FUTURE REFERENCE TRAJECTORY IMPROVEMENT IN SELF-TUNING I-PD CONTROLLER BASED ON GENERALIZED PREDICTIVE CONTROL LAW

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Abstract: In this paper a new design method of a self-tuning I-PD controller is proposed. The I-PD controller is designed based on a generalized predictive control (GPC) law. By using a conventional design method of the I-PD controller based on the GPC law, it is impossible to introduce future reference trajectory into the I-PD controller. The future reference trajectory of the GPC approaches a set-point value gradually from a present output, that is one of the features of the GPC. The proposed method can introduce the advantage of the GPC into the I-PD controller exactly because the proposed method expresses the future reference trajectory suitably when the set-point value is given as a step type signal.
This paper shows that the structure of a PID controller based on the GPC law becomes an I-PD type by using the proposed method and that a design parameter of the GPC which adjusts the future reference trajectory influences only an integral time of the I-PD controller.
In order to illustrate the effectiveness of the newly proposed method, numerical examples are shown.

Keywords: PID control, Generalized predictive control, Self-tuning control, Integral action

1. INTRODUCTION

This paper proposes a tuning method of proportional, integral and derivative (PID) parameters in an I-PD controller automatically. In this paper the PID parameters are decided by comparing the I-PD controller with a generalized predictive control (GPC) law (Clarke *et al.*, 1987; Camacho and Bordons, 1999). Using the relation of the GPC law to the I-PD controller many design methods have been already proposed. Ohshima et al. (Ohshima *et al.*, 1991) and Yamamoto et al. (Yamamoto *et al.*, 1994) showed that the control structure of the GPC includes the I-PD because the I-PD controller is equal to the GPC law on special conditions. Asano et al. (Asano *et al.*, 1998) proposed a design method of approximating the GPC law by the I-PD controller. Miller et al. designed the I-PD controller based on the GPC law including a steady state prediction which can reduce computation load for a long prediction (Miller *et al.*, 1995; Miller *et al.*, 1996; Miller *et al.*, 1999). Moradi et al. proposed a new reference trajectory different from that of the original GPC, and a PID controller was designed using the relation of the modified GPC (Moradi *et al.*, 2001).

In this study it is assumed that a controlled plant can be expressed as first-order lag plus dead-time or second-order lag plus dead-time. Because of a proposed control structure, the former is an I-P control and the latter is an I-PD control. Most chemical processes can be expressed as first-order lag or second-order lag plus dead-time. Therefore, it is possible to obtain good control performance under the condition.

To design the I-PD controller based on the GPC law, coefficient polynomials of output, control input and reference input in the GPC law need to be replaced with components in the I-PD controller. In this case, there is a problem about the order of the I-PD controller. The orders of the coefficient polynomials of the output, control input and reference input which constitute the I-PD controller are 2, 0 and 0 respectively. However, since the coefficient polynomials of the GPC law are higher than those of the I-PD controller generally, it is difficult to approximate the GPC law by the I-PD controller. The problem about the coefficient polynomial of the output is solvable by assuming that the order of controlled plant is 2 or less, and by replacing the coefficient polynomials of the control and reference inputs respectively by the steady state gains (Asano *et al.*, 1998). However, because the I-PD controller is designed using the steady state gains, the designed I-PD controller is inferior to the original GPC law. Thus, it is necessary to obtain better approximation about the coefficient polynomials of the control and reference inputs. In order to improve approximation about the coefficient of the control input, Asano and Yamamoto proposed the design method of using a time-varying proportional gain (Asano and Yamamoto, 2001). However, about the coefficient of the reference input, an improvement method is not yet proposed. The order of the coefficient polynomial of the reference input in the GPC law depends on a predictive horizon which is one of the design parameters of the GPC. If the constraint that the maximum predictive horizon is set by the minimum predictive horizon + 2 or less is assumed and if the I-PD or a PID controller is constructed in two-degree-of-freedom system, the GPC law is approximated by the I-PD or the PID controller correctly. However since it restricts control performance, it is not acceptable to add the constraint to the predictive horizon of the GPC. Hence, conventional methods designed the I-PD controller based on the GPC law by approximating the coefficient polynomial of the reference input by a steady state gain (Asano *et al.*, 1998; Asano and Yamamoto, 2001; Miller *et al.*, 1995; Miller *et al.*, 1996; Miller *et al.*, 1999).

Reference trajectory of the GPC, which approaches a set-point value gradually from a present output, is effective in reducing overshoot. The reference trajectory can be adjusted by the designable parameter "α". In this paper it is discussed how this parameter α is introduced into the I-PD controller. By the conventional design methods of the I-PD controller based on the GPC law (Asano *et al.*, 1998; Asano and Yamamoto, 2001; Miller *et al.*, 1995; Miller *et al.*, 1996; Miller *et al.*, 1999), it is impossible to make use of this advantage since the future reference trajectory is replaced by a constant set-point value and the coefficient polynomial of the reference input is approximated by the steady state gain. For the reasons stated above, in the conventional methods the I-PD or I-P controller is used instead of the PID controller and α is fixed as 0. On the other hand in the proposed method, it is made clear that the PID based on the GPC becomes the I-PD type essentially even if the design parameter α is more than 0, and that α influences only an integral time of the I-PD controller.

Finally, to illustrate the effectiveness of the proposed method, numerical simulations are shown. From these simulation results, it is seen that the increase of α makes the integral time increase in monotone and that the increase of integral time makes the overshoot decrease in monotone.

2. PLANT MODEL AND I-PD CONTROLLER

Consider the following discrete-time model

$$A[z^{-1}]y[k] = B[z^{-1}]u[k-1] + \frac{\xi[k]}{\Delta} \tag{1}$$

$$A[z^{-1}] = 1 + a_1 z^{-1} + a_2 z^{-2} \tag{2}$$

$$B[z^{-1}] = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m} \tag{3}$$

$$\Delta = 1 - z^{-1} \tag{4}$$

where $y[k]$, $u[k]$ and $\xi[k]$ are the output, control input and disturbance, respectively. z^{-1} denotes the backward shift operator. When a dead-time is the positive integer, the leading elements of the polynomial $B[z^{-1}]$ are 0 (Clarke *et al.*, 1987). Generally a process control system is of high-order. Because it is hard to identify higher-order elements correctly and the system can be represented and approximated by second-order lag plus dead-time, it is acceptable that the system (1) is of order less than 2 (Yamamoto *et al.*, 1999). In that case it is also hard to identify the dead-time correctly. By using the high-order polynomial $B[z^{-1}]$, an unknown dead-time is represented in (1) (Yamamoto *et al.*, 1999).

The structure of the I-PD controller is given by

$$\Delta u[k] = k_c \frac{T_s}{T_I}(r - y[k]) - k_c \left(\Delta + \frac{T_D}{T_s}\Delta^2\right) y[k] \tag{5}$$

$$= C[1]r - C[z^{-1}]y[k] \tag{6}$$

$$C[z^{-1}] = k_c \left(\Delta + \frac{T_s}{T_I} + \frac{T_D}{T_s}\Delta^2 \right) \tag{7}$$

where k_c, T_I and T_D are the proportional gain, integral time and derivative time, respectively. The sampling interval is denoted as T_s and r is the step type set-point value. The purpose of this study is to tune the PID parameters automatically in order to make the output follow the set-point r.

3. DERIVATION OF THE GENERALIZED PREDICTIVE CONTROL LAW

The GPC law is derived by minimizing the following performance index (Clarke *et al.*, 1987)

$$\begin{aligned} J = E \Bigg[& \sum_{j=N_1}^{N_2} \{y[k+j] - w[k+j]\}^2 \\ & + \sum_{j=1}^{N_u} \lambda_j \{\Delta u[k+j-1]\}^2 \Bigg] \end{aligned} \tag{8}$$

where, N_1, N_2, N_u and λ_j are the minimum predictive horizon, maximum predictive horizon, control horizon and weighting parameter toward deviation of the control input, respectively. Although the conventional design methods of the I-PD controller based on the GPC law (Asano *et al.*, 1998; Miller *et al.*, 1999; Sato *et al.*, 2005) uses $w[k]$ in place of $w[k+j]$ in (8), in this paper the following future reference input $w[k+j]$ is used in order to obtain control performance as good as the GPC

$$w[k] = y[k] \tag{9}$$
$$w[k+j] = (1-\alpha)r + \alpha w[k+j-1] \tag{10}$$
$$0 \le \alpha < 1. \tag{11}$$

(9)∼(11) makes it possible that the reference trajectory is adjusted with the design parameter α in the same way as the original GPC (Clarke *et al.*, 1987).

To derive the GPC law, the polynomials $F_j[z^{-1}]$, $R_j[z^{-1}]$ and $S_j[z^{-1}]$ are calculated by using the following Diophantine equation

$$1 = \Delta A[z^{-1}]E_j[z^{-1}] + z^{-j}F_j[z^{-1}] \tag{12}$$
$$E_j[z^{-1}] = 1 + e_1 z^{-1} + \cdots + e_{j-1}z^{-(j-1)} \tag{13}$$
$$F_j[z^{-1}] = f_{j,0} + f_{j,1}z^{-1} + f_{j,2}z^{-2} \tag{14}$$
$$E_j[z^{-1}]B[z^{-1}] = R_j[z^{-1}] + z^{-j}S_j[z^{-1}] \tag{15}$$
$$R_j[z^{-1}] = r_0 + r_1 z^{-1} + \cdots + r_{j-1}z^{-(j-1)} \tag{16}$$
$$\begin{aligned} S_j[z^{-1}] = & s_{j,0} + s_{j,1}z^{-1} \\ & + \cdots + s_{j,m-1}z^{-(m-1)}. \end{aligned} \tag{17}$$

Because the GPC uses only the first element of control input series, the GPC law is given by

$$\Delta u[k] = \frac{P[z^{-1}]}{G[z^{-1}]}w[k+N_2] - \frac{F[z^{-1}]}{G[z^{-1}]}y[k] \tag{18}$$

where the coefficient polynomials of the control law are given by the following

$$\begin{aligned} P[z^{-1}] = & p_{N_2} + p_{N_2-1}z^{-1} \\ & + \cdots + p_{N_1}z^{-(N_2-N_1)} \end{aligned} \tag{19}$$
$$[p_{N_1} \ \cdots \ p_{N_2}] = [1 \ 0 \ \cdots \ 0](R^T R + \Lambda)^{-1}R \tag{20}$$
$$\Lambda = \mathrm{diag}\{\lambda_1, \cdots, \lambda_{N_u}\} \tag{21}$$
$$R = \begin{bmatrix} r_{N_1-1} & \cdots & r_0 & & 0 \\ \vdots & & & \ddots & \\ r_{N_u-1} & & & & r_0 \\ \vdots & & & & \vdots \\ r_{N_2-1} & \cdots & \cdots & \cdots & r_{N_2-N_u} \end{bmatrix} \tag{22}$$
$$G[z^{-1}] = 1 + z^{-1}S[z^{-1}] \tag{23}$$
$$S[z^{-1}] = p_{N_1}S_{N_1}[z^{-1}] + \cdots + p_{N_2}S_{N_2}[z^{-1}] \tag{24}$$
$$F[z^{-1}] = p_{N_1}F_{N_1}[z^{-1}] + \cdots + p_{N_2}F_{N_2}[z^{-1}] \tag{25}$$
$$= f_0 + f_1 z^{-1} + f_2 z^{-2}. \tag{26}$$

4. DESIGN METHOD OF THE I-PD CONTROLLER BASED ON THE GPC LAW

By using the design parameter α which can adjust a rise-time of the output and is one of the features of the GPC, to design the I-PD controller based on the GPC law, the reference input $w[k+j]$ is rewritten by the following equation using (9) and (10)

$$w[k+j] = \alpha^j y[k] + (1-\alpha^j)r. \tag{27}$$

It is shown that the future reference input is decided by the set-point value and the present output. The use of (27) gives the following

$$P[z^{-1}]w[k+N_2] = p_r r + p_y y[k] \tag{28}$$
$$p_r = \sum_{j=N_1}^{N_2}(1-\alpha^j)p_j \tag{29}$$
$$p_y = \sum_{j=N_1}^{N_2}\alpha^j p_j \tag{30}$$

then using the above equations the GPC law (18) is rewritten by the following equation

$$\Delta u[k] = \frac{p_r}{G[z^{-1}]}r - \frac{F_p[z^{-1}]}{G[z^{-1}]}y[k] \tag{31}$$

where the polynomial $F_p[z^{-1}]$ is given by

$$F_p[z^{-1}] = f_0 - p_y + f_1 z^{-1} + f_2 z^{-2}. \tag{32}$$

In the GPC law (31) the steady state gain of the polynomial $G[z^{-1}]$ is defined as ν

$$\nu = G[1]. \tag{33}$$

and $G[z^{-1}]$ is replaced with ν (Asano *et al.*, 1998). Then comparing (6) with (31), the PID parameters are obtained. The proposed I-PD controller is designed so that the following equation is satisfied

$$C[z^{-1}] = \frac{F_p[z^{-1}]}{\nu}. \tag{34}$$

It follows from (34) that the PID parameters are calculated by the following

$$k_c = -\frac{1}{\nu}(f_1 + 2f_2) \tag{35}$$

$$T_I = -\frac{f_1 + 2f_2}{f_0 - p_y + f_1 + f_2}T_s \tag{36}$$

$$T_D = -\frac{f_2}{f_1 + 2f_2}T_s. \tag{37}$$

It is seen from (36) that the design parameter α of the GPC influences the integral time only. Furthermore the following equation is satisfied

$$p_r = F_p[1]. \tag{38}$$

Hence the output can follow the set-point value without steady state error. Because p_r which is the coefficient of the set-point value is a constant, when the PID parameters are designed as (35) $\sim$ (37), the I-PD controller can be designed based on the GPC law, without approximation about the coefficient polynomial of the set-point value r.

If the coefficients of $A[z^{-1}]$ and $B[z^{-1}]$ are known, the PID parameters can be calculated through (35) $\sim$ (37). Since in this paper the coefficients are assumed to be unknown, the following recursive least squares identification law (Goodwin and Sin, 1984) is used to obtain the estimated values of the unknown coefficients of $A[z^{-1}]$ and $B[z^{-1}]$

$$\hat{\theta}[k] = \hat{\theta}[k-1] + \frac{\Gamma[k-1]\psi[k-1]}{1 + \psi^T[k-1]\Gamma[k-1]\psi[k-1]}\varepsilon[k] \tag{39}$$

$$\Gamma[k] = \Gamma[k-1] + \frac{\lambda\Gamma[k-1]\psi[k-1]\psi^T[k-1]\Gamma[k-1]}{1 + \lambda\psi^T[k-1]\Gamma[k-1]\psi[k-1]} \tag{40}$$

$$\varepsilon[k] = \Delta y[k] - \hat{\theta}[k-1]\psi[k-1] \tag{41}$$

$$\hat{\theta}[k] = [\hat{a}_1[k], \hat{a}_2[k], \hat{b}_0[k], \hat{b}_1[k], \cdots, \hat{b}_m[k]] \tag{42}$$

$$\psi[k-1] = [-\Delta y[k-1], -\Delta y[k-2], \Delta u[k-1], \Delta u[k-2], \cdots, \Delta u[k-m-1]] \tag{43}$$

$$\Gamma[0] = \alpha_\Gamma I, (0 < \alpha_\Gamma < \infty) \tag{44}$$

where λ denotes the forgetting factor $(0 < \lambda < 2)$ and $\Gamma[k]$ is the estimated covariance matrix.

5. NUMERICAL EXAMPLES

To show the effectiveness of the proposed method, numerical examples are conducted. In this section, two numerical examples are shown. The one is to confirm the advantage of introducing the design parameter α used in the GPC into the design of the I-PD controller. The other is to show the proposed self-tuning I-PD controller works well in the case that plant parameters are unknown.

5.1 Effect of introducing the design parameter "α"

A controlled plant is given by the following transfer function

$$G(s) = \frac{1}{s^2 + 1.6s + 1}e^{-s}. \tag{45}$$

Using the sampling interval $T_s = 1$[s] the continuous-time system (45) is transformed into the following discrete-time system

$$(1 - 0.74z^{-1} + 0.20z^{-2})y[k] = z^{-1}(0.29 + 0.17z^{-1})u[k-1]. \tag{46}$$

The length of simulation is 50 steps. The design parameters of the GPC: $N_1 = 1$, $N_2 = 5$, $N_u = 2$ and $\lambda_j = 1$ $(j = 1, 2)$. To confirm the influence of introducing the design parameter α, α is set from 0 to 1.0, in 0.1 increments. The proportional gain k_c and the derivative time T_D are independently designed to the choice of α, thus $k_c = 0.22$ and $T_D = 0.56$ are obtained.

The output results are shown in Fig. 1. Because the proposed method with $\alpha = 0$ is equivalent to the conventional one, the output response by using $\alpha = 0$ is also the result by the conventional method. In the case of $\alpha = 0, 0.1, 0.2$, the output responses overlap since the integral times shown in Fig. 2 are almost the same. Fig. 1 and Fig. 2 show that the larger α is, the smaller the overshoot of the output is. Because $\alpha = 1$ makes it impossible for the output to follow the reference input, the range of the integral time is obtained as $0.169 \leq T_I < 0.218$ in the simulation conditions. Since the conventional method fixes α as 0, the overshoot emerges. On the other hand since the proposed method designs the I-PD controller using the adjustable α, the overshoot is decreased.

The integral times corresponding to the value of each α are shown in Fig. 2. It is shown that the larger the value of α is, the larger the integral time is. We see from the results that the increase of α increases the integral time and adjusts the output response.

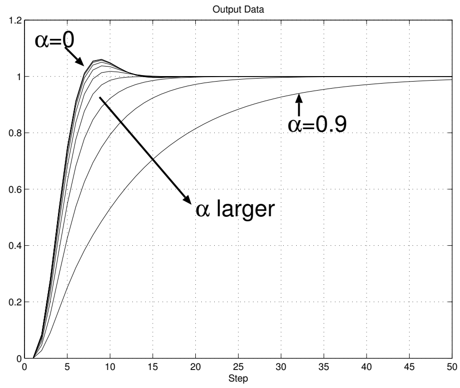

Fig. 1. Output result by using the proposed I-PD controller in the case of known plant parameters

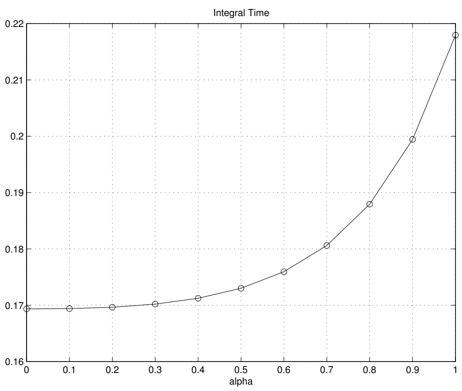

Fig. 2. Relationship between the design parameter α and the integral time

5.2 Self-Tuning Case

In this subsection, consider the following plant, the parameters of which are unknown

$$\begin{aligned}&(1 - 0.74z^{-1} + 0.20z^{-2})y[k]\\&= z^{-1}(0.29 + 0.17z^{-1})u[k-1] + \frac{\xi[k]}{\Delta}. \quad (47)\end{aligned}$$

As for (47), the integrated noise is added to (46).

Simulations are conducted under the conditions: simulation length is 200 steps, variance of random disturbance $\xi[k]$ is 10^{-6}, the the set-point value is a rectangular wave with amplitude 1.0 and period of 50 steps and the recursive least square identification law having reset with the forgetting factor 0.99 is used. The initial value of the estimated covariance matrix is $10^3 I$ and the initial values of the identified coefficients are the nominal values which are multiplied true values of (47) by 0.5. The design parameters of the GPC: $\alpha = 0.6$ and the others are the same as 5.1.

The output, control input, identified polynomials $A[z^{-1}]$ and $B[z^{-1}]$ and obtained PID parameters are shown in Fig. 3 $\sim$ Fig. 7. It is seen that good control performance is obtained in the case of which the plant parameters are also unknown.

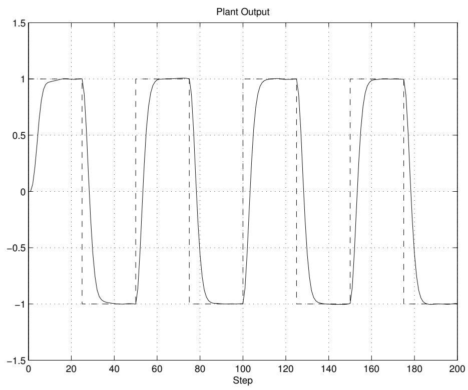

Fig. 3. Output result by using the proposed self-tuning I-PD controller

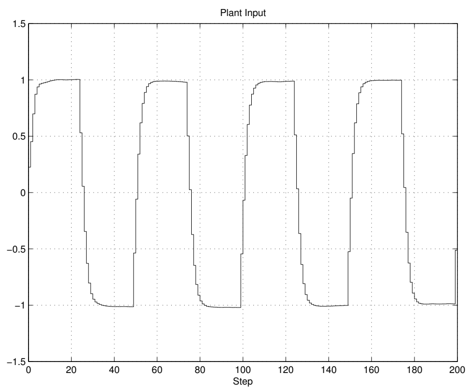

Fig. 4. Input result by using the proposed self-tuning I-PD controller

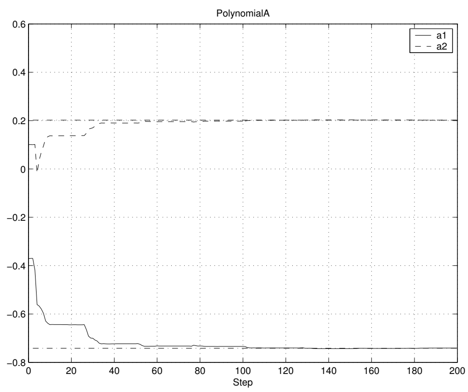

Fig. 5. Identified parameters of the polynomial $A[z^{-1}]$

6. CONCLUSION

Conventional methods (Asano *et al.*, 1998; Asano and Yamamoto, 2001; Miller *et al.*, 1995; Miller *et al.*, 1996; Miller *et al.*, 1999) designed I-PD controllers based on a GPC law by replacing a coefficient polynomial of a reference input in the GPC law with the steady state gain of that. On the other hand because the proposed method uses reference trajectory which approaches a set-point value gradually from a present output, the I-PD controller can be designed based on the GPC law without approximation about the coefficient

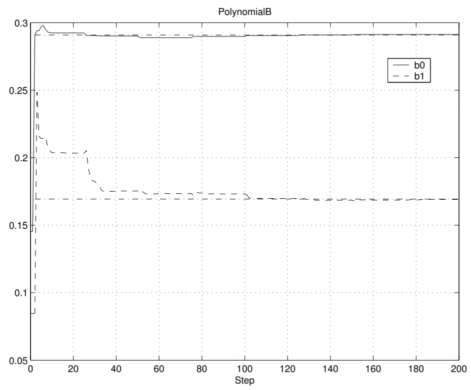

Fig. 6. Identified parameters of the polynomial $B[z^{-1}]$

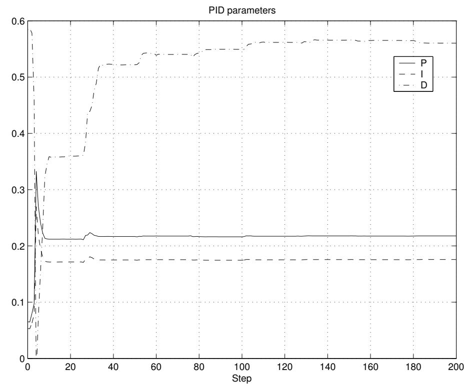

Fig. 7. Obtained PID parameters

polynomial of the reference input. We also have shown that the design parameter α of the GPC has an influence on only an integral time of the proposed I-PD controller and that a PID based on the GPC becomes the I-PD type essentially.

The proposed method approximates a coefficient polynomial of a control input in the GPC law by the steady state gain (Asano *et al.*, 1998). The future study will be apply Asano and Yamamoto's method (Asano and Yamamoto, 2001) for obtaining better approximation.

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