

H_∞ DISCRETE TIME FUZZY CONTROL WITH APPLICATION TO CHAOS CONTROL BASED ON PIECEWISE LYAPUNOV FUNCTIONS

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Abstract: This paper presents an H_∞ controller synthesis method for discrete time fuzzy dynamic systems based on a piecewise smooth Lyapunov function. The basic idea of the approach is to design a piecewise linear state feedback control law and use a piecewise smooth Lyapunov function to establish the global stability with H_∞ performance of the resulting closed loop fuzzy control systems. It is shown that the control laws can be obtained by solving a set of Linear Matrix Inequalities (LMI). Application to control chaotic systems is given to illustrate the performance and advantages of the proposed method. *Copyright © 2005 IFAC*

Key words: Controller synthesis, Fuzzy systems, H_∞ control, Linear matrix inequality, Piecewise Lyapunov functions

1. INTRODUCTION

Fuzzy logical control (FLC) has recently proved to be a successful control approach for certain complex nonlinear systems, see Zadeh (1973), Mamdani and Assilian (1974), Sugeno (1985), and Takagi and Sugeno (1985) for example. The *conventional* FLC techniques usually decompose the complex system into several subsystems according to the human expert's understanding of the system and use a simple control law to emulate the human control strategy in each local operating region. The global control law is then constructed by combining all local control actions through fuzzy membership functions. Though the method has been practically successful it has proved difficult to develop a general

analysis and design theory for *conventional* fuzzy control systems.

Recently, there have appeared a number of stability analysis and controller design results in fuzzy control literature (e.g. Tanaka and Sugeno, 1992; Tseng et al., 2001; Lian et al., 2001), where the Takagi-Sugeno's fuzzy models are used. The stability and stabilization of the overall fuzzy system is determined by solving a set of Linear Matrix Inequalities (LMI). It is required that a common positive definite matrix P can be found to satisfy the LMIs for all the local models. However this is a difficult problem to solve since such a matrix might not exist in many cases, especially for highly nonlinear complex systems. Most recently, a stability result of fuzzy systems using a piecewise quadratic Lyapunov function has

been reported (Johansson et al. 1999). It is also demonstrated in the paper that the piecewise Lyapunov function is a much richer class of Lyapunov function candidates than the common Lyapunov function candidates and thus it is able to deal with a larger class of fuzzy dynamic systems. In fact, the common Lyapunov function is a special case of the more general piecewise Lyapunov function.

During the last few years, we have proposed a number of new methods for the systematic analysis and design of fuzzy logic controllers based on the Takagi-Sugeno's model (Cao et al., 1996, 1997a, 1997b, 1999). These methods include designs based on a nominal model, a common Lyapunov function and a piecewise continuous Lyapunov function. However, for the methods based on the piecewise Lyapunov function, certain restrictive boundary conditions have to be imposed.

Motivated from the results of piecewise continuous Lyapunov functions in Johansson et al. (1999), we developed a stability theorem for discrete time fuzzy dynamic systems based on a novel piecewise Lyapunov function in Feng (2004). In this paper, we propose a new constructive H_∞ controller synthesis method for the discrete time fuzzy dynamic systems based on the piecewise Lyapunov function. It should be noted that with this kind of piecewise Lyapunov function, the restrictive boundary condition existing in our previous controller design can be removed and global stability of the resulting closed loop system can be established. Moreover, the design procedure is to solve a set of LMIs that is numerically feasible with commercially available software.

The rest of the paper is organised as follows. Section 2 introduces the discrete time fuzzy dynamic model and an alternative piecewise quadratic stability theorem. Section 3 presents a piecewise H_∞ controller synthesis method. Application to control of chaotic systems is shown in section 4. Finally, conclusions are given in section 5.

2. FUZZY DYNAMIC MODEL AND PIECEWISE QUADRATIC STABILITY

The following fuzzy dynamic model or the so-called T-S fuzzy model (Tseng et al., 2001; Cao et al., 1996, 1997b, 1999) can be used to represent a complex discrete-time system with both fuzzy inference rules and local analytic linear models as follows.

$$\begin{aligned} R^l: & \text{ IF } x_1 \text{ is } F_1^l \text{ AND } \dots x_n \text{ is } F_n^l \\ & \text{ THEN } x(t+1) = A_l x(t) + B_l u(t) + D_l v(t) \\ & z(t) = H_l x(t) + G_l u(t) \end{aligned} \quad (1)$$

$l \in L := \{1, 2, \dots, m\}$,

where R^l denotes the l -th fuzzy inference rule, m the number of inference rules, F_j^l ($j=1, 2, \dots, n$) the fuzzy sets, $x(t) \in \mathfrak{R}^n$ the state, $u(t) \in \mathfrak{R}^p$ the control, $z(t) \in \mathfrak{R}^r$ the controlled output, $v(t) \in \mathfrak{R}^q$ the disturbance which belongs to $l_2[0, \infty)$, and $(A_l, B_l, D_l, H_l, G_l)$ the l -th local model of the fuzzy system (1).

Let $\mu_l(x(t))$ be the normalized membership function of the inferred fuzzy set F^l where $F^l = \prod_{i=1}^n F_i^l$ and

$$\sum_{l=1}^m \mu_l = 1. \quad (2)$$

By using a centre-average defuzzifier, product inference and singleton fuzzifier, the dynamic fuzzy model (1) can be expressed by the following global model

$$\begin{aligned} x(t+1) &= A(\mu)x(t) + B(\mu)u(t) + D(\mu)v(t) \\ z(t) &= H(\mu)x(t) + G(\mu)u(t) \end{aligned} \quad (3)$$

where

$$\begin{aligned} A(\mu) &= \sum_{l=1}^m \mu_l A_l, \quad B(\mu) = \sum_{l=1}^m \mu_l B_l, \quad D(\mu) = \sum_{l=1}^m \mu_l D_l \\ H(\mu) &= \sum_{l=1}^m \mu_l H_l, \quad G(\mu) = \sum_{l=1}^m \mu_l G_l. \end{aligned}$$

Define m subspaces in the state space as follows,

$$\bar{S}_l = S_l \cup \partial S_l, \quad l = 1, 2, \dots, m \quad (4)$$

where

$$S_l = \{x \mid \mu_l(x) > \mu_i(x), \quad i = 1, 2, \dots, m, i \neq l\} \quad (5)$$

and its boundary

$$\partial S_l = \{x \mid \mu_l(x) = \mu_i(x), \quad i = 1, 2, \dots, m, i \neq l\}. \quad (6)$$

Then the global model of the fuzzy dynamic system can also be expressed in each subspace by

$$\begin{aligned} x(t+1) &= (A_l + \Delta A_l(\mu))x(t) + (B_l + \Delta B_l(\mu))u(t) \\ &+ (D_l + \Delta D_l(\mu))v(t) \end{aligned} \quad (7)$$

$$z(t) = (H_l + \Delta H_l(\mu))x(t) + (G_l + \Delta G_l(\mu))u(t)$$

for $x(t) \in \bar{S}_l$, where

$$\Delta A_l(\mu) = \sum_{i=1, i \neq l}^m \mu_i \Delta A_{li}, \quad \Delta B_l(\mu) = \sum_{i=1, i \neq l}^m \mu_i \Delta B_{li},$$

$$\Delta D_l(\mu) = \sum_{i=1, i \neq l}^m \mu_i \Delta D_{li}$$

$$\Delta H_l(\mu) = \sum_{i=1, i \neq l}^m \mu_i \Delta H_{li}, \quad \Delta G_l(\mu) = \sum_{i=1, i \neq l}^m \mu_i \Delta G_{li}$$

$$\Delta A_{li} = A_i - A_l, \quad \Delta B_{li} = B_i - B_l, \quad \Delta D_{li} = D_i - D_l$$

$$\Delta H_{li} = H_i - H_l, \quad \Delta G_{li} = G_i - G_l.$$

It should be noted that many membership functions could be equal to zero, that is, many fuzzy rules could be inactive when the l -th subsystem plays a dominant role.

For purpose of stability analysis and subsequent use, we introduce the following upper bounds for the uncertainty term of the fuzzy system (7),

$$[\Delta A_l(\mu)]^T [\Delta A_l(\mu)] \leq E_{lA}^T E_{lA},$$

$$\begin{aligned}
[\Delta B_l(\mu)]^T [\Delta B_l(\mu)] &\leq E_{IB}^T E_{IB}, \\
[\Delta D_l(\mu)] [\Delta D_l(\mu)]^T &\leq E_{ID} E_{ID}^T \\
[\Delta H_l(\mu)]^T [\Delta H_l(\mu)] &\leq E_{IH}^T E_{IH}, \\
[\Delta G_l(\mu)]^T [\Delta G_l(\mu)] &\leq E_{IG}^T E_{IG}.
\end{aligned} \quad (8)$$

It is noted that there are many ways to obtain these bounds, the interested readers can refer to Cao et al. (1996, 1997b, 1999) for details.

In this paper, we introduce a novel piecewise Lyapunov function. This function is guaranteed to be decreasing when the state of the system jumps from one region to another.

Theorem 2.1: Consider the free fuzzy dynamic system (1) with $u \equiv v \equiv 0$. If there exist a set of constants $\varepsilon_l, l=1,2,\dots,m$, a set of positive definite matrices $P_l, l \in L$ such that the following LMIs are satisfied,

$$0 > \begin{bmatrix} -P_l & P_l A_l^T & P_l E_{lA}^T \\ A_l P_l & -(P_l - \varepsilon_l I) & 0 \\ E_{lA} P_l & 0 & -\varepsilon_l I \end{bmatrix}, l \in L \quad (9)$$

$$0 > \begin{bmatrix} -P_l & P_l A_l^T & P_l E_{lA}^T \\ A_l P_l & -(P_j - \varepsilon_l I) & 0 \\ E_{lA} P_l & 0 & -\varepsilon_l I \end{bmatrix}, l, j \in \Omega \quad (10)$$

where the set Ω represents all possible transitions from one region to another, that is,

$$\Omega := \{l, j \mid x(t) \in S_l, x(t+1) \in S_j, j \neq l\},$$

then the fuzzy dynamic system is globally exponentially stable, that is, $x(t)$ tends to zero exponentially for every continuous piecewise trajectory in the state space.

Proof: Choose a Lyapunov candidate

$$V(t) = x^T(t) P_l^{-1} x(t), \quad x(t) \in \bar{S}_l, \quad l \in L. \quad (11)$$

Then the proof is straightforward and thus omitted.

Remark 2.1: It can be observed that if P_l is replaced by P_l / ε_l and let ε_l 's be the same for all l , then the parameter ε can be removed from the matrix inequality, and thus LMIs (9) and (10) can be simplified correspondingly.

3. CONTROLLER SYNTHESIS WITH H_∞ PERFORMANCE

In this section, we will address the H_∞ controller synthesis problem for the discrete time fuzzy dynamic systems introduced in the section 2. The objective of the controller synthesis is to design a suitable controller for the system (3) to be stable with a guaranteed performance in the H_∞ sense, that is,

given a prescribed level of disturbance attenuation $\gamma > 0$, find a controller such that the system is globally stable and the induced l_2 -norm of the operator from $v(t)$ to the controlled output $z(t)$ is less than γ under zero initial conditions,

$$\|z(t)\|_2 < \gamma \|v(t)\|_2$$

for all nonzero $v(t) \in l_2$. In this case, the closed loop control system is said to be globally stable with disturbance attenuation γ .

Consider the fuzzy system in each subspace

$$\begin{aligned}
x(t+1) &= (A_l + \Delta A_l(\mu))x(t) + (B_l + \Delta B_l(\mu))u(t) \\
&\quad + (D_l + \Delta D_l(\mu))v(t)
\end{aligned} \quad (12)$$

$$z(t) = (H_l + \Delta H_l(\mu))x(t) + (G_l + \Delta G_l(\mu))u(t)$$

for $x(t) \in \bar{S}_l$.

With the following piecewise controller,

$$u(t) = K(x)x(t) = K_l x(t) \quad x(t) \in \bar{S}_l, \quad l \in L, \quad (13)$$

the global closed loop system can be described by the following equation,

$$x(t+1) = A_c(\mu)x(t) + D_c(\mu)v(t) \quad (14)$$

$$z(t) = H_c(\mu)x(t)$$

where

$$A_c(\mu) = A(\mu) + B(\mu)K(x), \quad D_c(\mu) = D(\mu),$$

$$H_c(\mu) = H(\mu) + G(\mu)K(x).$$

The Eqn.(14) can also be expressed in each local subspace as,

$$\begin{aligned}
x(t+1) &= A_{cl}(\mu)x(t) + D_{cl}(\mu)v(t), \quad x(t) \in \bar{S}_l \\
z(t) &= H_{cl}(\mu)x(t)
\end{aligned} \quad (15)$$

where

$$A_{cl}(\mu) = A_l + \Delta A_l(\mu) + (B_l + \Delta B_l(\mu))K_l,$$

$$D_{cl} = D_l + \Delta D_l(\mu),$$

$$H_{cl}(\mu) = H_l + \Delta H_l(\mu) + (G_l + \Delta G_l(\mu))K_l.$$

Then we are ready to present the following lemma.

Lemma 3.1: Given a constant $\gamma > 0$, the fuzzy system (14) is globally stable with disturbance attenuation γ , if there exist a set of positive definite matrices $P_l, l \in L$ such that the following matrix inequalities are satisfied,

$$0 < \gamma^2 I - D_{cl}^T P_l^{-1} D_{cl}, \quad l \in L \quad (16)$$

$$\begin{aligned}
0 > A_{cl}^T P_l^{-1} A_{cl} - P_l^{-1} + A_{cl}^T P_l^{-1} D_{cl} \\
\times (\gamma^2 I - D_{cl}^T P_l^{-1} D_{cl})^{-1} D_{cl}^T P_l^{-1} A_{cl} + H_{cl}^T H_{cl}
\end{aligned}, \quad l \in L \quad (17)$$

$$0 < \gamma^2 I - D_{cl}^T P_j^{-1} D_{cl}, \quad l, j \in \Omega \quad (18)$$

$$\begin{aligned}
0 > A_{cl}^T P_j^{-1} A_{cl} - P_j^{-1} + A_{cl}^T P_j^{-1} D_{cl} \\
\times (\gamma^2 I - D_{cl}^T P_j^{-1} D_{cl})^{-1} D_{cl}^T P_j^{-1} A_{cl} + H_{cl}^T H_{cl}
\end{aligned}, \quad l, j \in \Omega. \quad (19)$$

Proof: It is straightforward and thus omitted.

Then via the lemma 3.1, we have the following result.

Theorem 3.1: Given a constant $\gamma > 0$, the system (14) is globally stable with disturbance attenuation γ , if there exist a set of constants $\varepsilon_l, l=1,2,\dots,m$, a set of positive definite matrices $P_l, l \in L$ and a set of matrices $Q_l, l \in L$ such that the LMIs (20)-(21) described at the end of the paper are satisfied, where

$$\Pi_l := P_l - 2\gamma^{-2}D_l D_l^T - 2\gamma^{-2}E_{lD}E_{lD}^T - \varepsilon_l I$$

$$\Pi_{lj} := P_j - 2\gamma^{-2}D_l D_l^T - 2\gamma^{-2}E_{lD}E_{lD}^T - \varepsilon_l I$$

Moreover, the controller gain for each local subsystem is given by

$$K_l = Q_l P_l^{-1}, \quad l \in L. \quad (22)$$

Proof: According to Lemma 3.1, we know that the system (14) is globally stable with disturbance attenuation γ , if the conditions (16)-(19) are satisfied. It follows from eqn. (15) that $\Pi_l > 0$. We will first show that the inequality $\Pi_l > 0$ implies (16). It follows from $\Pi_l > 0$ that $P_l - \gamma^{-2}D_{cl}D_{cl}^T > 0$, which is equivalent to the following LMI

$$\begin{bmatrix} P_l & D_{cl} \\ D_{cl}^T & \gamma^2 I \end{bmatrix} > 0$$

by Schur complement lemma. And by using the Schur complement lemma again with respect to the other term we can conclude that $\gamma^2 I - D_{cl}^T P_l^{-1} D_{cl} > 0$.

We then show that the inequality (20) implies the inequality (17). It is noted that via the matrix inversion lemma the right hand side of the inequality (17) can be expressed as,

$$\begin{aligned} RH &:= A_{cl}^T (P_l - \gamma^{-2} D_{cl} D_{cl}^T)^{-1} A_{cl} - P_l^{-1} + H_{cl}^T H_{cl} \\ &= [A_l + \Delta A_l + (B_l + \Delta B_l)K_l]^T [P_l - \gamma^{-2}(D_l + \Delta D_l)(D_l + \Delta D_l)^T]^{-1} \\ &\quad \times [A_l + \Delta A_l + (B_l + \Delta B_l)K_l] - P_l^{-1} \\ &\quad + [H_l + \Delta H_l + (G_l + \Delta G_l)K_l]^T [H_l + \Delta H_l + (G_l + \Delta G_l)K_l] \end{aligned}$$

Let $\Theta = [P_l - 2\gamma^{-2}(D_l D_l^T + E_{lD}E_{lD}^T)]^{-1}$, we have

$$\begin{aligned} RH &\leq (A_l + B_l K_l)^T \Theta (A_l + B_l K_l) \\ &\quad + (A_l + B_l K_l)^T \Theta (\Delta A_l + \Delta B_l K_l) + (\Delta A_l + \Delta B_l K_l)^T \Theta (A_l + B_l K_l) \\ &\quad + (\Delta A_l + \Delta B_l K_l)^T \Theta (\Delta A_l + \Delta B_l K_l) - P_l^{-1} \\ &\quad + 2(H_l + \Delta H_l)^T (H_l + \Delta H_l) + 2[(G_l + \Delta G_l)K_l]^T [(\Delta G_l + \Delta G_l)K_l] \\ &\leq (A_l + B_l K_l)^T \Theta (A_l + B_l K_l) \\ &\quad + (A_l + B_l K_l)^T \Theta \left(\frac{1}{\varepsilon_l} I - \Theta \right)^{-1} \Theta (A_l + B_l K_l) \\ &\quad + \frac{1}{\varepsilon_l} (\Delta A_l + \Delta B_l K_l)^T (\Delta A_l + \Delta B_l K_l) - P_l^{-1} \\ &\quad + 4(H_l^T H_l + \Delta H_l^T \Delta H_l) + 4K_l^T (G_l^T G_l + \Delta G_l^T \Delta G_l) K_l \\ &\quad \leq (A_l + B_l K_l)^T [\Theta - \varepsilon_l I]^{-1} (A_l + B_l K_l) \\ &\quad + \frac{2}{\varepsilon_l} (\Delta A_l^T \Delta A_l + K_l^T \Delta B_l^T \Delta B_l K_l) - P_l^{-1} \\ &\quad + 4(H_l^T H_l + \Delta H_l^T \Delta H_l) + 4K_l^T (G_l^T G_l + \Delta G_l^T \Delta G_l) K_l \\ &\leq (A_l + B_l K_l)^T \Pi_l^{-1} (A_l + B_l K_l) \\ &\quad + \frac{2}{\varepsilon_l} (E_{lA}^T E_{lA} + K_l^T E_{lB}^T E_{lB} K_l) - P_l^{-1} \\ &\quad + 4(H_l^T H_l + E_{lH}^T E_{lH}) + 4K_l^T (G_l^T G_l + E_{lG}^T E_{lG}) K_l \end{aligned}$$

It then follows that the following inequality implies the inequality (17), that is

$$\begin{aligned} &(A_l + B_l K_l)^T \Omega_l^{-1} (A_l + B_l K_l) - P_l^{-1} + \frac{2}{\varepsilon_l} (E_{lA}^T E_{lA} + K_l^T E_{lB}^T E_{lB} K_l) \\ &\quad + 4(H_l^T H_l + E_{lH}^T E_{lH}) + 4K_l^T (G_l^T G_l + E_{lG}^T E_{lG}) K_l < 0 \end{aligned}$$

or equivalently,

$$\begin{aligned} &P_l (A_l + B_l K_l)^T \Omega_l^{-1} (A_l + B_l K_l) P_l - P_l \\ &\quad + P_l \left(\frac{2}{\varepsilon_l} E_{lA}^T E_{lA} + 4H_l^T H_l + 4E_{lH}^T E_{lH} \right) P_l \\ &\quad + P_l K_l^T \left(\frac{2}{\varepsilon_l} E_{lB}^T E_{lB} + 4G_l^T G_l + 4E_{lG}^T E_{lG} \right) K_l P_l < 0 \end{aligned}$$

(23)

Let $Q_l = K_l P_l$, using the Schur complement formula a few times, we can easily show that the inequality (23) is equivalent to the LMI (20). Thus we have shown that the inequality (20) implies the inequality (17). Following the similar procedure, we can also show that the inequalities (21) imply the inequalities (18)-(19) respectively. Therefore, it can be concluded from Lemma 3.1 that the closed loop control system is globally stable with disturbance attenuation γ and thus the proof is completed.

It is noted that the similar property as in the Remark 2.1 does not hold in this case. Thus based on the above theorem, the following algorithm can be developed.

Algorithm 1:

Step 1. Set $\varepsilon_l, l=1,2,\dots,m$, to some positive constants.

Step 2. Solve the matrix inequalities (20)-(21) for a set of positive definite matrices $P_l, l \in L$, and matrices $Q_l, l \in L$. This can be facilitated by using the Matlab *LMI toolbox* (Gahinet et al., 1995).

Step 3. If the solutions are found, the controller parameters can be obtained by $K_l = Q_l P_l^{-1}, l \in L$, and stop. Otherwise, set $\varepsilon_l = \varepsilon_l / 2$, for those inequalities (20) or (21) having no solution, and check whether $\varepsilon_l (l=1,2,\dots,m)$ are greater than some given threshold. If it is the case, then go back to step 2. Otherwise, claim the present controller design fails.

4. APPLICATION TO CONTROL OF CHAOTIC SYSTEMS

In this section, we will apply the proposed controller synthesis approach into a chaotic system to demonstrate the effectiveness and advantage in contrast to the common Lyapunov function based approach.

Consider a chaotic map T whose representation of error system to the fixed point (9.6274, 0.6010, 1.0697) is of the form

R^1 : IF $e_1(t)$ is F_1
THEN $e(t+1) = A_1 e(t) + B_1 u(t) + D_1 v(t)$

R^2 : IF $e_1(t)$ is F_2
THEN $e(t+1) = A_2 e(t) + B_2 u(t) + D_2 v(t)$

with

$$A_i = \begin{bmatrix} 0 & 0.89 & 0.5 \\ h_i & 0.89 & 0 \\ -0.1 & 0 & 0.9 \end{bmatrix}, i=1,2, h_1 = -1.12, h_2 = 2,$$

$$B_1 = \begin{bmatrix} 1 \\ 0 \\ 1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix},$$

$$H_1 = H_2 = [0.1 \ 0.1 \ 0.1], G_1 = G_2 = 0.05, v = \sin(0.02\pi t).$$

Assume that $e_1(t) \in [-8, 20]$, and we choose the two fuzzy sets

$$F_1 = \frac{1}{2} \left(1 - \frac{e_1 - 6}{14} \right), F_2 = \frac{1}{2} \left(1 + \frac{e_1 - 6}{14} \right).$$

The uncertainties for each subsystem can be also obtained as

$$E_{1A} = -E_{2A} = \begin{bmatrix} 0 & 0 & 0 \\ 0.3 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, E_{1B} = -E_{2B} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0 \end{bmatrix},$$

$$E_{1D}^T = E_{2D}^T = E_{1H} = E_{2H} = [0 \ 0 \ 0], E_{1G} = E_{2G} = 0.$$

The unforced T-S system map T takes on chaotic behaviour shown in Fig. 1 with initial condition $x(0) = [10 \ -0.4 \ 1]^T$ and simulation time of 40 seconds.

It is noted that there is no solution to the common quadratic Lyapunov function based approach for this system. However, if using the piecewise Lyapunov function based approach in this paper, then the following solution has been found for (20)-(21) with $\gamma = 0.1$,

$$P_1 = \begin{bmatrix} 2.4721 & 1.9635 & -1.9584 \\ 1.9635 & 16.6339 & -19.3031 \\ -1.9584 & -19.3031 & 33.6801 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 3.5373 & -0.1092 & -1.3112 \\ -0.1092 & 15.1540 & -20.4240 \\ -1.3112 & -20.4240 & 31.4829 \end{bmatrix},$$

$$K_1 = [0.5190 \ -0.8286 \ -0.5491],$$

$$K_2 = [-1.9156 \ -0.8672 \ -0.5618],$$

$$\varepsilon_1 = 0.3873, \varepsilon_2 = 0.6350.$$

Simulation results with initial conditions $x(0) = [10 \ -0.4 \ 1]^T$ are shown in Fig. 2 where the control input is added after $t > 20$ seconds.

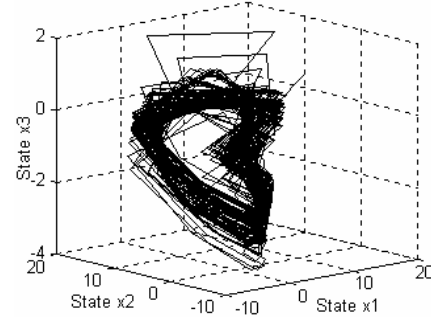


Fig. 1 The chaotic behaviour of the unforced map T

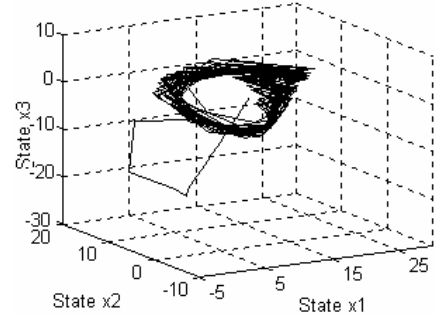


Fig. 2 The control results of the map T

From the illustrative example, the advantage of the piecewise Lyapunov function based approach is clearly demonstrated in contrast to the common Lyapunov function based approach.

5. CONCLUSIONS

In this paper, a new method has been developed to design robust H_∞ controllers for discrete time fuzzy dynamic systems based on a piecewise Lyapunov function. The solutions can be obtained by LMI techniques. Application to control of chaotic systems is presented to demonstrate the design procedure and the advantages of the proposed controller design method.

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REFERENCES

- Cao, S.G., N.W. Rees, and G. Feng (1996). Quadratic stability analysis and design of continuous time fuzzy control systems. *Int. J. Systems Science*, **27**, 193-203.
- Cao, S.G., N.W. Rees, and G. Feng (1997a). Analysis and design for a class of complex control systems, Part I: fuzzy modeling and identification. *Automatica*, **33**, 1017-1028.

- Cao, S.G., N.W. Rees, and G. Feng (1997b). Analysis and design for a class of complex control systems, Part II: fuzzy controller design. *Automatica*, **33**, 1029-1039.
- Cao, S.G., N.W. Rees, and G. Feng (1999). Analysis and design of fuzzy control systems using dynamic fuzzy state space models", *IEEE Trans. Fuzzy Systems*, **7**, 192-200.
- Feng, G. (2004). Stability analysis of discrete time fuzzy dynamic systems based on piecewise Lyapunov functions. *IEEE Trans. Fuzzy Systems*, **12**, 22-28.
- Gahinet, P., A. Nemirovski, A. Laub, and M. Chilali (1995). *The LMI Control Toolbox*, The Mathworks, Inc.
- Johansson, M., A. Rantzer, and K. Arzen (1999). Piecewise quadratic stability of fuzzy systems. *IEEE Trans. Fuzzy Systems*, **7**, 713-722.
- Lian, K.Y., T.S. Chiang, C.S. Chiu, and P. Liu (2001). Synthesis of fuzzy model-based designs to synchronization and secure communications for chaotic systems. *IEEE Transactions on Systems, Man and Cybernetics, Part B*, **31**, 66-83.
- Mamdani, E.H. and S. Assilian (1974). Applications of fuzzy algorithms for control of simple dynamic plant, *IEE Proc. Part-D*, **121**, 1585-1588.
- Sugeno, M. (1985). *Industrial applications of fuzzy control*. Elsevier Science Publishers, New York.
- Takagi, T. and M. Sugeno (1985). Fuzzy identification of systems and its application to modelling and control. *IEEE Trans. Systems Man Cybernetics*, **15**, 116-132.
- Tanaka, K. and M. Sugeno (1992). Stability analysis and design of fuzzy control systems. *Fuzzy Sets and Systems*, **45**, 135-156.
- Tseng, C.S., B.S. Chen, and H.J. Uang (2001). Fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model. *IEEE Trans. Fuzzy Systems*, **9**, 381-392.
- Zadeh, L.A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Trans. Systems Man Cybernetics*, **3**, 28-44.

$$0 > \begin{bmatrix} -P_l & (A_l P_l + B_l Q_l)^T & P_l E_{lA}^T & P_l H_l^T & P_l E_{lH}^T & Q_l^T E_{lB}^T & Q_l^T G_l^T & Q_l^T E_{lG}^T \\ A_l P_l + B_l Q_l & -\Pi_l & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{lA} P_l & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 & 0 & 0 & 0 \\ H_l P_l & 0 & 0 & -\frac{1}{4} I & 0 & 0 & 0 & 0 \\ E_{lH} P_l & 0 & 0 & 0 & -\frac{1}{4} I & 0 & 0 & 0 \\ E_{lB} Q_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 \\ G_l Q_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I & 0 \\ E_{lG} Q_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I \end{bmatrix}, l \in L \quad (20)$$

$$0 > \begin{bmatrix} -P_l & (A_l P_l + B_l Q_l)^T & P_l E_{lA}^T & P_l H_l^T & P_l E_{lH}^T & Q_l^T E_{lB}^T & Q_l^T G_l^T & Q_l^T E_{lG}^T \\ A_l P_l + B_l Q_l & -\Pi_{lj} & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{lA} P_l & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 & 0 & 0 & 0 \\ H_l P_l & 0 & 0 & -\frac{1}{4} I & 0 & 0 & 0 & 0 \\ E_{lH} P_l & 0 & 0 & 0 & -\frac{1}{4} I & 0 & 0 & 0 \\ E_{lB} Q_l & 0 & 0 & 0 & 0 & -\frac{\varepsilon_l}{2} I & 0 & 0 \\ G_l Q_l & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I & 0 \\ E_{lG} Q_l & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} I \end{bmatrix}, l, j \in \Omega \quad (21)$$