design objectives are satisfied. In this example  $\theta_1 = 3.1743$  and  $\theta_2 = 3.0374$ .

To solve the optimization problem, the Genetic Algorithms is used, which is used previously for the purpose of eigenstructure assignment (Patton and Liu, 1994; Esna Ashari and Khaki Sedigh, 2004). The closed-loop eigenvalues of subsystems are considered as the optimization parameters. Then at each step, the possible solutions for the eigenvalues of the isolated subsystems (chromosomes) will be produced using the random search operations. They are selected such that (44) is satisfied. Note that the chromosomes that are out of this range don't satisfy (20) and (21), obviously. After that, matrix  $K_i$  is computed using any output feedback pole assignment method according to the selected eigenvalues for the  $i$ -th subsystem. Then the cost function (45) must be computed and a new generation of chromosomes should be created. Employing the genetic algorithms for the optimization problem, the global optimal solution will be calculated easily.

Note that for the isolated subsystems:

$$\kappa(T_i) = 38.0263, 1.8312 \quad i=1,2$$

where  $\kappa(\cdot)$  denotes the condition number of  $(\cdot)$ . Also  $T_i$  is the modal matrix for the  $i$ -th subsystem, which is composed of the eigenvectors of the subsystem. It can be shown that in spite of the stability of the overall closed-loop system, the sufficient condition proposed in (Labibi, *et al.*, 2003) is not satisfied. It shows that the condition in (Labibi, *et al.*, 2003) is more conservative than the new condition for this example.

## 6. CONCLUSION

This paper has introduced a new approach to design a robust decentralized controller for large-scale systems. For each isolated subsystem, sufficient conditions are achieved, which guarantee that the closed loop poles of the overall system will be assigned in the desired region. It is shown that by assigning the eigenstructure of each isolated subsystem via output feedback or state feedback appropriately, these conditions can be satisfied and the closed-loop state matrices become normal. Since the eigenvectors of the subsystems are orthogonal, the closed-loop system has minimum sensitivity to

perturbations of the parameters of the system. Also a new robustness index is proposed, which leads to computationally effective methods for robust decentralized design.

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