# PREDICTIVE CONTROL OF NETWORKED SYSTEMS WITH RANDOM DELAYS

**G. P. Liu** [*,**] **Yuanqing Xia** [*] **D. Rees** [*]

[*] *School of Electronics, University of Glamorgan, Pontypridd, CF37 1DL, United Kingdom. Email: gpliu@glam.ac.uk;xia_yuanqing@163.net;drees@glam.ac.uk*
[**] *Lab of Complex Systems and Intelligence Science, Chinese Academy of Sciences, Beijing 100080, China*

Abstract: This paper is concerned with the design of networked control systems with random network delay in the forward and feedback channels and gives stability criteria of closed-loop networked predictive control systems. The principle of the predictor is adopted to overcome the effects of network delay. The necessary and sufficient conditions on the stability of the closed-loop networked control system are derived, which provide useful analytical stability criteria. It is shown that closed-loop system with bounded random network delay is stable if the corresponding switched system is stable. 

Keywords: Networked control system, predictive control, random network delay, stability.

## 1. INTRODUCTION

With the development of network technology, more and more networks (e.g., Internet) have been applied to distributed control systems, which are termed networked control systems (NCS) (Overstreet and Tzes, 1999). Although the networks make it convenient to control large distributed systems, there also exist many control issues, which are not considered by conventional control theory, such as network delay and data dropout, sampling and transmitting methods. To solve those problems, various methods are developed, e.g., augmented deterministic discrete-time model, queuing, optimal stochastic control, perturbation, sampling time scheduling, robust control, fuzzy logic modulation, event-based control, end-user control adaptation, data packet dropout analysis, and hybrid systems stability analysis. But, those methods have put some strict assumptions on NCS, e.g., the network time delay is less than a sampling period. Most of them simply treated the NCS as a system with time delay, which ignores NCS features, e.g., random network delay and data transmission in packets.

The random network delay in the forward channel in NCS has been studied in (Liu *et al.*, 2004). But, the random network delay in the forward and feedback channels makes the control design and stability analysis much more difficult. This paper proposes a predictive control scheme for networked control systems with random network delay both in the feedback and forward channels and also provides analytical stability criteria of closed-loop networked predictive control systems.

## 2. DESIGN OF NPCS WITH BOTH FORWARD AND FEEDBACK NETWORK DELAYS

A networked predictive control scheme for NCS with random network delay in the forward and feedback channels is proposed. The main part of the scheme is the networked predictive controller, which compensates the network delay in the forward and feedback channels and achieves the desired control performance. A very important characteristic of the network is that it can transmit a set of data at the same time. Thus, to avoid the data dropout, the output data are transmitted in an overlap sequence form at each time, that is, each output will be transmitted several times with other outputs consecutively. Thus, it is assumed that the output sequence $[y_t,\ y_{t-1},\ \cdots,\ y_{t-N}]$ at time $t$ is packed and sent to the controller side through the feedback channel. In order to compensate the network transmission delay in the forward channel, a network delay compensator is proposed. The predictive control sequence at time $t$, which consists of the future control predictions, is packed and sent to the plant side through the forward channel. The network delay compensator chooses the latest control value from the control prediction sequences available on the plant side. The networked predictive control system is shown in Figure 1.

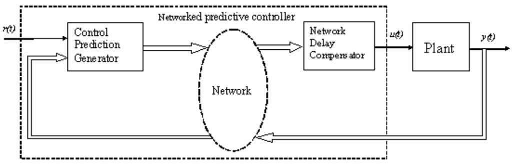


Figure 1: The networked predictive control system

Consider a MIMO (multi-input multi-output) discrete system described in the following state space form

$$
\begin{aligned}
x_{t+1} &= Ax_t + Bu_t \\
y_t &= Cx_t
\end{aligned}
\tag{1}
$$

where $x_t \in R^n$, $u_t \in R^m$, and $y_t \in R^l$ are the state, input, and output vectors of the system, respectively, $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $C \in R^{l\times n}$ the system matrices. For the simplicity of stability analysis, it is assumed that the reference input of the system is zero.

*Assumption 2.1.* The pair $(A, B)$ is completely controllable, and the pair $(A, C)$ is completely observable.

*Assumption 2.2.* The upper bound of the network delay in the forward channel is not greater than $M$ (a positive integer).

*Assumption 2.3.* The upper bound of the network delay in the feedback channel is not greater than $N$ (a positive integer).

*Assumption 2.4.* The number of consecutive data dropouts in the forward channel and feedback channel must be less than $M$ and $N$, respectively.

The state observer is designed as

$$
\hat{x}_{t+1|t} = A\hat{x}_{t|t-1} + Bu_t + L(y_t - C\hat{x}_{t|t-1}) \tag{2}
$$

where $\hat{x}_{t+1|t} \in R^n$ and $u_t \in R^m$ are the one-step ahead state prediction and the input of the observer at time t and the matrix $L \in R^{n\times l}$, which can be designed using observer design approaches.

Following the state observer described by (2) , based on the output data up to t-k, the state predictions from time t-k+1 to t are constructed as

$$
\begin{aligned}
\hat{x}_{t-k+1|t-k} &= A\hat{x}_{t-k|t-k-1} + Bu_{t-k} + \\
&\quad L(y_{t-k} - C\hat{x}_{t-k|t-k-1}) \\
\hat{x}_{t-k+2|t-k} &= A\hat{x}_{t-k+1|t-k} + Bu_{t-k+1} \\
&\vdots \\
\hat{x}_{t|t-k} &= A\hat{x}_{t-1|t-k} + Bu_{t-1}
\end{aligned}
\tag{3}
$$

which results in

$$
\begin{aligned}
\hat{x}_{t|t-k} = {} & A^{k-1}(A - LC)\hat{x}_{t-k|t-k-1} \\
& + \sum_{j=1}^{k} A^{k-j}Bu_{t-k+j-1} + A^{k-1}Ly_{t-k}
\end{aligned}
\tag{4}
$$

for , $k = 1, 2, 3, \cdots, k$.

The state prediction from time $t+1$ to $t+i$ are constructed by

$$
\begin{aligned}
\hat{x}_{t+1|t-k} &= A\hat{x}_{t|t-k} + Bu_{t|t-k} \\
\hat{x}_{t+2|t-k} &= A\hat{x}_{t+1|t-k} + Bu_{t+1|t-k} \\
&\vdots \\
\hat{x}_{t+i|t-k} &= A\hat{x}_{t+i-1|t-k} + Bu_{t+i-1|t-k}
\end{aligned}
\tag{5}
$$

In particular, the augmented system without time-delay, then, $u_t = K\hat{x}_{t|t-1}$, can be described as follows:

$$
\begin{aligned}
\hat{x}_{t+1|t} &= (A + BK - LC)\hat{x}_{t|t-1} + LCx_t \\
x_{t+1} &= Ax_t + BK\hat{x}_{t|t-1}
\end{aligned}
\tag{6}
$$

For the case of no network delay, it is assumed that the state-feedback controller is designed by a modern control method, for example, LQG, eigenstructure or pole assignment, $H_2$ and $H_\infty$ in the presence of disturbance, etc. For the case where there are both the forward network delay $i$ and feedback network delay $k$, the control predictions are calculated by

$$
u_{t+i|t-k} = K\hat{x}_{t+i|t-k} \tag{7}
$$

where the state feedback matrix $K \in R^{m\times n}$. Thus,

$$
\begin{aligned}
\hat{x}_{t+i|t-k} &= (A+BK)^i \hat{x}_{t|t-k} \\
&= (A+BK)^i (A^{k-1}(A-LC)\hat{x}_{t-k|t-k-1} \\
&+ \sum_{j=1}^{k} A^{k-j} B u_{t-k+j-1} + A^{k-1} L y_{t-k})
\end{aligned}
\tag{8}
$$

As a result, the output of the networked predictive control at time $t$ is determined by

$$
\begin{aligned}
u_{t|t-k} = &\ KA^{k-1}(A-LC)\hat{x}_{t-k|t-k-1} + \\
&\sum_{j=1}^{k} KA^{k-j} B u_{t-k+j-1} + KA^{k-1} L y_{t-k}
\end{aligned}
\tag{9}
$$

From equation (9), it is clear that the future control predictions depend on the state estimation $\hat{x}_{t-k|t-k-1}$ and the past control input up to $u_{t-1}$ and the past output up to $y_{t-k}$ of the system. Since there exist the forward delay $i$ and feedback delay $k$, the control input of the plant is designed as

$$
u_t = u_{t|t-i-k} \tag{10}
$$

Combined this predictive controller with networked delay compensator, both the forward and feedback network delays will be compensated for a certain amount of time-delay. In the next section, the analysis of stability of closed-loop system will be presented under this control scheme.

## 3. STABILITY CRITERIA OF CLOSED-LOOP NPC SYSTEMS

With the networked predictive control scheme proposed in this paper, a very important problem is to study the stability of the closed-loop system. First, the stability of the closed-loop system with constant network delay is investigated, and necessary and sufficient condition for the closed-loop system to be stable is derived. Second, for the random time delay, the problem is more interesting because this case is closer to the real network time-delays. In this case, the stability problem of the closed-loop system is solved using the theory of switched systems.

### *3.1 Constant delays in both forward and feedback channels*

In this case,it is assumed that the network delays $i$ and $k$ in the forward and feedback channels are constant. The first result is presented as follows.

*Theorem 3.1.* For the networked predictive control systems with constant network delays $i$ and $k$ in the forward and feedback channels, the closed-loop system is stable if and only if all eigenvalues of the following matrix are within the unit circle.

$$
\Psi =
\left[\begin{array}{ccccccccccccc}
A & 0 & 0 & \cdots & 0 & \cdots & 0 & B & 0 & 0 & \cdots & 0 & 0 \\
M_iA^{k-1}LC & 0 & 0 & \cdots & M_iB & \cdots & M_iA^{k-2} & M_iA^{k-1}B & 0 & 0 & \cdots & 0 & M_iA^{k-1}(A-LC) \\
0 & I & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & I & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & \cdots & I & 0 & 0 & 0 & \cdots & 0 & 0 \\
LCA^{k+i} & AB & LCAB & \cdots & LCA^{i+1}B & \cdots & LCA^{k+i-2}B & LCA^{k+i-1}B & A-LC & 0 & \cdots & 0 & 0 \\
0 & B & 0 & \cdots & 0 & \cdots & 0 & 0 & I & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & I & 0
\end{array}\right]
\tag{11}
$$

where $M_i = K(A+BK)^i$,
$\Psi \in R^{[(k+i)m+(k+i+2)n]\times[(k+i)m+(k+i+2)n]}$.

**Proof** The necessary and sufficient condition of the closed-loop system stability is that its closed-loop poles are stable. From the predictive control obtained in the previous section, it is clear that

$$
\begin{aligned}
u_t = &\ K(A+BK)^i (A^{k-1}(A-LC)\hat{x}_{t-i-k|t-i-k-1} \\
&+ \sum_{j=1}^{k} A^{k-j} B u_{t-i-k+j-1} + A^{k-1} LC x_{t-i-k})
\end{aligned}
$$

then,

$$
\begin{aligned}
x_{t+1} &= Ax_t + Bu_t \\
&= Ax_t + Bu_{t|t-i-k} \\
&= Ax_t + BK(A+BK)^i (A^{k-1} \times \\
&\quad (A-LC)\hat{x}_{t-i-k|t-i-k-1} + \\
&\quad \sum_{j=1}^{k} A^{k-j} B u_{t-i-k+j-1} + A^{k-1} LC x_{t-i-k})
\end{aligned}
\tag{12}
$$

and

$$
\begin{aligned}
\hat{x}_{t+1|t} &= A\hat{x}_{t|t-1} + Bu_t + L(y_t - C\hat{x}_{t|t-1}) \\
&= (A-LC)\hat{x}_{t|t-1} + LCx_t + BK(A+ \\
&\quad BK)^i (A^{k-1}(A-LC)\hat{x}_{t-i-k|t-i-k-1} + \\
&\quad \sum_{j=1}^{k} A^{k-j} B u_{t-i-k+j-1} + A^{k-1} LC x_{t-i-k})
\end{aligned}
\tag{13}
$$

Let

$$
\begin{aligned}
X(t+1) = [\, & x_{t+1}^T \; x_t^T \; \cdots \; x_{t-k+2}^T \; x_{t-k-i+1}^T \\
& u_t^T \; u_{t-1}^T \cdots \; u_{t-i}^T \; \cdots \; u_{t-k-i+1}^T \\
& \hat{x}_{t+1|t}^T \; \hat{x}_{t|t-1}^T \; \cdots \; \hat{x}_{t-k-i+2|t-k-i+1} \\
& \hat{x}_{t-k-i+1|t-k-i} \,]
\end{aligned}
$$

$$
\Lambda(k) =
$$

$$
\left[\begin{array}{cccccccccc}
A & 0 & \cdots & 0 & BM_iA^{k-1}LC & 0 & \cdots & BM_iB & \cdots \\
I & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
\vdots & \ddots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots \\
0 & 0 & \ddots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
0 & 0 & \cdots & I & 0 & 0 & \cdots & 0 & \cdots \\
0 & 0 & \cdots & 0 & M_iA^{k-1}LC & 0 & \cdots & M_iB & \cdots \\
0 & 0 & \cdots & 0 & 0 & I & \cdots & 0 & \cdots \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & I & \cdots \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \ddots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
LC & 0 & \cdots & 0 & BM_iA^{k-1}LC & 0 & \cdots & BM_iB & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots
\end{array}\right.
$$

$$
\left.\begin{array}{ccccccc}
BM_iA^{k-2}B & BM_iA^{k-1}B & 0 & 0 & \cdots & 0 & \Pi_{1i} \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
M_iA^{k-2} & M_iA^{k-1}B & 0 & 0 & \cdots & 0 & \Pi_{2i} \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\
I & 0 & 0 & 0 & \cdots & 0 & 0 \\
BM_iA^{k-2}B & BM_iA^{k-1}B & A-LC & 0 & \cdots & 0 & \Pi_{3i} \\
0 & 0 & I & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \cdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ddots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & I & 0
\end{array}\right] \qquad (14)
$$

where $\Pi_{1i} = BM_iA^{k-1}(A-LC)$, $\Pi_{2i} = M_iA^{k-1}(A-LC)$, $\Pi_{3i} = BM_iA^{k-1}(A-LC)$. Then, the closed-loop system can be written as

$$
X(t+1) = \Lambda(k)X(t) \qquad (15)
$$

By matrix algebraic manipulations, it can be proved that all eigenvalues of $\Lambda(k)$ are within the unit circle if and only if all eigenvalues of $\Psi$ are within the unit circle. Therefore, the closed-loop system is stable if and only if all eigenvalues of matrices (11) are within the unit circle (The details of the proof are omitted here due to the limitation of space).

### 3.2 Random Network Delay

It is assumed that the network delay in the forward and feedback channels are random but bounded by $M$ and $N$ respectively, that is, $i_t \in \{0, 1, 2, \cdots, M\}$, $k_t \in \{0, 1, 2, \cdots, N\}$. Then, $i$ and $k$ in constant case are replaced by $i_t$ and $k_t$, respectively. With the algorithm proposed in this paper, all predictive output sequence at time $t$ is packed and sent to the plant side through network. The output networked delay compensator choose the latest output value from the output prediction sequences available on the plant side. The input networked delay compensator chooses the latest control value from the control prediction sequences available on the plant side, the control signal will be obtained every sampling time if the time-delays are less than the corresponding up bounds.

Under this control scheme, the closed-loop system will be a switched linear system, then, the stability of all eigenvalues of $(M+1) \times (N+1)$ matrices will not guarantee that the closed-loop system is stable, based on the theory of switched system, we can design the observer gain $L$ and feedback gain $K$ such that the closed-loop system with random time-delay is stable (Decarlo *et al.*, 2000).

In order to present the results in this section , for simplicity, let

$$
\Lambda(0,0) =
\begin{bmatrix}
\overbrace{\begin{matrix} A & 0 & \cdots & 0 & 0 \end{matrix}}^{(M+N+1)n} & \overbrace{\begin{matrix} 0 & \cdots & 0 & 0 \end{matrix}}^{(M+N)m} & BK & 0 & \cdots & 0 & 0 \\
\begin{matrix} I & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & 0 & 0 & \cdots & 0 & 0 \\
\begin{matrix} \vdots & \vdots & \cdots & \vdots & \vdots \end{matrix} & \begin{matrix} \vdots & \cdots & \vdots & \vdots \end{matrix} & \vdots & \vdots & \cdots & \vdots & \vdots \\
\begin{matrix} 0 & 0 & \cdots & I & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & 0 & 0 & \cdots & 0 & 0 \\
\begin{matrix} 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & K & 0 & \cdots & 0 & 0 \\
\begin{matrix} 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} I & \cdots & 0 & 0 \end{matrix} & 0 & 0 & \cdots & 0 & 0 \\
\begin{matrix} \vdots & \vdots & \cdots & \vdots & \vdots \end{matrix} & \begin{matrix} \vdots & \cdots & \vdots & \vdots \end{matrix} & \vdots & \vdots & \cdots & \vdots & \vdots \\
\begin{matrix} 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & I & 0 \end{matrix} & 0 & 0 & \cdots & 0 & 0 \\
\begin{matrix} LC & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & A+BK-LC & 0 & \cdots & 0 & 0 \\
\begin{matrix} 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & I & 0 & \cdots & 0 & 0 \\
\begin{matrix} \vdots & \vdots & \cdots & \vdots & \vdots \end{matrix} & \begin{matrix} \vdots & \cdots & \vdots & \vdots \end{matrix} & \vdots & \vdots & \cdots & \vdots & \vdots \\
\begin{matrix} 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & 0 & 0 & \cdots & 0 & 0 \\
\begin{matrix} 0 & 0 & \cdots & 0 & 0 \end{matrix} & \begin{matrix} 0 & \cdots & 0 & 0 \end{matrix} & 0 & 0 & \cdots & I & 0
\end{bmatrix} \qquad (16)
$$

$$
\in R^{[2n(M+N)+m(M+N)+2n]\times[2n(M+N)+m(M+N)+2n]}
$$

$$
\Lambda(i_t, 0) =
\begin{bmatrix}
A & \overbrace{0 \; \cdots}^{i_t n} & M_1(i_t) & \cdots & 0 \\
I & 0 \; \cdots & 0 & \cdots & 0 \\
\vdots & \vdots \; \cdots & \vdots & \cdots & \vdots \\
0 & 0 \; \cdots & I & \cdots & 0 \\
0 & 0 \; \cdots & 0 & \cdots & 0 \\
0 & 0 \; \cdots & 0 & \cdots & 0 \\
\vdots & \vdots \; \cdots & \vdots & \cdots & \vdots \\
0 & 0 \; \cdots & 0 & \cdots & 0 \\
LC & 0 \; \cdots & 0 & \cdots & 0 \\
0 & 0 \; \cdots & 0 & \cdots & 0 \\
\vdots & \vdots \; \cdots & \vdots & \cdots & \vdots \\
0 & 0 \; \cdots & 0 & \cdots & 0 \\
0 & 0 \; \cdots & 0 & \cdots & 0
\end{bmatrix}
$$

$$
\left.\begin{matrix}
\overbrace{0 \cdots 0\ 0}^{(M+N)m} & 0 & \cdots & M_2(i_t) & \cdots\ 0\ 0 \\
0 \cdots 0\ 0 & 0 & \cdots & 0 & \cdots\ 0\ 0 \\
\vdots \cdots \vdots\ \vdots & \vdots & \cdots & \vdots & \cdots\ \vdots\ \vdots \\
0 \cdots 0\ 0 & 0 & \cdots & 0 & \cdots\ 0\ 0 \\
0 \cdots 0\ 0 & K & \cdots & 0 & \cdots\ 0\ 0 \\
I \cdots 0\ 0 & 0 \cdots & 0 & \cdots & 0\ \ 0 \\
\vdots \cdots \vdots\ \vdots & \vdots & \cdots & \vdots & \cdots\ \vdots\ \vdots \\
0 \cdots I\ 0 & 0 & \cdots & 0 & \cdots\ 0\ 0 \\
0 \cdots 0\ 0 & M_4(i_t) & \cdots & 0 & \cdots\ 0\ 0 \\
0 \cdots 0\ 0 & I & \cdots & 0 & \cdots\ 0\ 0 \\
\vdots \cdots \vdots\ \vdots & \vdots & \cdots & \vdots & \cdots\ \vdots\ \vdots \\
0 \cdots 0\ 0 & 0 & \cdots & 0 & \cdots\ 0\ 0 \\
0 \cdots 0\ 0 & 0 & \cdots & 0 & \cdots\ I\ 0
\end{matrix}\right] \tag{17}
$$

$$\in R^{[2n(M+N)+m(M+N)+2n]\times[2n(M+N)+m(M+N)+2n]}$$

where $i_t = 1, 2, \cdots, N_{ca}$.

$$
\Lambda(i_t, k_t) = \begin{bmatrix}
\Lambda_{11}(i, k_t) & \Lambda_{12}(i, k_t) & \Lambda_{13}(i, k_t) \\
\Lambda_{21}(i, k_t) & \Lambda_{22}(i, k_t) & \Lambda_{23}(i, k_t) \\
\Lambda_{31}(i, k_t) & \Lambda_{32}(i, k_t) & \Lambda_{33}(i, k_t)
\end{bmatrix} \tag{18}
$$

$$\in R^{[2n(M+N)+m(M+N)+2n]\times[2n(M+N)+m(M+N)+2n]}$$

for $i = 0, 1, 2, \cdots, M$ and $k_t = 1, 2, \cdots, N$,

$$
\Lambda_{11}(i_t, k_t) = \begin{bmatrix}
A & \overbrace{0 \cdots 0}^{(i_t+k_t-1)n} & BM_{i_t}A^{k_t-1}LC & 0 \cdots 0\ 0 \\
I & 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots & \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0 & 0 \cdots I & 0 & 0 \cdots 0\ 0 \\
\vdots & \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0 & 0 \cdots 0 & 0 & 0 \cdots I\ 0
\end{bmatrix}
$$

$$\in R^{[n(M+N)+n]\times[n(M+N)+n]} \tag{19}$$

$$
\Lambda_{12}(i_t, k_t) = \begin{bmatrix}
\overbrace{0 \cdots 0}^{i_t n} & \Pi(1) & \cdots & \Pi(2) & \Pi(3) & 0 \cdots 0\ 0 \\
0 \cdots 0 & 0 & \cdots & 0 & 0 & 0 \cdots 0\ 0 \\
\vdots \cdots \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0 \cdots 0 & 0 & \cdots & 0 & 0 & 0 \cdots 0\ 0 \\
0 \cdots 0 & 0 & \cdots & 0 & 0 & 0 \cdots 0\ 0 \\
\vdots \cdots \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0 \cdots 0 & 0 & \cdots & 0 & 0 & 0 \cdots 0\ 0 \\
0 \cdots 0 & 0 & \cdots & 0 & 0 & 0 \cdots 0\ 0
\end{bmatrix}
$$

$$\in R^{[n(M+N)+n]\times[m(M+N)]} \tag{20}$$

$$
\Lambda_{13}(i_t, k_t) = \begin{bmatrix}
\overbrace{0\ 0 \cdots 0}^{k_t n} & \Pi(4) & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots\ \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots\ \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0
\end{bmatrix} \tag{21}
$$

$$\in R^{[n(M+N)+n]\times[n(M+N)+n]}$$

$$
\Lambda_{21}(i_t, k_t) = \begin{bmatrix}
\overbrace{0\ 0 \cdots 0}^{k_t n} & M_{i_t}A^{k_t-1}LC & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots\ \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots\ \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0
\end{bmatrix} \tag{22}
$$

$$\in R^{[m(M+N)]\times[n(M+N)+n]}$$

$$
\Lambda_{22}(i_t, k_t) = \begin{bmatrix}
\overbrace{0 \cdots 0}^{i_t n} & \Pi(5) & \Pi(6) & \cdots & \Pi(7) & \Pi(8) & 0 \cdots & 0 & 0 \\
I \cdots 0 & 0 & 0 & \cdots & 0 & 0 & 0 \cdots & 0 & 0 \\
\vdots \cdots \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \cdots & \vdots & \vdots \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & 0 \cdots & 0 & 0 \\
0 \cdots 0 & 0 & 0 & \cdots & I & 0 & 0 \cdots & 0 & 0 \\
\vdots \cdots \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots\ \vdots \cdots & \vdots & \vdots \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & 0 \cdots & 0 & 0 \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & 0 \cdots & I & 0
\end{bmatrix}
$$

$$\in R^{[m(M+N)]\times[m(M+N)]} \tag{23}$$

$$
\Lambda_{23}(i_t, k_t) = \begin{bmatrix}
\overbrace{0\ 0 \cdots 0}^{k_t n} & \Pi(9) & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots\ \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
\vdots\ \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots\ \vdots \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0 \\
0\ 0 \cdots 0 & 0 & 0 \cdots 0\ 0
\end{bmatrix} \tag{24}
$$

$$\in R^{[m(M+N)]\times[n(M+N)+n]}$$

$$\Lambda_{31}(i_t, k_t) =$$

$$
\begin{bmatrix}
LC & \overbrace{0 \cdots 0}^{(i_t+k_t-1)n} & M_{i_t}A^{k_t-1}LC & 0 \cdots 0 & 0 \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0 & 0 \\
\vdots & \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots & \vdots \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0 & 0 \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0 & 0 \\
\vdots & \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots & \vdots \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0 & 0 \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0 & 0
\end{bmatrix}
$$

$$
\in R^{[n(M+N)+n]\times[n(M+N)+n]} \tag{25}
$$

$$
\Lambda_{32}(i_t,k_t) =
\begin{bmatrix}
\overbrace{0 \cdots 0}^{i_t n} & \Omega(1) & \Omega(2) & \cdots & \Omega(3) & \Omega(4) & \cdots 0 & 0 \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & \cdots 0 & 0 \\
\vdots \cdots \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots \vdots & \vdots \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & \cdots 0 & 0 \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & \cdots 0 & 0 \\
\vdots \cdots \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots \vdots & \vdots \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & \cdots 0 & 0 \\
0 \cdots 0 & 0 & 0 & \cdots & 0 & 0 & \cdots 0 & 0
\end{bmatrix}
$$

$$
\in R^{(n(M+N)+n)\times(mN)} \tag{26}
$$

$$
\Lambda_{33}(i_t,k_t) =
\begin{bmatrix}
A-LC & \overbrace{0 \cdots 0}^{(i_t+k_t-1)n} & \Omega(5) & 0 \cdots 0 & 0 \\
I & 0 \cdots 0 & 0 & 0 \cdots 0 & 0 \\
\vdots & \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots & \vdots \\
0 & 0 \cdots I & 0 & 0 \cdots 0 & 0 \\
0 & 0 \cdots 0 & I & 0 \cdots 0 & 0 \\
\vdots & \vdots \cdots \vdots & \vdots & \vdots \cdots \vdots & \vdots \\
0 & 0 \cdots 0 & 0 & 0 \cdots 0 & 0 \\
0 & 0 \cdots 0 & 0 & 0 \cdots I & 0
\end{bmatrix}
\in R^{(n(M+N)+n)\times(n(M+N)+n)} \tag{27}
$$

where $M_{i_t} = K(A+BK)^{i_t}$, $M_1(i_t) = BK(A+BK)^{i_t-1}LC$, $M_2(i_t) = BK(A+BK)^{i_t-1}(A+BK-LC)$, $M_3(i_t) = LC$, $M_4(i_t) = A+BK-LC$, $\Pi(1) = M_{i_t}B$, $\Pi(2) = BM_{i_t}A^{k_t-2}B$, $\Pi(3) = BM_{i_t}A^{k_t-1}B$,$\Pi(4) = BM_{i_t}A^{k_t-1}(A-LC)$, $\Pi(5) = M_{i_t}A^{k_t-1}LC$, $\Pi(6) = M_{i_t}AB$, $\Pi(7) = M_{i_t}A^{k_t-2}B$, $\Pi(8) = M_{i_t}A^{k_t-1}B$, $\Pi(9) = M_{i_t}A^{k_t-1}(A-LC)$, $\Omega(1) = BKB$, $\Omega(2) = BKAB$, $\Omega(3) = BK^{k_t-2}AB$, $\Omega(4) = BKA^{k_t-1}AB$ , $\Omega(5) = BKA^{k_t-1}(A-LC)$.

Then, the main results in this section can be stated as follows:

*Theorem 3.2.* For the networked predictive control systems with random network delay in both forward and feedback channels, the closed-loop system is stable if there exists a positive definite matrix

$$
P \in R^{[2n(M+N)+m(M+N)+2n]\times[2n(M+N)+m(M+N)+2n]}
$$

such that

$$
\Lambda^T(i_t,k_t)P\Lambda(i_t,k_t) - P < 0 \tag{28}
$$

for $i_t = 0,1,2,\cdots,M$ and $k_t = 0,1,2,\cdots,N$.
**Proof**. The proof is omitted due to the limitation of space.

*Remark 3.1.* It can be deduced easily from (28) that each subsystem is stable. If $K$ and $L$ are designed previously, then (28) is a set of LMIs. LMI toolbox can be used to find feasible solution $P$ (Boyd *et al.*, 1994)

## 4. CONCLUSION

The design and stability analysis of networked predictive control systems have been discussed in this paper. The network delays in in both forward and feedback channels have been considered in two cases: the fixed network delay and the random network delay. For both cases, the stability criteria have been obtained for networked predictive control. Particularly, in the case of random network delay, it has been concluded that the closed-loop networked predictive control system is stable if the corresponding switched system is stable. This gives fundamental results for the design and analysis of networked control systems.

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