A NETWORKED CONTROL SYSTEM WITH STOCHASTICALLY VARYING TRANSMISSION DELAY AND UNCERTAIN PROCESS PARAMETERS

Ye Yang[1], Yongji Wang[1], and Shuang-Hua Yang[2*]

1Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan, China
2Department of Computer Science, Loughborough University, Loughborough, LE11 3TU

Abstract: This paper discusses the compensation of the transmission delay in a networked state feedback control system (NCS), which possess a randomly varying transmission delay and uncertain process parameters. The compensation is implemented by using a buffer in the actuator node and a state estimator in the controller node. A Linear Matrix Inequality (LMI) based sufficient condition for the stability of the NCS under the designed compensation is proposed. The simulation results illustrate the efficiency of our compensation method.

Keywords: delay compensation, network, time delay, stability analysis, uncertain linear system.

1. INTRODUCTION

As the development of network technologies, more and more communication networks are used in industrial control. Applications of NCSs include internet-based process control (Yang et al., 2002, 2003; Overstreet and Tzes, 1999), internet-based robotics (Oboe and Fiorini, 1998), field-bus based NCS and Ethernet based NCS (Lian, 2001) etc. The advantages of NCSs are reducing cost of cabling, ease system diagnosis and maintenance, increasing modularity and flexibility in system design.

However, the network transmission delays degrade the system dynamic performance and affect the stability of NCSs. The network transmission delay is time varying and stochastic. There are two ways to overcome the transmission delay. One is to improve the quality of network transmission by optimising communication protocols and adopting hardware devices with high performance so that the network-induced delays can be ignored. The other is to counteract the effect of network-induced delays on the system by using control theoretical approaches such as time-delay compensation, stochastic optimal control, predictive control, and robust control etc.

Rich literatures have been published on the NCSs. Zhang et al. (2001) analysed the stability of NCSs, and achieved some important results based on the assumption that transmission delay is less than a sampling period and the data are transmitted in a single packet. Walsh et al (2002) considered a NCS in which the network is inserted between continuous plant and continuous controller, and introduced the notion of Maximal Allowable Transfer Interval (MATI), which is the maximum time interval between transfers of data from sensors to a controller. Their goal is to find the MATI that guarantees the stability of NCSs. Montestruque and Antsaklis (2003) focused on reducing the network usage by using the knowledge of the plant dynamics. Necessary and sufficient conditions for stability of

* Corresponding author.

NCSs with a state feedback and an output feedback were derived respectively. Luck and Ray (1990) modelled the network-induced delays as a constant by building buffers in the controller node and the actuator node respectively. The disadvantage of this method is prolonging the network-induced delay.

This paper is organized as follows. In Section 2, a model of NCSs is given with several assumptions. A transmission delay compensation method is proposed in Section 3. The stability analysis for NCSs is addressed in Section 4. Section 5 illustrates the simulation results, which demonstrate the accuracy of the proposed method. Section 6 is the conclusions. The appendix gives the detailed proof for the stability theorem.

2. MODELLING OF THE NCS

Consider a class of ordinary NCS as shown in Fig. 1. It consists of a plant described in an uncertain discrete linear model.

$$\begin{cases} X(k+1) = (A+\Delta A)X(k) + (B+\Delta B)U(k) \\ Y(k) = CX(k) \end{cases} \tag{1}$$

and a discrete controller

$$U(k) = -KX(k), \qquad k = 1,2,\cdots, \tag{2}$$

where $X \in R^{n_1}$ is the state vector; $U_k \in R^{n_2}$ is the control input vector; $A \in R^{n_1 \times n_1}$ and $B \in R^{n_1 \times n_2}$ are known constant real matrices; ΔA *and* ΔB are matrix-valued functions of appropriate dimension representing time-varying parameter uncertainties in the plant model. The parameter uncertainties considered are assumed to be norm bounded and satisfy

$$[\Delta A \quad \Delta B] = DF(k)[E_1 \quad E_2] \tag{3}$$

where D, E_1, E_2 are known real matrixes of appropriate dimension that represent the structure of uncertainties, and $F(k) \in R^{s_1 \times s_2}$ is unknown matrix function and satisfies

$$F^T(k)F(k) \le I \tag{4}$$

in which I is the identity matrix with an appropriate dimension.

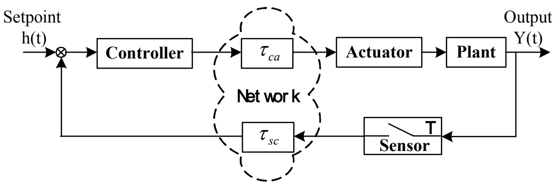

Fig.1. Networked control system

There are two transmission delays involved, one is the sensor-to-controller delay denoted as τ_{sc}, and another is the controller-to-actuator delay denoted as τ_{ca}. The total transmission delay τ equals $\tau_{sc}+\tau_{ca}$. The following assumptions are made for the NCS:

- The total transmission delay τ is bounded and stochastically varying, i.e. $0< \tau \le mT$, where m is an integer and T is the sampling interval.
- The sensor data and the control signals are transmitted in two single packets respectively.
- The controller is working in the event-driven mode, i.e. the control signal is calculated as soon as the sensors data arrive.
- The sensors are working in the clock-driven mode, i.e. the plant outputs are taken and sent from the sensors periodically with the interval T.
- The actuators are working in both the clock-driven and event modes, i.e. the actuators receive the control signals on network event, and refresh the control signals to the process only at the sampling time.

The actuators may receive zero, one, or more than one (up to m) control signals from the controller during a single sampling period of time. If the actuators receive no control signal during any sampling interval $[t_k, t_{k+1})$, the current control signal u_k will continue acting on the plant during the next sampling interval $[t_{k+1}, t_{k+2})$. If the actuators receive more than one control signal during any sampling interval $[t_k, t_{k+1})$, only the most recent control signal is kept and the actuators will discard the others.

Concerning the random transmission time delay, the state feedback controller shown in the equation (2) can be re-presented as follows

$$U(k) = -K\sum_{i=1}^{m} \delta(r_k - i)X(k), \qquad k = 1,2,\cdots \tag{5}$$

$$\delta(r_k - i) = \begin{cases} 0 & r_k \ne i \\ 1 & r_k = i \end{cases}$$

$$\sum_{i=1}^{m} \delta(r_k - i) = 1$$

r_k represents the random network transmission delay of the NCS, $r_k \in \wp = \{T, 2T, \cdots mT\}$.

A further rational assumption is made as follows:

Let $\{r_k, k > 0\}$ be a Markov chain with the state space $\wp = \{T, 2T, \cdots mT\}$ and the transition probabilities are

$$\Pr\{r_{k+1} = j \mid r_k = i\} = \Pr_{ij}, \forall i, j \in \wp \quad (6)$$

Here $\Pr_{ij} \geq 0$ *and* $\sum_{j=1}^{m} \Pr_{ij} = 1.$

3. TRANSMISSION DELAY COMPENSATION

Fig. 2 illustrates the principle of the transmission delay compensation for the NCS. A process model is located in the controller node in order to predict the performance of the plant. A buffer is located in the actuator node in order to compensate the transmission time delay.

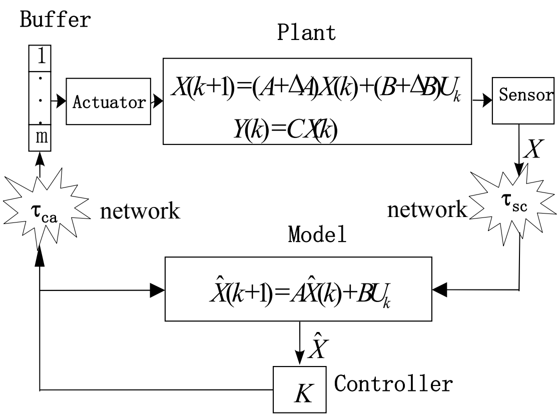

Fig. 2. Transmission delay compensation in the NCS

If the latest plant state received by the controller node is $X(k)$, the controller will predict the next m plant states based on this measured plant state $X(k)$: $\hat{X}(k+1)$, $\hat{X}(k+2)$, to $\hat{X}(k+m)$, calculate the future m control signals: $U(k+1), \cdots U(k+m)$ and then transmit them to the actuator node together with the time stamp received from the sensor node. The prediction of the plant states and the future control actions based on the measured plant state $X(k)$ are as follows:

$$\begin{cases} \hat{X}_{k+1|1} = AX(k) + BU_k \\ U_{k+1|1} = -K\hat{X}_{k+1|1} \\ \hat{X}_{k+2|2} = A\hat{X}_{k+1|1} + BU_{k+1|1} \\ U_{k+2|2} = -K\hat{X}_{k+2|2} \\ \quad\vdots \\ \hat{X}_{k+m|m} = A\hat{X}_{k+m-1|m-1} + BU_{k+m-1|m-1} \\ U_{k+m|m} = -K\hat{X}_{k+m|m} \end{cases} \quad (7)$$

Similarly, the prediction of the plant states and the future control actions based on the latest available plant states received by the controller node $X(k-1), \cdots, X(k-m+1)$ can be easily obtained.

Denote $\hat{X}_{k+1|i}, i \in [1,2,\cdots,m]$ as the prediction of the plant state at the instant $k+1$ based on the measured state X_{k+1-i}, and $U_{k+1|i}, i \in [1,2,\cdots,m]$ as the future control action at the instant $k+1$, which is obtained based on the prediction of the plant state $\hat{X}_{k+1|i}$. The future control actions at the instant *k+1* based on the latest available plant states $X(k), X(k-1), \cdots, X(k-m+1)$ can be described as follows:

$$\begin{cases} U_{k+1|1} = -K\hat{X}_{k+1|1} \\ U_{k+1|2} = -K\hat{X}_{k+1|2} \\ \quad\vdots \\ U_{k+1|m} = -K\hat{X}_{k+1|m} \end{cases} \quad (8)$$

$$\begin{cases} \hat{X}_{k+1|1} = AX(k) + BU_k \\ \hat{X}_{k+1|2} = A\hat{X}_{k|1} + BU_{k|1} \\ \quad\vdots \\ \hat{X}_{k+1|m} = A\hat{X}_{k|m-1} + BU_{k|m-1} \end{cases} \quad (9)$$

All the control actions and the estimations of the state shown in the equations 8 and 9 are at the instant *k+1* only, but based on the latest available measured states received from the sensor node at the instant *k*, *k-1*, …or *k-m+1* respectively. The actual control action applied into the plant at the instant *k+1,* i.e. the compensated control action U_{k+1} depends on the actual total transmission delay r_k.

Once the control signal is received by the actuator node the total transmission delay r_k will be calculated by comparing the current time stamp and the latest time stamp received from the sensor node. The control actions available for the plant will be $U(k+1-r_k)$ $U(k+2-r_k)$ $\cdots$ $U(k+m-r_k)$, which are saved in the buffer at the actuator node. The actuator node will choose *U(k+1)* from the buffer as the control signal acting on the plant at the instant *k+1*. In the next sampling interval, if no any control signal is received from the control node, *U(k+2)* will be used for the plant. If more than one control signal packets are received the latest packet will be saved in the buffer and used for the plant.

Being similar with the uncompensated control action shown in the equation (5) the compensated control action shown in the equation (8) can be formulised as follows:

$$U_{k+1} = \sum_{i=1}^{m} \delta(r_{k+1} - i) U_{k+1|i} \quad (10)$$

U_{k+1} is the control action acting on the plant at the instant $k+1$.

4. STABILITY ANALYSIS

Define $e_{k+1|i} = X_{k+1} - \hat{X}_{k+1|i}$ as the state error between the real plant state at the instant *k+1* and the state estimation for the instant *k+1* based on the real state at the instant $k+1-i$. Combining the equations 1, 8, 9 and 10 the state errors at the instant *k+1* based on the real state at the instant $k+1-i, i=1,2,\cdots,m$ can be given as follows:

$$\begin{cases} e_{k+1|1} = (\Delta A - \Delta BK)X_k + \Delta BK\sum_{i=1}^{m}\delta(r_k - i)e_{k|i} \\ \vdots \\ e_{k+1|m} = (\Delta A - \Delta BK)X_k + (A - BK)e_{k|m-1} + \\ (B+\Delta B)K\sum_{i=1}^{m}\delta(r_k - i)e_{k|i} \end{cases} \tag{11}$$

The plant state and these predictive state errors form a new extended state vector $Z(k)$:

$$Z(k) = \begin{bmatrix} X(k) \\ e_{k|m} \\ e_{k|m-1} \\ \vdots \\ e_{k|1} \end{bmatrix} \tag{12}$$

Therefore, the dynamics of the NCS with the time-delay compensation can be described by

$$Z(k+1) = [\Lambda + \Delta\Lambda + (\Gamma(r_k) + \Delta\Gamma(r_k))L]Z(k) \tag{13}$$

where,

$$\Lambda = \begin{bmatrix} A & 0 & \cdots & \cdots & 0 \\ 0 & 0 & A & 0 & 0 \\ 0 & 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \vdots & 0 & A \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\Delta\Lambda = \begin{bmatrix} \Delta A & 0 & \cdots & 0 \\ \Delta A & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \Delta A & 0 & \dots & 0 \end{bmatrix}$$

$$\Gamma(r_k) = \begin{bmatrix} -B & \delta(r_k - m)B & \delta(r_k - m+1)B & \cdots & \delta(r_k - 1)B \\ 0 & \delta(r_k - m)B & \delta(r_k - m+1)B - B & \cdots & \delta(r_k - 1)B \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \delta(r_k - m)B & \delta(r_k - m+1)B & \cdots & \delta(r_k - 1)B - B \\ 0 & 0 & \cdots & \cdots & 0 \end{bmatrix}$$

$$\Delta\Gamma(r_k) = \begin{bmatrix} -\Delta B & \delta(r_k - m)\Delta B & \cdots & \delta(r_k - 1)\Delta B \\ -\Delta B & \delta(r_k - m)\Delta B & \cdots & \delta(r_k - 1)\Delta B \\ \vdots & \vdots & \vdots & \vdots \\ -\Delta B & \delta(r_k - m)\Delta B & \dots & \delta(r_k - 1)\Delta B \end{bmatrix}$$

$$L = diag[\overbrace{K, K, \cdots K}^{m+1}] \tag{14}$$

With the assumption shown in the equations 3 and 4, $\Delta\Lambda$ *and* $\Delta\Gamma(r_k)$ can be expressed as:

$$[\Delta\Lambda \ \ \Delta\Gamma(r_k)] = \hat{D}\hat{F}(k)[\hat{E}_1 \ \ \hat{E}_2(r_k)] \tag{15}$$

where

$$\hat{D} = diag[\overbrace{D, D, \cdots D}^{m+1}]$$

$$\hat{F} = diag[\overbrace{F, F, \cdots F}^{m+1}]$$

$$\hat{E}_1 = \begin{bmatrix} E1 & 0 & \cdots & 0 \\ E1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ E1 & 0 & \dots & 0 \end{bmatrix}$$

$$\hat{E}_2(r_k) = \begin{bmatrix} -E2 & \delta(r_k - m)E_2 & \cdots & \delta(r_k - 1)E_2 \\ -E2 & \delta(r_k - m)E_2 & \cdots & \delta(r_k - 1)E_2 \\ \vdots & \vdots & \vdots & \vdots \\ -E2 & \delta(r_k - m)E_2 & \dots & \delta(r_k - 1)E_2 \end{bmatrix}$$

and

$$\hat{F}^T(k)\hat{F}(k) \le I \tag{16}$$

The following sufficient stability condition is achieved for the NCS with the predictive compensation.

Theorem: The NCS shown in the equations 1 to 4 with the time-delay compensation described by the equation 13 or by the equations 8, 9, and 10 is robust stochastically stable if there exist $P_i > 0, i = 1,\cdots m$, a matrix L described by the equation 14 and a scalar $\varepsilon_i > 0, i = 1,\cdots m$ satisfying the following m linear matrix inequalities:

$$M_i = \begin{bmatrix} -P_i & P_i L^T \Gamma_i^T W_i + P_i \Lambda^T W_i & P_i L^T \hat{E}_2^T(i) + P_i \hat{E}_1^T \\ * & -Q + \varepsilon_i W_i^T \hat{D}\hat{D}^T W_i & 0 \\ * & * & -\varepsilon_i I \end{bmatrix} < 0 \tag{17}$$

where

$W_i = [\sqrt{\Pr_{i1}} I \cdots \sqrt{\Pr_{im}} I], i = 1,\cdots m.$

$Q = diag\{P_1, \cdots P_m\}$

$\Pr_{ij}$ is the corresponding element of the state transition matrix of the Markov process r_k, as shown in the equation 6. The proof of the theorem is given in the appendix.

5. SIMULATION RESULTS

Consider a simple discrete plant described in the equations 1 to 4 with the sampling interval $T = 0.5$ second and the following parameters.

$$A = \begin{bmatrix} 1.0013 & 0.05 \\ 0.05 & 1.0013 \end{bmatrix}, \ B = \begin{bmatrix} 0.0013 \\ 0.05 \end{bmatrix}$$

$$C = [0.5, \ \ 0.2], \ D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$E1 = \begin{bmatrix} 0.02 & 0.17 \\ 0.15 & 0.12 \end{bmatrix}, \ E2 = \begin{bmatrix} 0.1 \\ 0.09 \end{bmatrix}$$

$$F(k) = \sin(k) .$$

The network transmission delay is $r_k \in \wp = \{T, 2T, 3T\}$, and the state transition matrix is

$$\Pr = \begin{bmatrix} 0.4 & 0.6 & 0 \\ 0.2 & 0.3 & 0.5 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}.$$

The state feedback controller is $U_k = -KX_k$, where $K = [10.3 \ \ 3.38]$ was designed in advance without considering the presence of the network.

According to the stability theorem in the section 4, the NCS with the time-delay compensation is stable for the state feedback matrix $K = [10.3 \ \ 3.38]$ by using the LMI toolbox in the MatLab. The responses of the states x_1, x_2, and the output Y under the square wave setpoint change are shown in Figs. 3, 4, and 5 respectively. The system was initially at a steady state, i.e. $x_1(0) = 0$; $x_2(0) = 0$; $Y(0) = 0$. The setpoint shown in Fig. 5 is changed from 0 to 1.0 at the instant $k = 0$, and then back from 1.0 to 0 at the instant $k = 100$. In Figs. 3 and 4 the solid and dash lines represent the responses of the two state variables without and with the transmission delay compensation respectively. It is obvious that the responses with the transmission delay compensation are quicker in approaching to the new steady states and have much less overshoot. Fig. 5 illustrates the same conclusion achieved in the output response. The square wave setpoint is shown in Fig. 5 as a reference. The output with the transmission delay compensation has much less overshoot and approaches to the setpoint much quicker than the one without the compensation. The comparison concludes that the transmission delay compensation method introduced in this paper can improve the system performance.

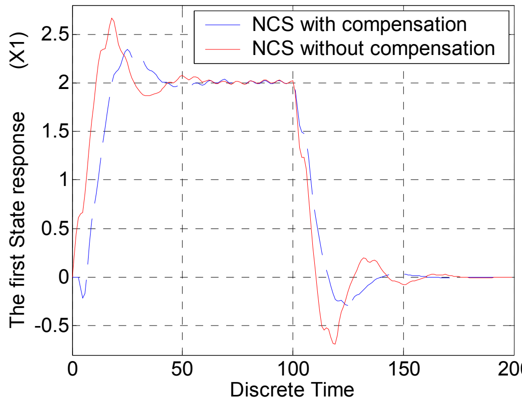

Fig. 3. State x_1 response

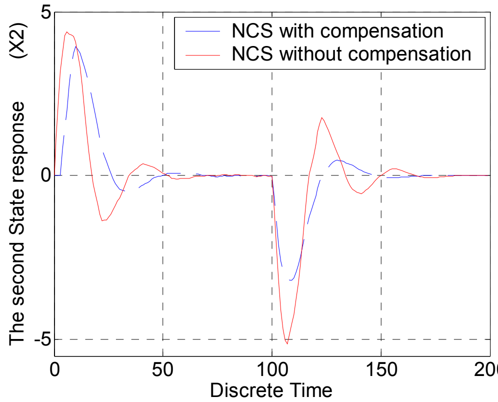

Fig. 4. State x_2 response

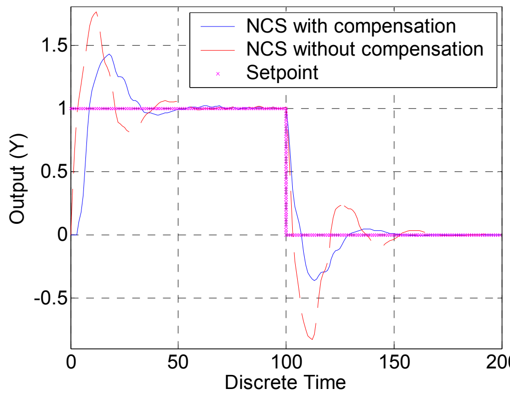

Fig. 5. Output response

6. CONCLUSIONS

This paper discusses the NCS with a stochastic transmission delay and uncertain process parameters. The stochastic transmission delay is assumed to be a Markov chain and be integer times of the sampling interval. The uncertain parameters are assumed to be norm bounded. A binary variable is introduced to represent the control action with a random transmission delay. A state feedback controller is firstly designed without considering the involvement of the network transmission delay. A buffer is then located in the actuator node to save the future control actions sent from the controller node. The control action actually applied to the plant at the instant *k+1* is chosen from the buffer based on the total transmission delay. The buffer is designed to compensate the influence of the transmission delay. An LMI-based sufficient condition for the stability of the NCS with the above compensation is derived in this paper. The simulation results also illustrate the potential of the transmission delay compensation method.

There are still a number of problems to be addressed. Firstly, the stability theorem proposed here is only a sufficient condition. A necessary condition is under investigation. The state feedback controller is used in this paper for the NCS. If the plant states are un-measurable a output feedback controller for the NCS should be investigated.

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APPENDIX

Lemma (see Branicky et al., 2000): Let $Z, D, F(t), E$ be matrices with appropriate dimensions. Suppose Z is symmetric and $F^T(t)F(t) \le I$, then

$$Z + DF(t)E + E^T F^T(t) D^T < 0$$

if and only if there exists scalar $\varepsilon > 0$ satisfying

$$Z + \varepsilon DD^T + \frac{1}{\varepsilon} E^T E < 0 \qquad \text{(A.1)}$$

Proof of the theorem:
Combining the equations 13 and 15, and taking the piecewise quadratic stochastic Lyapunov function:

$$V(\bullet) = Z_k^T S(r_k) Z_k \qquad \text{(A.2)}$$

Where, $S(r_k) = S_i > 0, \;\; when \;\; r_k = i.$
Let

$\bar{S}_i = \sum_{j=1}^{m} \Pr_{ij} S_j$, $G = diag\{S_1, \cdots S_m\}$, and

$W_i = [\sqrt{\Pr_{i1}} I \cdots \sqrt{\Pr_{im}} I], i = 1, \cdots m,$ where $\Pr_{ij}$ is the corresponding element of the state transition matrix of the Markov process r_k. Thus, we have $\bar{S}_i = W_i G W_i^T$.

The mean square stable theory of stochastic systems gives (Cao and Lam, 1999):

$$\begin{aligned} &E\{V_{k+1}(Z_{k+1}, r_{k+1}) \mid Z_k, r_k = i\} - V_k(Z_k, r_k = i) \\ &= Z_k^T (\Pi_i^T \bar{S}_i \Pi_i - S_i) Z_k \\ &= Z_k^T (\Pi_i^T W_i G W_i^T \Pi_i - S_i) Z_k \qquad \text{(A.3)} \end{aligned}$$

Obviously, if (A.3) <0, the discrete uncertain system is robust stochastically stable. Using the Schur complement the inequality (A.3)<0 can be represented as follows:

$$\begin{bmatrix} -S_i & \Pi_i^T W_i \\ * & -G^{-1} \end{bmatrix} < 0 \qquad \text{(A.4)}$$

According to the Lemma (A.1) and the assumption (16) the inequality (A.4) is true, if and only if there exits a scalar $\varepsilon_i > 0$ satisfying the equation (17). The theorem is proved.