CONDITIONS OF OUTPUT STABILIZATION FOR UNCERTAIN DISCRETE TS FUZZY MODELS.

Alexandre KRUSZEWSKI, Thierry Marie GUERRA

LAMIH, UMR CNRS 8530, Université de Valenciennes et du Hainaut-Cambrésis
Le Mont Houy, 59313 Valenciennes Cedex 9, France
Tel: (+33) 3 27 51 14 87 fax: (+33) 3 27 51 13 18
E-Mail: {akruszew,Guerra}@univ-valenciennes.fr

Abstract: Output stabilization of uncertain discrete-time non linear models via an observer is a problem that can be considered through a Takagi-Sugeno framework. This work provides systematic design procedures by using direct Lyapunov's method with a non quadratic Lyapunov's function. This leads to LMI conditions that can be efficiently solved. This result always includes the quadratic case.

Keywords: fuzzy, output stabilization, observer design, stabilization, robust control.

1. INTRODUCTION

The stabilization of uncertain nonlinear models is a large domain. A way to get systematic procedures to derive control laws is to use the model in one of its Takagi-Sugeno (TS) (Takagi and Sugeno 1985) form. This way commonly uses the direct Lyapunov's method to get LMI (Linear Matrix Inequalities) conditions. Solutions that can satisfy such a problem can be found efficiently by using the interior point algorithm (Boyd et al. 1994). The first works used a quadratic Lyapunov function and a parallel distributed compensation (PDC) control law (Wang et al. 1996; Tanaka et al. 1998). Other Lyapunov functions were also proposed. In the continuous case a fuzzy Lyapunov function were used (Blanco, et al. 2001; Tanaka et al. 2001) but the presence of the derivative of the activation function makes the results "poor". Interesting works are related to piecewise Lyapunov functions (Johansson et al. 1999). Finally in the discrete case, some results are presented in (Guerra and Vermeiren 2004) which use a non quadratic fuzzy Lyapunov function that shares the same rules than the models. The use of this Lyapunov function ensures that the solutions obtained in the quadratic case are included.

The stabilization of uncertain models received an increasing attention. Several results are available in the continuous and the discrete case. Examples of results using norm bounded linear models are given in (Xie and De Souza 1992). For uncertain Takagi Sugeno models some results are available too. An example, in the continuous case with the utilization of a quadratic Lyapunov function, the stabilization via sate feedback is given in (Lee et al. 2001). And finally in the discrete case with or without state delays with the utilization of a non quadratic Lyapunov function, some results are given in (Guerra et al. 2004)

In this work, we are interested in the uncertain discrete Takagi Sugeno's fuzzy models case. A non quadratic Lyapunov's function is designed to obtain LMI conditions for the output stabilization of uncertain TS fuzzy models via an observer and a non PDC control law.

The paper is organized as follows. A first part presents the notations and some basic properties. These ones are mainly useful matrix properties that allow dealing with uncertainties for discrete models. A relaxation scheme for LMI conditions is also proposed (Liu and Zhang 2003). The main result is then presented. This one gives the LMI conditions to be satisfied for a control law together an observer to stabilize an uncertain discrete fuzzy model. They are obtained using a non quadratic Lyapunov function. The obtained results are shown to always include the quadratic case. At last an example is provided to show the effectiveness of the proposed approach.

2. NOTATIONS AND MATERIAL

We consider the following notations, with $h_i(\bullet) \geq 0$ scalar positive functions and matrices of the same dimension Y_i $i \in \{1,\dots,r\}$:

$$Y_z = \sum_{i=1}^{r} h_i(z(t)) Y_i, \; Y_{z+} = \sum_{i=1}^{r} h_i(z(t+1)) Y_i,$$

$$Y_{\bullet}^{-1} = \left(\sum_{i=1}^{r} h_i(\bullet) Y_i \right)^{-1}.$$

A star $(*)$ indicates a transpose quantity in a symmetric matrix. Congruence of a symmetric definite positive matrix $P = P^T > 0$ with Y corresponds to the following quantity: $YPY^T > 0$.

The next lemmas will be useful in the paper. The first one is well known (Xie and De Souza, 1992). The second one corresponds to a property given in (De Oliveira et al. 1999) and modified in the context of non-quadratic stabilization of discrete TS models (Guerra and Vermeiren 2004).

Lemma 1: With X, Y and $F = F^T > 0$ matrices of appropriate dimension the following inequality holds.

$$X^T Y + Y^T X \leq X^T F X + Y^T F^{-1} Y \tag{1}$$

Lemma (Schur's complement): With $P = P^T > 0$, $R > 0$ and X matrices of appropriate dimensions, the two following properties are equivalent:

(i) $P - X^T R^{-1} X > 0$ $\Leftrightarrow$ (ii) $\begin{bmatrix} P & (*) \\ X & R \end{bmatrix} > 0$

Lemma 2 (extension to the Schur's complement): With $P = P^T > 0$, $R > 0$ and X matrices of appropriate dimensions, the two following properties are equivalent.

(i) $P - X^T R X > 0$ (2)

(ii) There exists Ψ a matrix of appropriate dimensions such that:

$$\begin{bmatrix} P & (*) \\ \Psi X & \Psi + \Psi^T - R \end{bmatrix} > 0 \tag{3}$$

Proof: Sufficiency: If $P - X^T R X > 0$, then $P - X^T R R^{-1} R X > 0$ and with Schur's complement (3) is obtained with $\Psi = R$. Necessity: Using the congruence with the row full rank matrix $\begin{bmatrix} I & -X^T \end{bmatrix}$ on the expression (3) gives immediately the result.

Remark 1: If Ψ is under constraint, the sufficiency is not more true.

The models considered are nonlinear affine models. Takagi-Sugeno's models can exactly represent such models in a compact set of the state variables (Tanaka and Wang 2001). There exists a systematic way to put affine nonlinear models in the TS form:

$$x(t+1) = f(x(t)) + g(x(t)) u(t) \tag{4}$$

into:

$$x(t+1) = \sum_{i=1}^{r} h_i(z(t)) \left(A_i x(t) + B_i u(t) \right) \tag{5}$$

With $z(t)$ the premises vector which depends on state vector $x(t)$ and r represents the number of linear models. This last number grows exponentially according to the number of nonlinearities involved in the nonlinear model (Tanaka et al. 1998; Taniguchi et al. 2001). Note also that the TS representation of (4) is not unique (Taniguchi et al. 2001). By using this representation of the nonlinear models, the goal is to derive output stabilization conditions that only depend on the parameters of the linear models and the parameters of the control law and the observer. Excepted the convex sum property, no specific property of the membership functions $h_i(z(t))$ are used. This means that the conditions will always be sufficient ones. Then, another goal is to get the less conservative conditions, i.e.: find conditions that solve the biggest set of problems.

The obtained conditions derived from the stabilization of TS models are usually in the following form:

$$\Upsilon_{zz}^{z+} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{k=1}^{r} h_i(z(t)) h_j(z(t)) h_k(z(t+1)) \Upsilon_{ij}^{k} > 0 \tag{6}$$

A way to get LMI conditions from (6) is to use the following relaxation scheme (Liu and Zhang 2003) based on a first one due to (Kim and Lee 2000) which is a good compromise between conservatism and complexity. This one will be used in all the theorems for the stabilization conditions. In our case, it corresponds to the following writing, with:

$$Q_i^k = \left(Q_i^k \right)^T > 0,$$

$$Q_{ij}^k = \left(Q_{ji}^k \right)^T, \; i, j, k \in \{1,\dots,r\}, \; j > i$$

$$\Upsilon_{ii}^k > Q_i^k \qquad i, j \in \{1,\dots,r\} \tag{7}$$

$$\Upsilon_{ij}^k + \Upsilon_{ji}^k > Q_{ij}^k + Q_{ji}^k \quad j > i, \, i, j, k \in \{1,\dots,r\} \tag{8}$$

$$\Psi^k = \begin{bmatrix} Q_1^k & Q_{12}^k & \cdots & Q_{1r}^k \\ Q_{21}^k & Q_2^k & & \vdots \\ \vdots & & \ddots & Q_{(r-1)r}^k \\ Q_{r1}^k & \cdots & Q_{r(r-1)}^k & Q_r^k \end{bmatrix} > 0 \tag{9}$$

3. STATEMENT OF THE PROBLEM AND MAIN RESULT

Consider the following TS uncertain fuzzy model with measurable premises:

$$\begin{cases} x(t+1) = (A_z + \Delta A_z) x(t) + (B_z + \Delta B_z) u(t) \\ y(t) = C_z x(t) \end{cases}$$

With the uncertainties written as:

$$\begin{bmatrix} \Delta A_z & \Delta B_z \end{bmatrix} = \begin{bmatrix} Ha & Hb \end{bmatrix} \begin{bmatrix} \Delta a(t) & 0 \\ 0 & \Delta b(t) \end{bmatrix} \begin{bmatrix} Ea_z \\ Eb_z \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \Delta a^T(t) \Delta a(t) & 0 \\ 0 & \Delta b^T(t) \Delta b(t) \end{bmatrix} < \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{11}$$

Consider also the non PDC control law:

$$u(t) = -F_z G_z^{-1} \hat{x}(t) \tag{12}$$

and the following TS fuzzy observer:

$$\hat{x}(t+1) = A_z \hat{x}(t) + B_z u(t) + S_z^{-1} K_z \left(y(t) - \hat{y}(t) \right) \quad (13)$$

With: $\tilde{x} = x - \hat{x}$ the error of prediction we can write:

$$\begin{aligned}\tilde{x}(t+1) &= \left(A_z - S_z^{-1} K_z C_z + \Delta B_z F_z G_z^{-1} \right) \tilde{x}(t) \\ &\quad + \left(\Delta A_z - \Delta B_z F_z G_z^{-1} \right) x(t)\end{aligned} \quad (14)$$

The closed loop (TS model + control law + observer) is written as:

$$\begin{bmatrix} x_+ \\ \tilde{x}_+ \end{bmatrix} = \begin{bmatrix} \bar{A}_z - \bar{B}_z F_z G_z^{-1} & \bar{B}_z F_z G_z^{-1} \\ \Delta A_z - \Delta B_z F_z G_z^{-1} & \begin{pmatrix} A_z - S_z^{-1} K_z C_z \\ + \Delta B_z F_z G_z^{-1} \end{pmatrix} \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} \quad (15)$$

with $\bar{A}_z = A_z + \Delta A_z$ and $\bar{B}_z = B_z + \Delta B_z$.

The following Lyapunov function which is clearly a non quadratic one is chosen:

$$V(x, \tilde{x}) = x^T(t) G_z^{-T} P_z G_z^{-1} x(t) + \tilde{x}^T(t) \tilde{P}_z \tilde{x}(t) \quad (16)$$

Remark 2: This function is a Lyapunov one and the proof can be found in (Guerra and Vermeiren 2004).

The variation of the Lyapunov function (16) is negative if (19) holds (bottom of the page).

After congruence with $\begin{bmatrix} G_z^T & 0 \\ 0 & I \end{bmatrix}$ $\Delta V(x(t)) < 0$ is true if (20) holds (bottom of the page). Then by using the lemma 2, (20) is verified if (21) is (bottom of the page). The goal is to remove all terms that can not be put in a linear form. First the terms with G_z^{-1} is treated. Consider from (21) the following terms:

$$W = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & (*) & (*) \\ 0 & \begin{pmatrix} (B_z + \Delta B_z) F_z G_z^{-1} \\ + R_z \Delta B_z F_z G_z^{-1} \end{pmatrix} & 0 & 0 \\ 0 & S_z \Delta B_z F_z G_z^{-1} & 0 & 0 \end{bmatrix} \quad (17)$$

(17) can be written: $W = N + N^T$ where

$$N = \left\{ \begin{bmatrix} 0 \\ 0 \\ (B_z + \Delta B_z) F_z + R_z \Delta B_z F_z \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ S_z \Delta B_z F_z \end{bmatrix} \right\} \begin{bmatrix} 0 \\ G_z^{-T} \\ 0 \\ 0 \end{bmatrix}^T \quad (18)$$

Then using lemma 1 on $N + N^T$, (21) is verified if (22) is satisfied (bottom of the page) with $\Gamma > 0$ and:

$$\Omega_{(3,3)} = -G_{z+} - G_{z+}^T + P_{z+} + \begin{pmatrix} (B_z + \Delta B_z) F_z \\ + R_z \Delta B_z F_z \end{pmatrix} \Gamma_1^{-1} \begin{pmatrix} (B_z + \Delta B_z) F_z \\ + R_z \Delta B_z F_z \end{pmatrix}^T$$

$$\Omega_{(4,4)} = -S_z - S_z^T + \tilde{P}_{z+} + S_z \Delta B_z F_z \Gamma_2^{-1} F_z^T \Delta B_z^T S_z^T$$

Using lemma 2 for the expression $G_z^{-T} (\Gamma_1 + \Gamma_2) G_z^{-1}$ (with $X = G_z^{-1}$ and $\Psi = G_z$) and the Schur's complement for the last parts of terms $\Omega_{(3,3)}$ and $\Omega_{(4,4)}$ leads to the condition (23) (top of the next page).

$$\begin{aligned}&\begin{bmatrix} A_z + \Delta A_z - (B_z + \Delta B_z) F_z G_z^{-1} & (B_z + \Delta B_z) F_z G_z^{-1} \\ \Delta A_z - \Delta B_z F_z G_z^{-1} & A_z - S_z^{-1} K_z C_z + \Delta B_z F_z G_z^{-1} \end{bmatrix}^T \times \begin{bmatrix} G_{z+}^{-T} P_{z+} G_{z+}^{-1} & 0 \\ 0 & \tilde{P}_{z+} \end{bmatrix} \\ &\times \begin{bmatrix} A_z + \Delta A_z - (B_z + \Delta B_z) F_z G_z^{-1} & (B_z + \Delta B_z) F_z G_z^{-1} \\ \Delta A_z - \Delta B_z F_z G_z^{-1} & A_z - S_z^{-1} K_z C_z + \Delta B_z F_z G_z^{-1} \end{bmatrix} - \begin{bmatrix} G_z^{-T} P_z G_z^{-1} & 0 \\ 0 & \tilde{P}_z \end{bmatrix} < 0\end{aligned} \quad (19)$$

$$\begin{aligned}&\begin{bmatrix} (A_z + \Delta A_z) G_z - (B_z + \Delta B_z) F_z & (B_z + \Delta B_z) F_z G_z^{-1} \\ \Delta A_z G_z - \Delta B_z F_z & A_z - S_z^{-1} K_z C_z + \Delta B_z F_z G_z^{-1} \end{bmatrix}^T \times \begin{bmatrix} G_{z+}^{-T} P_{z+} G_{z+}^{-1} & 0 \\ 0 & \tilde{P}_{z+} \end{bmatrix} \\ &\times \begin{bmatrix} (A_z + \Delta A_z) G_z - (B_z + \Delta B_z) F_z & (B_z + \Delta B_z) F_z G_z^{-1} \\ \Delta A_z G_z - \Delta B_z F_z & A_z - S_z^{-1} K_z C_z + \Delta B_z F_z G_z^{-1} \end{bmatrix} - \begin{bmatrix} P_z & 0 \\ 0 & \tilde{P}_z \end{bmatrix} < 0\end{aligned} \quad (20)$$

$$\begin{bmatrix} P_z & 0 & (*) & (*) \\ 0 & \tilde{P}_z & (*) & (*) \\ \begin{pmatrix} (A_z + \Delta A_z) G_z - (B_z + \Delta B_z) F_z \\ + R_z \Delta A_z G_z - R_z \Delta B_z F_z \end{pmatrix} & \begin{pmatrix} (B_z + \Delta B_z) F_z G_z^{-1} + R_z A_z \\ - R_z S_z^{-1} K_z C_z + R_z \Delta B_z F_z G_z^{-1} \end{pmatrix} & G_{z+} + G_{z+}^T - P_{z+} & R_z \\ S_z \Delta A_z G_z - S_z \Delta B_z F_z & S_z A_z - K_z C_z + S_z \Delta B_z F_z G_z^{-1} & R_z^T & S_z + S_z^T - \tilde{P}_{z+} \end{bmatrix} > 0 \quad (21)$$

$$\begin{bmatrix} -P_z & 0 & (*) & (*) \\ 0 & -\tilde{P}_z + G_z^{-T} (\Gamma_1 + \Gamma_2) G_z^{-1} & 0 & (*) \\ \begin{pmatrix} (A_z + \Delta A_z) G_z - (B_z + \Delta B_z) F_z \\ + R_z \Delta A_z G_z - R_z \Delta B_z F_z \end{pmatrix} & R_z A_z - R_z S_z^{-1} K_z C_z & \Omega_{(3,3)} & R_z \\ S_z \Delta A_z G_z - S_z \Delta B_z F_z & S_z A_z - K_z C_z & R_z^T & \Omega_{(4,4)} \end{bmatrix} < 0 \quad (22)$$

$$
\begin{bmatrix}
-P_z & 0 & 0 & (*) & 0 & (*) & 0 \\
0 & -\tilde{P}_z & I & (*) & 0 & (*) & 0 \\
0 & I & \begin{pmatrix} -G_z - G_z^T \\ +\Gamma_1 + \Gamma_2 \end{pmatrix} & 0 & 0 & 0 & 0 \\
\begin{pmatrix} \left(A_z + \Delta A_z\right) G_z \\ -\left(B_z + \Delta B_z\right) F_z \\ +R_z \Delta A_z G_z \\ -R_z \Delta B_z F_z \end{pmatrix} & \begin{pmatrix} R_z A_z \\ -R_z S_z^{-1} K_z C_z \end{pmatrix} & 0 & -G_{z+} - G_{z+}^T + P_{z+} & (*) & R_z & 0 \\
0 & 0 & 0 & \begin{pmatrix} \left(B_z + \Delta B_z\right) F_z \\ +R_z \Delta B_z F_z \end{pmatrix}^T & -\Gamma_1 & 0 & 0 \\
S_z \Delta A_z G_z - S_z \Delta B_z F_z & S_z A_z - K_z C_z & 0 & R_z^T & 0 & -S_z - S_z^T + \tilde{P}_{z+} & (*) \\
0 & 0 & 0 & 0 & 0 & F_z^T \Delta B_z^T S_z^T & -\Gamma_2
\end{bmatrix} < 0 \quad (23)
$$

The matrix in the inequality (23) can be split into two terms. The second one recovers all the uncertainties and the term $-R_z S_z^{-1} K_z C_z$. Let us define the second term noted $\Delta M + \Delta M^T$ where:

$$\Delta M_{(4,1)} = \left[\left(R_z + I\right) Ha\Delta a\right]\left[Ea_z G_z\right] - \left[\left(R_z + I\right) Hb\Delta b\right]\left[Eb_z F_z\right]$$

$$\Delta M_{(4,3)} = \left[-R_z\right]\left[S_z^{-1} K_z C_z\right]$$

$$\Delta M_{(5,4)} = \left[Hb\Delta b\right]\left[Eb_z F_z\right] + \left[R_z Hb\Delta b\right]\left[Eb_z F_z\right]$$

$$\Delta M_{(6,1)} = \left[S_z Ha\Delta a\right]\left[Ea_z G_z\right] - \left[S_z Hb\Delta b\right]\left[Eb_z F_z\right]$$

$$\Delta M_{(7,6)} = \left[F_z^T Eb^T \Delta b_z^T\right]\left[Hb^T S_z^T\right]$$

Using lemma 1 on $\Delta M + \Delta M^T$ the following inequality is obtained, $\Delta M + \Delta M^T \le \vartheta$ with ϑ defined equation (26) (next page) and with $\varepsilon > 0$, $\mu > 0$, $\eta > 0$, $\lambda > 0$, $Y > 0$, $\mathrm{X} > 0$ and $\delta > 0$.

So the inequality (23) is verified if (27) (next page) is satisfied with:

$$\Theta_{1,1} = -P_z + \varepsilon G_z^T Ea_z^T Ea_z G_z + \eta F_z^T Eb_z^T Eb_z F_z$$

$$\Theta_{4,4} = -G_{z+} - G_{z+}^T + P_{z+} + \lambda^{-1} R_z HbHb^T R_z^T + R_z \mathrm{X}^{-1} R_z^T + \eta^{-1}\left(I + R_z\right) HbHb^T \left(I + R_z\right)^T + \delta HbHb^T + \varepsilon^{-1}\left(I + R_z\right) HaHa^T \left(I + R_z\right)^T$$

$$\Theta_{5,5} = -\Gamma_1 + \delta^{-1} F_z^T Eb_z^T Eb_z F_z + \lambda F_z^T Eb_z^T Eb_z F_z$$

$$\Theta_{6,6} = -S_z - S_z^T + \tilde{P}_{z+} + \varepsilon^{-1} S_z HaHa^T S_z^T + \eta^{-1} S_z HbHb^T S_z^T + \mu^{-1} S_z HbHb^T S_z^T$$

$$\Theta_{7,7} = -\Gamma_2 + \mu F_z^T Eb_z^T Eb_z F_z$$

Using the Schur's complement on the terms of Θ excepted the expression $-\tilde{P}_{z+} + C_z^T K_z^T S_z^{-T} \mathrm{X} S_z^{-1} K_z C_z$ on which the lemma 2 is applied, (27) is satisfied if:

$$\Upsilon_{zz}^{z+} = \begin{bmatrix} \Xi_{1,1}^{zzz+} & (*) & (*) & 0 \\ \Xi_{2,1}^{zzz+} & \Xi_{2,2}^{zzz+} & (*) & 0 \\ \Xi_{3,1}^{zzz+} & \Xi_{3,2}^{zzz+} & \Xi_{3,3}^{zzz+} & 0 \\ 0 & 0 & 0 & \Xi_{4,4}^{zzz+} \end{bmatrix} < 0$$

Where all the matrices are given in (28), (29), (30) and (31) next page.

At last, notice that $\delta > 0$, $\Gamma_1 > 0$, $\Gamma_2 > 0$ and $\mathrm{X} > 0$ are only multiplied with terms who are not depending on a LMI variable. Then they can be considered as unspecified sums of matrices as δ_{zz}^{z+}, Γ_{1zz}^{z+}, Γ_{2zz}^{z+} and X_{zz}^{z+}. These sums have to be positive definite but not each of their elements. Let us define:

$$\Upsilon_{ij}^{k} = \begin{bmatrix} \Xi_{1,1}^{ijk} & (*) & (*) & 0 \\ \Xi_{2,1}^{ijk} & \Xi_{2,2}^{ijk} & (*) & 0 \\ \Xi_{3,1}^{ijk} & \Xi_{3,2}^{ijk} & \Xi_{3,3}^{ijk} & 0 \\ 0 & 0 & 0 & \Xi_{4,4}^{ijk} \end{bmatrix} \quad (25)$$

$$\vartheta = diag\begin{pmatrix} \varepsilon G_z^T Ea_z^T Ea_z G_z + \eta F_z^T Eb_z^T Eb_z F_z \\ C_z^T K_z^T S_z^{-T} \mathrm{X} S_z^{-1} K_z C_z \\ 0 \\ \begin{bmatrix} \varepsilon^{-1}\left(I + R_z\right) HaHa^T \left(I + R_z\right)^T + \eta^{-1}\left(I + R_z\right) HbHb^T \left(I + R_z\right)^T \\ +\delta HbHb^T + \lambda^{-1} R_z HbHb^T R_z^T + R_z \mathrm{X}^{-1} R_z^T \end{bmatrix} \\ \delta^{-1} F_z^T Eb_z^T Eb_z F_z + \lambda F_z^T Eb_z^T Eb_z F_z \\ \varepsilon^{-1} S_z HaHa^T S_z^T + \eta^{-1} S_z HbHb^T S_z^T + \mu^{-1} S_z HbHb^T S_z^T \\ \mu F_z^T Eb_z^T Eb_z F_z \end{pmatrix} \quad (26)$$

$$\begin{bmatrix} \Theta_{1,1} & 0 & 0 & (*) & 0 & 0 & 0 \\ 0 & -\tilde{P}_z + C_z^T K_z^T S_z^{-T} \mathrm{X} S_z^{-1} K_z C_z & I & (*) & 0 & 0 & 0 \\ 0 & I & -G_z - G_z^T + \Gamma_1 + \Gamma_2 & 0 & (*) & 0 & 0 \\ A_z G_z - B_z F_z & R_z A_z & 0 & \Theta_{4,4} & 0 & (*) & 0 \\ 0 & 0 & 0 & F_z^T B_z^T & \Theta_{5,5} & 0 & 0 \\ 0 & S_z A_z - K_z C_z & 0 & R_j^T & 0 & \Theta_{6,6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \Theta_{7,7} \end{bmatrix} < 0 \tag{27}$$

$$\Xi_{1,1}^{ijk+} = \begin{bmatrix} -P_i & (*) & (*) & 0 & 0 & 0 \\ Ea_i G_j & -\varepsilon^{-1} I & 0 & 0 & 0 & 0 \\ Eb_i F_j & 0 & -\eta^{-1} I & 0 & 0 & 0 \\ 0 & 0 & 0 & -\tilde{P}_i & (*) & I \\ 0 & 0 & 0 & K_j C_i & -S_i^T - S_i + \mathrm{X} & 0 \\ 0 & 0 & 0 & I & 0 & -G_i - G_i^T + \Gamma_1 + \Gamma_2 \end{bmatrix} \tag{28}$$

$$\Xi_{2,2}^{ijk} = \begin{bmatrix} \left(-G_k - G_k^T + P_k + HbHb^T\right) & (*) & (*) & (*) & (*) & (*) & 0 & 0 \\ R_i^T & -\mathrm{X} & 0 & 0 & 0 & 0 & 0 & 0 \\ Ha^T \left(I + R_j\right)^T & 0 & -\varepsilon I & 0 & 0 & 0 & 0 & 0 \\ Hb^T \left(I + R_j\right)^T & 0 & 0 & -\eta I & 0 & 0 & 0 & 0 \\ Hb^T R_j^T & 0 & 0 & 0 & -\lambda I & 0 & 0 & 0 \\ F_j^T B_i^T & 0 & 0 & 0 & 0 & -\Gamma_1 & (*) & (*) \\ 0 & 0 & 0 & 0 & 0 & Eb_i F_j & -\delta I & 0 \\ 0 & 0 & 0 & 0 & 0 & Eb_i F_j & 0 & -\lambda^{-1} I \end{bmatrix} \tag{29}$$

$$\Xi_{3,3}^{ijk} = \begin{bmatrix} -S_j - S_j^T + \tilde{P}_k & (*) & (*) & (*) \\ Ha^T S_j^T & -\varepsilon I & 0 & 0 \\ Hb^T S_j^T & 0 & -\eta I & 0 \\ Hb^T S_j^T & 0 & 0 & -\mu I \end{bmatrix} \quad \Xi_{4,4}^{ijk} = \begin{bmatrix} -\Gamma_2 & (*) \\ Eb_i F_j & -\mu^{-1} I \end{bmatrix} \tag{30}$$

$$\Xi_{2,1}^{ijk} = \Xi_{3,1}^{ijk} = \Xi_{3,2}^{ijk} = 0 \text{ excepted } \begin{cases} \Xi_{2,1}^{ijk}(1,1) = A_i G_j - B_i F_j \\ \Xi_{2,1}^{ijk}(1,4) = R_j A_i \end{cases}, \ \Xi_{3,1}^{ijk}(1,4) = S_j A_i - K_j C_i \text{ and } \Xi_{3,2}^{ijk}(1,1) = R_j^T \tag{31}$$

Theorem 1: Consider the uncertain discrete fuzzy model (10) together the control law (12) and the observer (13). With Υ_{ij}^k defined in (25) if there exists for given scalars $\varepsilon > 0$, $\mu > 0$, $\eta > 0$ and $\lambda > 0$ matrices $P_i = P_i^T > 0$, $\tilde{P}_i = \tilde{P}_i^T > 0$, X_{ij}^k, Γ_{1ij}^k, Γ_{2ij}^k, $Q_i^k = \left(Q_i^k\right)^T > 0$, $Q_{ij}^k = \left(Q_{ji}^k\right)^T$, S_i, K_i, F_i, R_i and G_i, $i, j, k \in \{1, \ldots, r\}$, $j > i$ and scalars δ_{ij}^k such that the problem defined in (7), (8) and (9) is satisfied, then the closed loop fuzzy model is globally asymptotically stable.

Proof: Existence of the G_z^{-1}. Let us notice that with Υ_{ij}^k defined in and condition (7) we have $G_k + G_k^T > 0$. Therefore $\sum_{k=1}^{r} h_k \left(G_k + G_k^T\right) > 0$ which ensures that G_z^{-1} exists. Then we must check that the function defined in (16) is a Lyapunov function candidate (See Guerra & Vermeiren 2004). Then, according to (24) and to the expression of the Υ_{ij}^k the variation of the Lyapunov function is negative if:

$$\sum_{k=1}^{r} h_{k+} \left(\sum_{i=1}^{r} h_i^2 \Upsilon_{ii}^k + \sum_{i=1}^{r} \sum_{j>i}^{r} h_i h_j \left(\Upsilon_{ij}^k + \Upsilon_{ji}^k\right) \right) < 0$$

With equations (7) and (8), (32) is verified if:

$$\sum_{k=1}^{r} h_{k+} \left(\sum_{i=1}^{r} h_i^2 Q_i^k + \sum_{i=1}^{r} \sum_{j>i}^{r} h_i h_j \left(Q_{ij}^k + Q_{ji}^k\right) \right) < 0 \text{ or:}$$

$$\forall s \neq 0, \ s^T \left(\sum_{i=1}^{r} h_i^2 Q_i^k + \sum_{i=1}^{r} \sum_{j>i}^{r} h_i h_j \left(Q_{ij}^k + Q_{ji}^k\right) \right) s < 0$$

and with the definition of Ψ_k and condition (9), (33) is true.

4. EXAMPLE

Considering the following nonlinear model in one of its Takagi-Sugeno's form:

$$\begin{cases} x(t+1) = \left(A_z + \Delta A_z\right) x(t) + \left(B_z + \Delta B_z\right) u(t) \\ y(t) = C_z x(t) \end{cases}$$

$$A_1 = \begin{bmatrix} -0.1 & -0.2 \\ -1 & 0.9 \end{bmatrix}, A_2 = \begin{bmatrix} -0.5 & -1.2 \\ -0.09 & -0.9 \end{bmatrix}, B_1 = \begin{bmatrix} 4.1 \\ 0.8 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix} \ C_1 = \begin{bmatrix} 0.5 & 2.5 \end{bmatrix} \ C_2 = \begin{bmatrix} 0.5 & 2 \end{bmatrix} \ Ha = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$Ea_1 = Ea_2 = \begin{bmatrix} 0.2 & 0 \end{bmatrix}, \ Hb = 0, \ Eb_1 = Eb_2 = 0$$

$$h_1(z) = 1 - h_2(z) = \left(1 - \cos\left(x_1(t)\right)\right)/2 \in \begin{bmatrix} 0 & 1 \end{bmatrix}$$

The LMI solver SEDUMI gives a solutions in the non quadratic case but not in the quadratic one (i.e. $P_i = G_i = P$ and $S_i = \tilde{P}_i = \tilde{P}$) and this solution gives the following observer and control law parameters:

$F_1 = \begin{bmatrix} -0.3395 & -0.7796 \end{bmatrix}$, $F_2 = \begin{bmatrix} -1.999 & -2.820 \end{bmatrix}$

$G_1 = \begin{bmatrix} 12.61 & 9.3 \\ 4.609 & 10.94 \end{bmatrix}$, $G_2 = \begin{bmatrix} 9.629 & 2.132 \\ 1.888 & 9.312 \end{bmatrix}$

$K_1 = \begin{bmatrix} -0.1084 \\ 0.1128 \end{bmatrix}$ and $K_2 = \begin{bmatrix} -0.2471 \\ 0.0191 \end{bmatrix}$.

The next figures show the evolution of the state and its estimate, figure 1 and the evolution of the estimation error for each state, figure 2, for the following initial conditions: $x(0) = \begin{bmatrix} 5 & 3 \end{bmatrix}^T$ for the model and $\hat{x}(0) = \begin{bmatrix} -3 & 5 \end{bmatrix}^T$ for the observer.

Equipotentials of the Lyapunov function are also plotted figure 1 to show they are far to be ellipsoids.

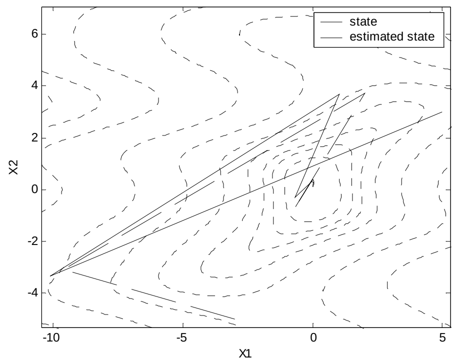

Fig. 1. State trajectory and its estimate.

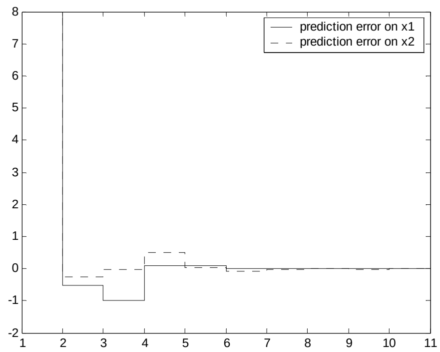

Fig. 2. Evolution of the estimation errors

5. CONCLUSION

In this paper, conditions for the output stabilization of a class of uncertain nonlinear discrete models are given. They are developed using a Takagi-Sugeno fuzzy model representative of the nonlinear model, this one being exact in a compact set of the state variables. These conditions were obtained by the utilization of a non quadratic Lyapunov function and some matrix properties and are given in the form of a LMI problem. This work seems to be the first to propose a non quadratic approach for the output stabilization via a nonlinear observer and a non PDC control law. This approach includes the classical quadratic approach. An example is given to show the effectiveness of the approach.

REFERENCES

Y. Blanco, W. Perruquetti, P. Borne (2001). Non quadratic stability of nonlinear systems in the Takagi-Sugeno form. *ECC'01*, Porto, Portugal.

S. Boyd, L. El Ghaoui, E. Féron, V. Balakrishnan (1994). Linear matrix inequalities in system and control theory. *Studies in Applied Mathematics*. Philadelphia, PA: SIAM, June.

J.V. De Oliveira, J. Bernussou, J.C.Geromel (1999). A new discrete-time robust stability condition. *Systems & Control Letters*, 37, 261-265

T.M. Guerra, A. Kruszewski, J. P. Richard (2004). Non-quadratic stabilization conditions for a class of uncertain discrete-time nonlinear models with time-varying delays. *IFAC Workshop TDS*, Leuven, Belgium.

T.M. Guerra, L. Vermeiren (2004). LMI-based relaxed non-quadratic stabilization conditions for nonlinear systems in Takagi-Sugeno's form. *Automatica*, 40(5), 823-829.

M. Johansson, A. Rantzer, K. Arzen (1999). Piecewise quadratic stability of fuzzy systems. *IEEE Transactions on Fuzzy Systems* 7, 713-722

E. Kim, H. Lee (2000). New Approaches to Relaxed Quadratic Stability Condition of Fuzzy Control Systems, *IEEE Transactions on Fuzzy Systems*, vol. 8, n°5, pp. 523-533

K.R. Lee, E.T. Jeung, H.B. Park (2001). Robust fuzzy H_∞ control for uncertain nonlinear systems via state feedback: an LMI approach. *Fuzzy Sets & Systems*, 120, 123-134.

X. Liu, Q. Zhang (2003). New approaches to H_∞ controller designs based on fuzzy observers for T-S fuzzy systems via LMI. *Automatica*, 39(9), 1571-1582.

T. Takagi, M. Sugeno (1985). Fuzzy identification of systems and its application to modeling and control. *IEEE transactions on System, Man and Cybernetics*, 15(1), 116-132.

K. Tanaka, T. Ikeda, H.O. Wang (1998). Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs. *IEEE Trans. on Fuzzy Systems*, 6(2), 1-16.

K. Tanaka, H.O. Wang (2001). *Fuzzy control systems design and analysis. A linear matrix inequality approach*. John Wiley & Sons, New York.

T. Taniguchi, K. Tanaka, H.O. Wang (2001). Model construction, rule reduction and robust compensation for generalized form of Takagi-Sugeno fuzzy systems. *IEEE Transactions on Fuzzy Systems*, vol. 9, n°4, 525-537

H.O. Wang, K. Tanaka, M. Griffin (1996). An approach to fuzzy control of nonlinear systems: stability and design issues. *IEEE Trans. on Fuzzy Systems*, 4 (1), 14-23.

L. Xie, C.E. De Souza (1992), Robust H_∞ control for linear systems with norm-bounded time-varying uncertainty. *IEEE Trans. on Automatic Control*, 37, 1188-1191