MOTION RECOVERY BY USING DYNAMIC VISION

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Abstract: The recovery of motion for a class of movements in the space by using the perspective observation of one point is considered in this paper. The motion equation can cover a wide class of practical movements in the space. The estimations of the position and motion parameters which are all time-varying are simultaneously developed in the proposed algorithm. The formulated problem can be converted into the observation of a dynamical system with nonlinearities. The proposed observer is based on the second method of Lyapunov. First, the parameters relating to the rotation of the motion are identified, where only one camera is needed. Then the position of the moving object is identified, where the stereo vision is necessary. In the third step, the parameters relating to the straight movement are identified. The assumptions about the perspective system are reasonable, and the convergence conditions are intuitive and have apparently physical interpretations. The proposed method requires minor *a priori* knowledge about the system and can cope with a much more general class of perspective systems. Furthermore, the algorithm is modified to deal with the occlusion phenomenon.

Keywords: Motion recovery, perspective observation, dynamic vision, Lyapunov method, occlusion.

1. INTRODUCTION

In the study of machine vision, observing the motion and the structure of a moving object in the space by using the image data with the aid of CCD camera(s) has been studied recently. The motion treated in this field is composed of a rotation part and a translation part. A very typical method is the application of the extended Kalman filter (EKF). Numerous successful results have been reported (Azarbayejani, *et al,* 1995) where the formulation is based on a discrete expression of the motion, and the observability conditions are derived based on the perspective observations of a group of points (Chiuso, *et al*, 2002) (where the "*correspondence problem*" becomes a critical issue in practice). Such a recursive algorithm obviously alleviates the noises in the image data in contrast to the non-recursive methods (Kanatani, 1990) based on solving a set of nonlinear algebraic equations. It should be mentioned that some theoretical convergence conditions of discrete EKF have been established recently both as observer and filter (Reif, *et al*, 1998).

The observation problem for continuous time perspective systems has been studied in the point of view of dynamical system theory (Ghosh, *et al*, (2000), Loucks (1994)). A necessary and sufficient condition for the perspective observability is given by Dayawasa *et al* (1994) for the case that the motion parameters are constants. For the movements with piecewise constant motion parameters, the perspective observability problems are clarified in Soatto (1997) for the cases of observing one point or a group of points (in this case, the "*correspondence problem*" still remains). Further, for the observer design, some simple formulations for observing the position of a moving object are proposed by Chen and Kano (2002, 2004) and Jankovic *et al* (1995). The proposed observers are guaranteed to converge in an arbitrarily large (but bounded) set of initial conditions, and since the convergence is exponential it is believed that the performance of the new observers are reliable, robust and would quickly compute the position on real data.

This paper considers the problem of motion recovery for a class of movements under perspective observation. Naturally, the motions are formulated in continuous-time settings and the so-called motion parameters are assumed to be all time-varying. The 3-D position and motion parameters are estimated by using image data observed through pin-hole camera with constant focal length (normalized to unity). The basic and important idea is to analyze the extent to which we can develop a scheme that is guaranteed to converge by observing minimum number of points, namely a single point. The point correspondence problem then does not arise, and such an analysis contributes to further studies on the effects by increasing the number of observation points for the purpose of more accurate estimation as well as improving the observability condition. A dynamical systems approach is employed since it provides us with powerful mathematical tools, and a nonlinear observer is developed based on the second method of Lyapunov.

The considered motion equation can cover a wide class of practical movements in the space. The estimation of the position and the motion parameter are simultaneously developed in the proposed algorithm. The formulated problem can be converted into the observation of a dynamical system with nonlinearities. First, the parameters relating to the rotation of the motion are identified, where only one camera is needed. Then the position of the moving object is estimated by using a nonlinear observer, where the stereo vision is necessary. It should be noted that, even though the position can be computed by using triangular methods from stereo vision, it is not used here due to its shortcoming in alleviating the noises in the image data (Kanatani, 1990). In the third step, the parameters relating to the straight movement are identified. The assumptions about the perspective system are reasonable, and the convergence conditions are intuitive and have apparently physical interpretations. The attraction of the new method lies in that the algorithm is very simple, easy to be implemented practically. Furthermore, the proposed method requires minor *a priori* knowledge about the system and can cope with a much more general class of perspective systems. Finally, in order to deal with the occlusion phenomenon, the algorithm is modified. It should be noted that the changing of focal length is not considered in this paper.

2. PROBLEM STATEMENT

Consider the movement of the object described by

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} = \begin{bmatrix} 0 & \omega_1(t) & \omega_2(t) \\ -\omega_1(t) & 0 & \omega_3(t) \\ -\omega_2(t) & -\omega_3(t) & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \end{bmatrix} \quad (1)$$

where $x(t) = [x_1, \quad x_2, \quad x_3]^T$ is the position; $\omega_i(t)$ and $b_i(t)\,(i = 1,2,3)$ are the motion parameters.

It is supposed that the observed position in one image plane is defined by

$$y(t) = [y_1(t), \quad y_2(t)] = \left[\frac{x_1}{x_3}, \quad \frac{x_2}{x_3}\right]. \quad (2)$$

If it is necessary, it is supposed that the observed position in another image plane is defined by

$$y^*(t) = [y_1^*(t), \quad y_2^*(t)] = \left[\frac{x_1 - m}{x_3}, \quad \frac{x_2 - n}{x_3}\right], \quad (2')$$

where m and n are constants. The perspective observations are defined in (2) and (2'). The combination of the observations in (2) together with (2') is called "stereo vision".

Let

$$\omega(t) = [\omega_1, \quad \omega_2, \quad \omega_3]^T, \; b(t) = [b_1, \quad b_2, \quad b_3]^T \quad (3)$$

In this paper, we make the following assumptions.

(A1). m and n are known constants with $m^2 + n^2 \neq 0$.

(A2). The motion parameters $\omega_i(t)$ and $b_i(t)\,(i = 1,2,3)$ are bounded.

(A3). $x_3(t)$ meets the condition $x_3(t) > \eta > 0$, where η is a constant.

(A4). $y(t)$ and $y^*(t)$ are bounded.

Remark 1: It is easy to see that assumptions (A3) and (A4) are reasonable by referring to the practical systems.

The purpose of this paper is to estimate the position $x(t)$ and the motion parameters $\omega_i(t)$ and $b_i(t)\,(i = 1,2,3)$ by using the perspective observations.

3 FORMULATION OF THE OBSERVER

Define

$$y_3(t) = \frac{1}{x_3(t)}. \quad (4)$$

Then, equation (1) can be transformed as

$$\begin{cases}\dot{y}_1(t)=\omega_2+\omega_1 y_2+\omega_2 y_1^2+\omega_3 y_1 y_2+b_1 y_3-b_3 y_1 y_3\\ \dot{y}_2(t)=\omega_3-\omega_1 y_1+\omega_2 y_1 y_2+\omega_3 y_2^2+b_2 y_3-b_3 y_2 y_3\\ \dot{y}_3(t)=\omega_2 y_1 y_3+\omega_3 y_2 y_3-b_3 y_3^2\end{cases} \quad (5)$$

Let

$$\theta(t)=[b_1 y_3,\, b_2 y_3,\, b_3 y_3,\, \omega_1,\, \omega_2,\, \omega_3]^T \triangleq [\theta_1,\, \theta_2,\, \theta_3,\, \theta_4,\, \theta_5,\, \theta_6]^T, \quad (6)$$

and

$$\phi(t)=\begin{bmatrix}1 & 0 & -y_1 & y_2 & 1+y_1^2 & y_1 y_2\\ 0 & 1 & -y_2 & -y_1 & y_1 y_2 & 1+y_2^2\end{bmatrix} \triangleq \begin{bmatrix}\phi_1(t)\\ \phi_2(t)\end{bmatrix}. \quad (7)$$

It should be noted that $\phi(t)$ is available. In order to estimate the parameters $\omega_i(t)$ and $b_i(t)\,(i=1,2,3)$, the following assumption are needed.

(A5). The dynamics of $\omega_i(t)$ and $b_i(t)\,(i=1,2,3)$ are known as

$$\dot{\omega}(t)=q(\theta,t),\ \dot{b}(t)=p(\theta,t)\cdot b. \quad (8)$$

Further, the functions $q(\theta,t)$ and $p(\theta,t)b$ are piecewise differentiable with respect to θ, and the partial derivatives are bounded (at the undifferentiable points, we mean the left and right side derivatives).

From (5) and the definitions in (6)-(8), we have

$$\begin{cases}\begin{bmatrix}\dot{y}_1(t)\\ \dot{y}_2(t)\end{bmatrix}=\phi(t)\theta(t)\\ \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\end{bmatrix}=\left((y_1\theta_5+y_2\theta_6-\theta_3)I+p(\theta,t)\right)\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix}\\ \begin{bmatrix}\dot{\theta}_4\\ \dot{\theta}_5\\ \dot{\theta}_6\end{bmatrix}=q(\theta,t)\end{cases} \quad (9)$$

It is obvious that the position of the object in the space can be calculated as

$$x_1(t)=\frac{y_1(t)}{y_3(t)},\ x_2(t)=\frac{y_2(t)}{y_3(t)},\ x_3(t)=\frac{1}{y_3(t)} \quad (10)$$

if $y_3(t)$ is available. So, if $y_3(t)$ can be estimated, the position $x(t)$ can also be estimated.

Firstly, the vector $\theta(t)$ is estimated in section 3.1 by using only the perspective observation defined in (2). Thus, the motion parameters $\omega_i(t)$ are estimated. Then, $y_3(t)$ is estimated in section 3.2 by appealing to the stereo vision. Finally, the parameters $b_i(t)\,(i=1,2,3)$ are estimated in section 3.3 .

3.1 Estimation of the vector $\theta(t)$

In the following, the observer of system (9) is formulated where only the observation defined in (2) is employed. We consider the system described by

$$\begin{cases}\dot{\hat{y}}_1(t)=\phi_1(t)\hat{\theta}(t)+w_{11}(t)\\ \dot{\hat{y}}_2(t)=\phi_2(t)\hat{\theta}(t)+w_{12}(t)\\ \dot{\hat{\theta}}(t)=\begin{bmatrix}\left((y_1\hat{\theta}_5+y_2\hat{\theta}_6-\hat{\theta}_3)I+p(\hat{\omega},t)\right)\begin{bmatrix}\hat{\theta}_1\\ \hat{\theta}_2\\ \hat{\theta}_3\end{bmatrix}\\ q(\hat{\omega},t)\end{bmatrix}\\ \quad +\Gamma_1\cdot\phi^T(t)w_1(t),\\ \hat{\theta}(t_j+0)=M\dfrac{\hat{\theta}(t_j-0)}{\left\|\hat{\theta}(t_j-0)\right\|_2}\end{cases} \quad (11)$$

where the initial condition is determined as

$$\hat{y}_1(0)=y_1(0),\ \hat{y}_2(0)=y_2(0),\ \hat{\theta}(0)=\hat{\theta}_0, \quad (12)$$

$\hat{\theta}_0$ is a vector with constant entries; $e_1(t)$ and $e_2(t)$ are respectively defined as

$$e_1=y_1-\hat{y}_1,\ e_2=y_2-\hat{y}_2; \quad (13)$$

$\delta_i>0\,(i=1,2)$ are design parameters; t_i is defined as

$$t_i=\min\left\{t: t>t_{i-1}\ and\ \left\|\hat{\theta}(t)\right\|_2\geq 2M\right\}, \quad (14)$$

and $t_0=0$; $M>0$ is a large constant; $\hat{\omega}(t)$ is defined as

$$\hat{\omega}(t)=[\hat{\omega}_1,\,\hat{\omega}_2,\,\hat{\omega}_3]\triangleq[\hat{\theta}_4(t),\,\hat{\theta}_5(t),\,\hat{\theta}_6(t)]^T; \quad (15)$$

Γ_1 is a positive design parameter; $w_1(t)$ which is defined by

$$w_1(t)=[w_{11}(t),\, w_{12}(t)]^T \quad (16)$$

will be given later.

Remark 2: t_i defined in (14) are the discontinuous points of the system (11). By observing (11) and (14), it can be easily seen that $\hat{\theta}(t)$ is bounded by $\left\|\hat{\theta}(t)\right\|_2\leq 2M$.

Combining (9) and (11) yields

$$\begin{cases}\dot{e}_1(t)=\phi_1 e_3-w_{11}(t),\ e_1(0)=0\\ \dot{e}_2(t)=\phi_2 e_3-w_{12}(t),\ e_2(0)=0\\ \dot{e}_3(t)=\begin{bmatrix}\left((y_1\theta_5+y_2\theta_6-\theta_3)I+p(\omega,t)\right)\begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\end{bmatrix}\\ q(\omega,t)\end{bmatrix}\\ \quad -\begin{bmatrix}\left((y_1\hat{\theta}_5+y_2\hat{\theta}_6-\hat{\theta}_3)I+p(\hat{\omega},t)\right)\begin{bmatrix}\hat{\theta}_1\\ \hat{\theta}_2\\ \hat{\theta}_3\end{bmatrix}\\ q(\hat{\omega},t)\end{bmatrix}-\Gamma_1\phi^T w_1\end{cases} \quad (17)$$

where

$$e_3=\theta-\hat{\theta} \quad (18)$$

Let

$$r(t)=[r_1(t),\, r_2(t)]^T=[\dot{e}_1+\alpha_1 e_1,\, \dot{e}_2+\alpha_2 e_2]^T \quad (19)$$

where α_1 and α_2 are positive constants. Thus, it yields

$$\dot{r}(t) = \begin{bmatrix} \frac{d}{dt}(\phi_1 e_3 - w_{11}) + \alpha_1(\phi_1 e_3 - w_{11}) \\ \frac{d}{dt}(\phi_2 e_3 - w_{12}) + \alpha_2(\phi_2 e_3 - w_{12}) \end{bmatrix} \quad (20)$$

Now, define $w_1(t)$ as

$$\dot{w}_{11}(t) = -(k_1 + \alpha_1)w_{11}(t) + \hat{\lambda}_1(t) \cdot sign(e_1) + k_1\alpha_1 e_1 \quad (21)$$

$$\dot{w}_{12}(t) = -(k_2 + \alpha_2)w_{12}(t) + \hat{\lambda}_2(t) \cdot sign(e_2) + k_2\alpha_2 e_2 \quad (22)$$

where $\hat{\lambda}_1(t)$ and $\hat{\lambda}_2(t)$ are respectively defined as

$$\hat{\lambda}_1(t) = |e_1| + \alpha_1 \int_0^t |e_1(\tau)| d\tau, \quad (23)$$

$$\hat{\lambda}_2(t) = |e_2| + \alpha_2 \int_0^t |e_2(\tau)| d\tau, \quad (24)$$

$w_{11}(0)$ and $w_{12}(0)$ can be any small constants; $sign(\cdot)$ denotes the signum function. Therefore, (20) can be expressed as

$$\dot{r}(t) = \eta(t) - \begin{bmatrix} k_1 r_1 + \hat{\lambda}_1(t) sign(e_1) \\ k_2 r_2 + \hat{\lambda}_2(t) sign(e_2) \end{bmatrix} \quad (25)$$

where $\eta(t)$ is defined as

$$\eta(t) = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} \frac{d}{dt}(\phi_1 e_3) + (k_1 + \alpha_1)(\phi_1 e_3) \\ \frac{d}{dt}(\phi_2 e_3) + (k_2 + \alpha_2)(\phi_2 e_3) \end{bmatrix} \quad (26)$$

Furthermore, the derivative of $\eta(t)$ is given by

$$\dot{\eta}(t) = \begin{bmatrix} \frac{d^2}{dt^2}(\phi_1 e_3) + (k_1 + \alpha_1)\frac{d}{dt}(\phi_1 e_3) \\ \frac{d^2}{dt^2}(\phi_2 e_3) + (k_2 + \alpha_2)\frac{d}{dt}(\phi_2 e_3) \end{bmatrix} \quad (27)$$

By the assumptions and Remark 2, it can be checked that $\eta(t)$ and $\dot{\eta}(t)$ are uniformly bounded. Thus, there exist constants $\lambda_i > 0$ such that

$$|\eta_i| + \frac{1}{\alpha_i}|\dot{\eta}_i| < \lambda_i \quad (28)$$

for $i = 1, 2$. However, the upper bounds λ_i are unknown. The upper bounds λ_i are estimated by (23) and (24), respectively.

In order to derive the main result, the next lemma is introduced for the constructed system (11).

Lemma 1. About the constructed system (11), the following results hold for $i = 1, 2$.

(1). All the generated signals are uniformly bounded.

(2). $|e_i(t)| \to 0$ and $|\dot{e}_i(t)| \to 0$ as $t \to \infty$.

Remark 3: In the constructed system (11), the dynamics of $\hat{y}_1(t)$ and $\hat{y}_2(t)$ are introduced to estimate the unknown inner products $\phi_1 e_3$ and $\phi_2 e_3$.

Theorem 1. Suppose there exist positive constants $\rho, \beta > 0$ such that

$$O(t, \rho) \triangleq \int_t^{t+\rho} \phi^T(\tau)\phi(\tau) d\tau \geq \beta I \quad (29)$$

for all $t > 0$. If the constant $\rho > 0$ is very small, then $e_3(t)$ is uniformly bounded and decreases to zero exponentially, i.e., $\hat{\theta}(t)$ generated in (11) is the estimate of $\theta(t)$ for sufficient large *t*.

Proposition 1: The matrix $O(t, \rho)$ is positive definite if and only if there do not exist constants a_i $(i = 1, 2, 3)$ with $a_1^2 + a_2^2 + a_3^2 \neq 0$ such that

$$a_1 y_1(\tau) + a_2 y_2(\tau) + a_3 = 0 \quad (30)$$

for almost all $\tau \in [t, t + \rho]$.

Remark 4: The positive definiteness of the matrix $O(t, \rho)$ for all *t* means that the observed image data on the image plane does not move along a straight line during any time interval $[t, t + \rho]$.

3.2 Estimation of $y_3(t)$

By observing the entries of $\theta(t)$, it can be seen that only the product $[b_1 \quad b_2 \quad b_3] y_3$ can be estimated. The remaining task is to estimate y_3 and $[b_1 \quad b_2 \quad b_3]^T$. In this section, y_3 is estimated by appealing to the stereo vision and applying the estimate of $\theta(t)$. Now, by differentiating $y^*(t)$ defined in (2'), it yields

$$\begin{cases} \dot{y}_1^* = \begin{bmatrix} 1 & 0 & -y_1^* & y_2^* & 1 + (y_1^*)^2 & y_1^* y_2^* \end{bmatrix} \theta(t) \\ \quad + (n\omega_1 + m\omega_2 y_1^* + n\omega_3 y_1^*) y_3 \\ \dot{y}_2^* = \begin{bmatrix} 0 & 1 & -y_2^* & -y_1^* & y_1^* y_2^* & 1 + (y_2^*)^2 \end{bmatrix} \theta(t) \\ \quad + (-m\omega_1 + m\omega_2 y_2^* + n\omega_3 y_2^*) y_3 \\ \dot{y}_3 = (\omega_2 y_1^* + \omega_3 y_2^* + (m\omega_2 + n\omega_3 - b_3) y_3) y_3 \end{cases} \quad (31)$$

By mimicking the formulation in Section 3.1 and using the estimate of $\theta(t)$, the observer $\hat{y}_3$ of y_3 can be formulated. Let

$$e_6(t) = y_3 - \hat{y}_3. \quad (32)$$

The next theorem gives the second result of this paper.

Theorem 2. Suppose there exist positive constants $\rho, \rho', \beta, \beta' > 0$ such that (29) holds and

$$N(t, \rho') \triangleq \int_t^{t+\rho'} \left((n\omega_1 + m\omega_2 y_1^* + n\omega_3 y_1^*)^2 + (-m\omega_1 + m\omega_2 y_2^* + n\omega_3 y_2^*)^2 \right) d\tau \geq \beta' \quad (33)$$

for all $t \geq 0$. If ρ and ρ' are very small, then $e_6(t)$ is uniformly bounded and decreases exponentially to zero.

3.3 Estimation of b

Now, based on the results in Sections 3.1 and 3.2, we consider the estimation of b based on the dynamics

$$\begin{cases} \dot{y}_1(t) = [1 \quad 0 \quad -y_1] y_3 \cdot b + [y_2 \quad 1+y_1^2 \quad y_1 y_2] \omega(t) \\ \dot{y}_2(t) = [0 \quad 1 \quad -y_2] y_3 \cdot b + [-y_1 \quad y_1 y_2 \quad 1+y_2^2] \omega(t) \\ \dot{b}(t) = p(\omega, t) \cdot b \end{cases} \quad (34)$$

where the first two equation is the rewritten form of the first two equations in (5). By employing the estimates of $y_3(t)$ and $\omega(t)$, the observer $\hat{b}$ of b can be similarly formulated. Let

$$e_9(t) = b(t) - \hat{b}(t) . \quad (35)$$

Theorem 3. If there exist very small positive constants ρ and ρ' such that (29) and (33) hold for all $t \geq 0$, then $e_9(t)$ is uniformly bounded and decreases exponentially to zero.

Remark 5: The design parameters $\alpha_i > 0$, $k_i > 0$ and Γ_i determine the estimating speed and the estimating precision. There is no general guidance about the choice of the parameters Γ_i. This is a well-known problem in the adaptive control literature.

4 CONSIDERATION OF OCCLUSION

In the practical applications, the occurrence of occlusion is inevitable. Thus, the algorithm should be modified in order to cope with this phenomenon. Since the estimation of $y_3(t)$ and $b(t)$ is based on the estimate of $\theta(t)$ and the estimation of $\theta(t)$ is based on the data obtained by Camera 1, the following three cases are considered.

Case 1: The image data is available from two cameras on the time intervals $\left[\tau_k^{(0)},\ \tau_k^{(1)}\right)$ $(k = 0,1,2,\cdots)$. It is assumed that $\tau_0^{(0)} = 0$.

Case 2: Camera 1 is occluded on the time intervals $\left[\tau_k^{(1)},\ \tau_k^{(2)}\right)$ $(k = 0,1,2,\cdots)$.

Case 3: The data from Camera 1 is available, however, Camera 2 is occluded on the time intervals $\left[\tau_k^{(2)},\ \tau_k^{(3)}\right)$, where $\tau_k^{(3)} = \tau_{k+1}^{(0)}$ $(k = 0,1,2,\cdots)$.

For case 1, the observer can be constructed as stated in Section 3. The initial values should be chosen as

$\hat{y}_i(\tau_k^{(0)}) = \hat{y}_i(\tau_k^{(0)} - 0)$, $\hat{\dot{y}}_i(\tau_k^{(0)}) = y_i(\tau_k^{(0)})$,

$\hat{y}_i^*(\tau_k^{(0)}) = y_i(\tau_k^{(0)})$,

$$\hat{\lambda}_i(\tau_k^{(0)}) = \begin{cases} \hat{\lambda}_i(\tau_{k-1}^{(3)} - 0) & if\ \tau_{k-1}^{(2)} \neq \tau_{k-1}^{(3)} \\ \hat{\lambda}_i(\tau_{k-1}^{(1)} - 0) & if\ \tau_{k-1}^{(2)} = \tau_{k-1}^{(3)}\ and\ \tau_{k-1}^{(0)} \neq \tau_{k-1}^{(1)} \\ \hat{\lambda}_i(\tau_{k-2}^{(3)} - 0) & else \end{cases},$$

$\hat{\lambda}_j(\tau_k^{(0)}) = \hat{\lambda}_j(\tau_{k-1}^{(1)} - 0)$, $\hat{\theta}(\tau_k^{(0)}) = \hat{\theta}(\tau_k^{(0)} - 0)$,

$\hat{y}_3(\tau_k^{(0)}) = \hat{y}_3(\tau_k^{(0)} - 0)$, $\hat{b}(\tau_k^{(0)}) = \hat{b}(\tau_k^{(0)} - 0)$

for $i = 1,2$; $j = 3, \cdots, 6$; $k = 1,2,\cdots$.

For case 2, the observer is constructed based on the dynamics in (9) and the dynamics of $y_3(t)$ and $b(t)$ in the form of

$$\dot{\hat{y}}_1(t) = \hat{\phi}_1(t)\hat{\theta}(t),\quad \hat{y}_1(t_{1j}^{(1)} + 0) = M \frac{\hat{y}_1(t_{1j}^{(1)} - 0)}{\left|\hat{y}_1(t_{1j}^{(1)} - 0)\right|}, \quad (36a)$$

$$\hat{y}_1(\tau_k^{(1)}) = \hat{y}_1(\tau_k^{(1)} - 0) \quad (36b)$$

$$\dot{\hat{y}}_2(t) = \hat{\phi}_2(t)\hat{\theta}(t),\quad \hat{y}_2(t_{1j}^{(1)} + 0) = M \frac{\hat{y}_2(t_{2j}^{(1)} - 0)}{\left|\hat{y}_2(t_{2j}^{(1)} - 0)\right|}, \quad (37a)$$

$$\hat{y}_2(\tau_k^{(1)}) = \hat{y}_2(\tau_k^{(1)} - 0) \quad (37b)$$

$$\dot{\hat{\theta}}(t) = \begin{bmatrix} \left((\hat{y}_1\hat{\theta}_5 + \hat{y}_2\hat{\theta}_6 - \hat{\theta}_3) I + p(\hat{\theta}, t) \right) \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \\ \hat{\theta}_3 \end{bmatrix} \\ q(\hat{\theta}, t) \end{bmatrix}, \quad (38a)$$

$$\hat{\theta}(t_{3j}^{(1)} + 0) = M \frac{\hat{\theta}(t_{3j}^{(1)} - 0)}{\left\|\hat{\theta}(t_{3j}^{(1)} - 0)\right\|_2},\quad \hat{\theta}(\tau_k^{(1)}) = \hat{\theta}(\tau_k^{(1)} - 0) \quad (38b)$$

$$\dot{\hat{y}}_3(t) = \left(\hat{\theta}_5\hat{y}_1 + \hat{\theta}_6\hat{y}_2 - \hat{\theta}_3\right)\hat{y}_3, \quad (39a)$$

$$\hat{y}_3(t_{4j}^{(1)} + 0) = M \frac{\hat{y}_3(t_{4j}^{(1)} - 0)}{\left|\hat{y}_3(t_{4j}^{(1)} - 0)\right|},\ \hat{y}_3(\tau_k^{(1)}) = \hat{y}_3(\tau_k^{(1)} - 0) \quad (39b)$$

$$\dot{\hat{b}}(t) = p(\hat{\theta}(t), t)\hat{b}(t),\ \hat{b}(t_{5j}^{(1)} + 0) = M \frac{\hat{b}(t_{bj}^{(1)} - 0)}{\left\|\hat{b}(t_{5j}^{(1)} - 0)\right\|_2}, \quad (40a)$$

$$\hat{b}(\tau_k^{(1)}) = \hat{b}(\tau_k^{(1)} - 0) \quad (40b)$$

where $\hat{\phi}_i(t)$ denote the corresponding vectors defined in (7) in which y_i are respectively replaced by $\hat{y}_i$ for $i = 1,2$; $t_{ij}^{(1)}$ are defined as

$$t_{1j}^{(1)} = \min\left\{t : t > t_{1,j-1}^{(1)}\ and\ \left|\hat{y}_1(t)\right| \geq 2M\right\},$$
$$t_{2j}^{(1)} = \min\left\{t : t > t_{2,j-1}^{(1)}\ and\ \left|\hat{y}_2(t)\right| \geq 2M\right\},$$
$$t_{3j}^{(1)} = \min\left\{t : t > t_{3,j-1}^{(1)}\ and\ \left\|\hat{\theta}(t)\right\|_2 \geq 2M\right\},$$
$$t_{4j}^{(1)} = \min\left\{t : t > t_{4,j-1}^{(1)}\ and\ \left|\hat{y}_3(t)\right| \geq 2M\right\},$$
$$t_{5j}^{(1)} = \min\left\{t : t > t_{5,j-1}^{(1)}\ and\ \left\|\hat{b}(t)\right\|_2 \geq 2M\right\}.$$

For case 3, the observer of $\theta(t)$ can be constructed as in Section 3. The initial value is chosen as

$\hat{y}_i(\tau_k^{(2)}) = \hat{y}_i(\tau_k^{(2)} - 0)$, $\hat{\theta}(\tau_k^{(2)}) = \hat{\theta}(\tau_k^{(2)} - 0)$,

$$\hat{\lambda}_i(\tau_k^{(2)}) = \begin{cases} \hat{\lambda}_i(\tau_k^{(1)} - 0) & if\ \tau_k^{(0)} \neq \tau_k^{(1)} \\ \hat{\lambda}_i(\tau_{k-1}^{(3)} - 0) & if\ \tau_k^{(0)} = \tau_k^{(1)}\ and\ \tau_{k-1}^{(2)} \neq \tau_{k-1}^{(3)} \\ \hat{\lambda}_i(\tau_{k-1}^{(1)} - 0) & else \end{cases}$$

for $i = 1,2$. The observers of $y_3(t)$ and $b(t)$ can be similarly constructed as in Case 2.

The convergence of the observer can be proved, if the total length of the intervals on which the data from two cameras is available is much longer than the total length of the intervals on which at least one camera is occluded.

5 EXAMPLE AND SIMULATION RESULTS

In this section, we present the simulation results for an example. The simulation is done by the software Simulink in Matlab. The sampling period Δ is chosen as $\Delta = 0.05$. The measured image data at each sampling interval $[k\Delta, (k+1)\Delta)$ is corrupted by a random noise which is the product of $0.01y(k\Delta)$ (or correspondingly $0.01y^*(k\Delta)$) and $v(k\Delta)$, where $v(t)$ is a band-limited white noise with power 0.01 and sampling period 0.05. Consider the movement of the object described by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -4 & 0.5 \\ 4 & 0 & 0.4 \\ -0.5 & -0.4 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2\pi \sin 2\pi t \\ 2\pi \cos 2\pi t \end{bmatrix},$$

$$[x_1(0), x_2(0), x_3(0)]^T = [2, 2, 4]^T.$$

The simulation results are shown in Figures 1-3. The simulation results of the differences $\omega_2(t) - \hat{\omega}_2(t)$ and $\omega_3(t) - \hat{\omega}_3(t)$ are very similar to that in Figure 1. The simulation result of $b_3(t) - \hat{b}_3(t)$ is very similar to that of $b_2(t) - \hat{b}_2(t)$.

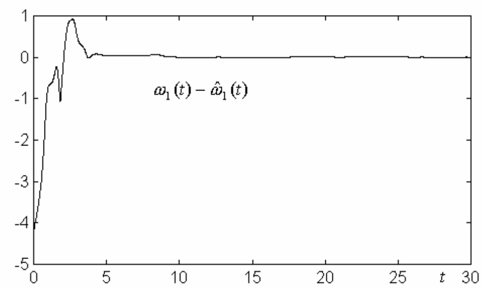

Fig. 1 The difference between ω_1 and its estimate $\hat{\omega}_1$.

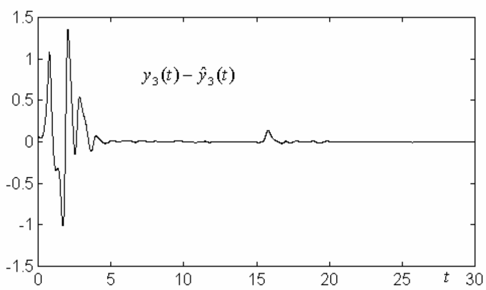

Fig. 2 The difference between y_3 and its estimate $\hat{y}_3$.

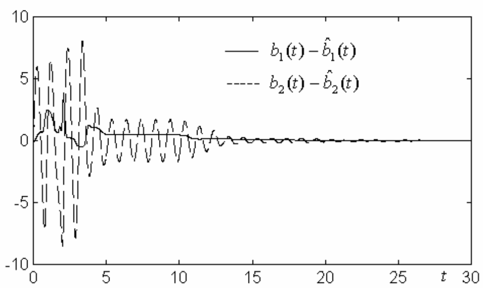

Fig. 3 The difference between b_1 and its estimate $\hat{b}_1$ and the difference between b_2 and its estimate $\hat{b}_2$.

6 CONCLUSIONS

In this paper, the identification problem of a general class of system by using the perspective observation is considered. The formulated problem can be converted into the observation of a dynamical system with nonlinearties. A new discontinuous observer, which is motivated by the sliding mode control method, is proposed to identify the perspective systems. The parameters in $\omega(t)$ can be identified by using one camera. However, the stereo vision is necessary in order to identify the variable $y_3(t)$. The identification of $b(t)$ is performed based on the identifications of $\omega(t)$ and $y_3(t)$. The proposed method is also modified in the occurrence of occlusion. The proposed method is robust to measurement noises.

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