TIME DOMAIN DECOMPOSITION IN SOLUTION OF SINGULAR NONLINEAR OPTIMAL CONTROL PROBLEMS

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Abstract: Method for optimal control calculation for discrete optimal control problems characterized by non-quadratic criterion, nonlinear model with affine control, state and control delays and constraints is developed. An augmented functional of Lagrange is applied and its decomposition in time domain is proposed using new coordinating vector for dual decomposition in order to calculate the optimal state and control trajectories. The method is applied to solve the problems for maximum production of batch and minimum start-up time of continuous fermentation processes.

Keywords: nonlinear systems, time delays, constraints, optimal control, singular control, augmented Lagrange;s functional, time domain decomposition, fermentation processes

1. INTRODUCTION

The industrial processes are characterised by high dimension and complexity, non-linearity, multiple time delays, goals, uncertainties, constraints. It is well known that the methods of the optimal control theory have some difficulties in solution of optimal control problems for plants with such characteristics (Brayson and Ho, 1969). They take also considerable computational time. New methods, algorithms and programmes are needed. Large scale systems theory and methodology gives a possibility the problems for optimal control of the technological processes to be formulated in the frameworks of hierarchical structures (Singh and Titli, 1978), based on the information flows, the principle of given hierarchy, and the analytical and numerical methods to be used. This approach is realised by the central concepts of decomposition and coordination (Singh and Titli, 1978; Bertsekas, 1979). An overall optimisation problem might be decomposed into governed by interconnected subsystems series of sub-problems with lower dimension or lower complexity. The solution of the overall problem can be obtained through a coordination process of iterative computations between the levels of hierarchy.

The decomposition method of Tamura (Tamura, 1975) is very useful because it allows the global problem to be decomposed in time domain. It gives very simple solution for systems with state and control delays as the delays can be considered as predictions in the coordinating vector of the conjugate variables. The method is developed for a quadratic criterion. When the criterion is non-quadratic one the optimal control problem can be solved using augmented towards the model equations Lagrange functional (Bertsekas, 1979). In this case it appears that application of the coordination procedure of prediction of the conjugate variables can not decompose the augmented functional in time domain because its dual functional is not any more separable (Lin, 1992). It is necessary to find additional variables to be selected as coordinating ones in order to obtain full time domain decomposition. This possibility is investigated in the paper. The results obtained for different types of optimal control problems (Tsoneva and Patarinska 1995; Tsoneva *et al.*, 1998) are generalised in the paper as a method for nonlinear optimal control problems solution. The application of the method for two fermentation processes is given.

2. PROBLEM STATEMENT

The optimal control problem is: find the control $\mathbf{u}(k)$, $k=\overline{0,K-1}$, such that the criterion

$$J=\mathbf{f}_0(\mathbf{x},K)\,, \qquad (1)$$

is maximised, subject to the model equations

$$\mathbf{x}(k+1)=\mathbf{x}(k)+\Delta t\{\mathbf{f}_1[\mathbf{x}(k),k,\mathbf{a}]+$$
$$+\mathbf{f}_2[\mathbf{x}(k-\tau),k,\mathbf{a}]+\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]\mathbf{u}(k-\theta)\}=\boldsymbol{f}(k), \qquad (2)$$

$$\mathbf{x}(k)=\varphi_x(k),\ k\in[-\tau,\ 0]\,,\ \mathbf{x}(0)=\varphi_x(0)\,,$$
$$\mathbf{u}(k)=\varphi_u(k),\ k\in[-\theta,-1]\,, \qquad (3)$$

and constraints

$$\mathbf{x}_{\min}(k)\le\mathbf{x}(k)\le\mathbf{x}_{\max}(k)\,,\ k=\overline{0,K},$$
$$\mathbf{u}_{\min}(k)\le\mathbf{u}(k)\le\mathbf{u}_{\max}(k)\,,\ k=\overline{0,K-1}, \qquad (4)$$

where $\mathbf{x}\in R^n, \mathbf{u}\in R^m$ are the vectors of state and control variables, $\mathbf{f}_0:R^n\to R$ is the criterion function determined at the end point of the optimisation interval, $\mathbf{f}_1,\mathbf{f}_2\in R^n$, $\mathbf{f}_3\in R^{nxm}$ are the continuous and continuously differentiable functions of state, τ and θ are the constant state and control delays, φ_x,φ_u are the initial functions, Δt is the discretization period, K is the number of steps in the optimization horizon, $\mathbf{x}_{\min},\mathbf{x}_{\max},\mathbf{u}_{\min},\mathbf{u}_{\max}$ are the bound values of the constraints, $\mathbf{a}\in R^r$ is the vector of the parameters.

3. DECOMPOSITION METHOD

The optimal control problems can be solved using a functional of Lagrange

$$L=\mathbf{f}_0(\mathbf{x},K)+\sum_{k=0}^{K-1}\{\lambda^T(k)[-\mathbf{x}(k+1)+\mathbf{f}(k)]\,, \qquad (5)$$

where $\lambda(k)\in R^n$ is the conjugate variables' vector. As the control vector does not appear in (1) and enters linearly the model equations, the considered problem is a singular one. The control vector does not appear in the necessary conditions for optimality

$$\frac{\partial L}{\partial\mathbf{u}(k)}=\Delta t\mathbf{f}_3[\mathbf{x}(k+\theta),k+\theta,\mathbf{a}]^T\,\lambda(k+\theta)=0$$

and it has to be calculated using other approaches (Brayson and Ho, 1969; Bertsekas, 1979). If an augmented Lagrange's functional with penalty part according to the model equations is used

$$L_a=\mathbf{f}_0(\mathbf{x},K)+\sum_{k=0}^{K-1}\{\boldsymbol{\lambda}^T(k)[-\mathbf{x}(k+1)+\mathbf{f}(k)]+$$
$$+(1/2)\mu[-\mathbf{x}(k+1)+\mathbf{f}(k)]^2\}\,, \qquad (6)$$

where μ is a penalty coefficient, the control vector appears in a quadratic term. In this case the functional of Lagrange is quadratic according to the control vector and the necessary condition of optimality is an analytical function of the control vector. Then as the considered problem is characterised with state and control time delays, the method of Tamura (Tamura, 1975) can be applied. The optimal control problem (6) can be solved on the basis of the necessary conditions for optimality

$$\frac{\partial L_a}{\partial\boldsymbol{\lambda}(k)}=-\mathbf{x}(k+1)+\mathbf{x}(k)+\Delta t\{\mathbf{f}_1[\mathbf{x}(k),k,\mathbf{a}]+$$
$$\mathbf{f}_2[\mathbf{x}(k-\tau),k,\mathbf{a}]+\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]\mathbf{u}(k-\theta)\}=\mathbf{e}_\lambda(k),$$

$$\frac{\partial L_a}{\partial\mathbf{x}(k)}=-\boldsymbol{\lambda}(k-1)+\{\mathbf{I}+\Delta t\{\frac{\partial\mathbf{f}_1[\mathbf{x}(k),k,\mathbf{a}]}{\partial\mathbf{x}(k)}+$$
$$+[\partial\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]/\partial\mathbf{x}(k)]\mathbf{u}(k-\theta)\}^T\}\boldsymbol{\lambda}(k)+$$
$$+\Delta t[\partial\mathbf{f}_2[\mathbf{x}(k),k,\mathbf{a}]/\partial\mathbf{x}(k)]^T\boldsymbol{\lambda}(k+\tau)+$$
$$+\mu\{-\mathbf{x}(k+1)+\mathbf{x}(k)+\Delta t\{\mathbf{f}_1[\mathbf{x}(k),k,\mathbf{a}]+$$
$$+\mathbf{f}_2[\mathbf{x}(k-\tau),k,\mathbf{a}]+\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]\mathbf{u}(k-\theta)\}\}^T.$$
$$.\{[\partial\mathbf{x}(k+1)/\partial\mathbf{x}(k)]+\mathbf{I}+\Delta t\{[\partial\mathbf{f}_1[\mathbf{x}(k),k,\mathbf{a}]/\partial\mathbf{x}(k)]+$$
$$+[\partial\mathbf{f}_2[\mathbf{x}(k-\tau),k,\mathbf{a}]/\partial\mathbf{x}(k)]+$$
$$[\partial\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]\mathbf{u}(k-\theta)]/\partial\mathbf{x}(k)\}=\mathbf{e}_x(k)=0\,,$$
$$k=\overline{0,K-1}\,.$$

$$\frac{\partial L_a}{\partial\mathbf{x}(K)}=\partial\mathbf{f}_0(x(K),K)/\partial\mathbf{x}(K)=\mathbf{e}_x(K)=0\,,$$

$$\frac{\partial L_a}{\partial\mathbf{u}(k)}=\Delta t\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]^T\,\boldsymbol{\lambda}(k+\theta)+$$
$$+\mu\{-\mathbf{x}(k+1)+\mathbf{x}(k)+\Delta t\{\mathbf{f}_1[\mathbf{x}(k),k,\mathbf{a}]+$$
$$\mathbf{f}_2[\mathbf{x}(k-\tau),k,\mathbf{a}]+\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]\mathbf{u}(k-\theta)\}\}^T.$$
$$.[\partial\mathbf{f}_3[\mathbf{x}(k),k,\mathbf{a}]\mathbf{u}(k-\theta)/\partial\mathbf{u}(k)]=\mathbf{e}_u(k)=0\,,$$
$$k=\overline{0,K-1}\,, \qquad (7)$$

where the conjugate variables are selected as coordinating ones. But the separability of the dual problem can not be reached as the values of the variables in the time moments k can not be separated because of the vector of variables $\mathbf{x}(k+1)$, which can be considered as vector of interconnections in time domain. In order to overcome these difficulties it is proposed to extend the coordination vector with the interconnections vector in time domain

$$\boldsymbol{\rho}(k)=\mathbf{x}(k+1), k=\overline{0,K-1}, \qquad (8)$$

Their values will be set at the beginning and calculated in the course of problem solution process. Then the optimal control problem can be solved in two level calculating structure using the new coordinating vector.

The values of the coordinating variables are set from the second level of the two level calculating structure:

$\boldsymbol{\lambda}(k) = \boldsymbol{\lambda}^j(k), k = \overline{0, K}$, $\boldsymbol{\rho}(k) = \boldsymbol{\rho}^j(k), k = \overline{0, K-1}$,(9)

where j is the index of the coordinating process iterations. When the values of the coordinating variables are substituted into the Lagrange's functional, its full decomposition according to the discrete time moments k is obtained. The functional is decomposed into $K+1$ sub-functionals $L_a(k)$ and each of them determines the optimal control and state at the given moment k. The coordinating sub-problem is obtained and solved on the basis of the necessary conditions for optimality

$$\partial L_a / \partial \boldsymbol{\lambda}^j(k) = \mathbf{e}_\lambda^j(k) = 0, k = \overline{0, K},$$
$$\partial L_a / \partial \boldsymbol{\rho}^j(k) = \mathbf{e}_\rho^j(k) = 0, k = \overline{0, K-1}. \quad (10)$$

The coordinating sub-problem is solved using a gradient procedure for $\boldsymbol{\lambda}$ and direct expressing of $\boldsymbol{\rho}$ from (10). The solutions have the form:

$$\boldsymbol{\lambda}^{j+1}(k) = \boldsymbol{\lambda}^j(k) - \mu^j \mathbf{e}_\lambda^j(k), k = \overline{0, K-1}, \quad (11)$$

$$\mathbf{e}_\lambda^j(k) = -\boldsymbol{\rho}^j(k) + \mathbf{x}^j(k) + \Delta t\{\mathbf{f}_1[\mathbf{x}^j(k), k, \mathbf{a}] +$$
$$\mathbf{f}_2[\mathbf{x}^j(k-\tau), k, \mathbf{a}] + \mathbf{f}_3[\mathbf{x}^j(k), k, \mathbf{a}]\mathbf{u}^j(k-\theta)\},$$
$$k = \overline{0, K-1}, \ \mathbf{e}_\lambda^j(K) = 0, \quad (12)$$

$$\boldsymbol{\rho}^{j+1}(k) = \boldsymbol{\lambda}^j(k)/\mu^j + \mathbf{e}_\lambda^j(k) + \boldsymbol{\rho}^j(k), \ k = \overline{0, K-1} \quad (13)$$

In (11)-(13) the values of $\mathbf{x}^j(k), \mathbf{u}^j(k)$ are obtained after solving the first level sub-problems with the set values of $\boldsymbol{\lambda}^j(k), k = \overline{0, K}, \boldsymbol{\rho}^j(k), k = \overline{0, K-1}$.

Coordination process terminates upon satisfaction of some error conditions:

$$\left\| e_\lambda^{j+1}(k) - e_\lambda^j(k) \right\| \le \varepsilon_\lambda, \ \varepsilon_\lambda > 0, \ k = \overline{0, K}, \quad (14)$$
$$\left\| e_\rho^{j+1}(k) - e_\rho^j(k) \right\| \le \varepsilon_\rho, \ \varepsilon_\rho > 0, \ k = \overline{0, K-1}. \quad (15)$$

If these conditions are not satisfied, the first level sub-problems are solved with the obtained values of the co-ordinating variables and new values of the penalty coefficient. Its value for the new iteration can be calculated according to Algorithm 1, using the obtained coordinating vector gradients in order to achieve quick convergence of the coordinating sub-problem solution

Algorithm 1:

1.The error is computed

$$e^{j+1} = \{ \left\| e_\lambda^{j+1}(k) \right\|^2 + \left\| e_\rho^{j+1}(k) \right\|^2 \}^{1/2}, \quad (16)$$

2. The new penalty coefficient is calculated from the conditions:

if j=1 or $e^{j+1} < e^j$,then $\mu^{j+1} = \mu^j$, (17)

if j>1 and $e^{j+1} \ge e^j$ then $\mu^{j+1} = a\mu^j$, α=[0.1, 10.0]

The first level sub-problems are determined under the set from the second level coordinating variables according to the necessary conditions for optimality of the sub-functionals $L_a(k)$:

$$\partial L_a(k) / \partial \mathbf{x}^{j,l}(k) = \mathbf{e}_x^{j,l}(k) = 0, \ k = \overline{0, K}, \quad (18)$$
$$\partial L_a(k) / \partial \mathbf{u}^{j,l}(k) = \mathbf{e}_u^{j,l}(k) = 0, \ k = \overline{0, K-1}$$

and are solved by gradient procedures:

- for the state variables:

$$\mathbf{x}^{j,l+1}(k) = \mathbf{x}^{j,l}(k) + \alpha_x \mathbf{e}_x^{j,l}(k), \quad (19)$$

$$e^{j,l}{}_x(k) = \{\mathbf{I} + \Delta t\{\partial \mathbf{f}_1[\mathbf{x}^{j,l}(k), k, \mathbf{a}] / \partial \mathbf{x}(k) +$$
$$+ [\partial \mathbf{f}_3[\mathbf{x}^{j,l}(k), k, \mathbf{a}] / \partial \mathbf{x}(k)] \mathbf{u}^{j,l}(k-\theta)\}\}^T \times$$
$$\times [\lambda^j(k) - \mu \mathbf{e}_\lambda^j(k)] +$$
$$+ \Delta t [\partial \mathbf{f}_2[\mathbf{x}^{j,l}(k), k, \mathbf{a}] / \partial \mathbf{x}(k)]^T \times$$
$$\times [\lambda^j(k+\tau) - \mu \mathbf{e}_\lambda^j(k+\tau)], \ k = \overline{0, K-1}, \quad (20)$$

where $\mathbf{u}^{j,l}(k) = \varphi_u(k), k \in [-\theta, -1]$,

$\mathbf{x}^{j,l}(k) = \varphi_x(k), k \in [-\tau, -1]$,

$\boldsymbol{\lambda}^j(k+\tau) = 0, \ \mathbf{x}^{j,l}(k+\tau) = 0 \ if \ k+\tau > K$,

$\boldsymbol{\rho}^j(k+\tau) = 0 \ if \ k+\tau > K-1$,

$\mathbf{u}^{j,l}(k-\theta+\tau) = 0 if\, k-\theta+\tau > K-1$

-for the control variables:

$$\mathbf{u}^{j,l+1}(k) = \mathbf{u}^{j,l}(k) + \alpha_u \mathbf{e}_u^{j,l}(k), \quad (21)$$

$$e_u^{j,l}(k) = \Delta t \{\mathbf{f}_3[\mathbf{x}^{j,l}(k+\theta), k+\theta, \mathbf{a}]\}^T \times$$
$$\times [\boldsymbol{\lambda}^j(k+\theta) + \mu^j e_\lambda^{j,l}(k+\theta)], k = \overline{0, K-1}, \quad (22)$$

where $\mathbf{x}^{j,l}(k) = \varphi_x(k), k \in [-\tau, -1]$,

$\mathbf{x}^{j,l}(k+\theta) = 0, \boldsymbol{\lambda}^j(k+\theta) = 0 \ if \ k+\theta > K$,

$\boldsymbol{\rho}^j(k+\theta) = 0 \ if \ k+\theta > K-1$,

$\mathbf{x}^{j,l}(k-\tau+\theta) = 0 if\, k-\tau+\theta > K$,

l is the iteration index. Calculations terminate upon satisfaction of error conditions. To account for the constraints the obtained values of the state and control trajectories are projected over their domains (4) respectively:

$$\mathbf{x}^{j,l}(k) = \begin{cases} \mathbf{x}_{min}(k), \ \mathbf{x}^{j,l}(k) < \mathbf{x}_{min}(k), \\ \mathbf{x}^{j,l}(k), \mathbf{x}_{min}(k) \le \mathbf{x}^{j,l}(k) \le \mathbf{x}_{max}(k), \\ \mathbf{x}_{max}(k), \mathbf{x}^{j,l}(k) > \mathbf{x}_{max}(k), \ k = \overline{0, K}. \end{cases} \quad (23)$$

The projection of the calculated values of the control trajectory over the constraint domain is done as in (23), only the time horizon is $k = \overline{0, K-1}$.

The computational procedure for solving the optimal control problem is organised in two level structure Fig.1., according to the following:

Algorithm 2:

1.The values of the coordinating variables and those of the penalty coefficients are set at the second level $\lambda^j(k), \rho^j(k), \mu^j, k=\overline{0,K-1}$ and are transferred to the first one, j=*1*.
2.At the first level, the initial control trajectory is set and the initial trajectory of the state is calculated, *l=1, j=1*.
3.At the first level the gradients $e_x^{j,l}(k),\ k=\overline{0,K-1}, e_u^{j,l}(k),\ k=\overline{K-1}$ are calculated and the new state and control trajectories are obtained from equations (19)-(21). They are projected onto the constraint domain (23).
4.The state and control error conditions are checked. If they are satisfied, the obtained state and control trajectories are transferred to the second level. If the conditions are not satisfied, items 3), 4) are repeated, *l=l+1*.
5.The new values of the coordinating variables are calculated from (11)-(13), j=*j+1*. The conditions (14),(15) are checked. If they are satisfied, the optimal solutions of the coordinating sub-problem and of the global problem are obtained. If these conditions are not satisfied, new values of the penalty coefficients are calculated according to (16),(17) and items 3),4),5) are repeated, and so on.
The convergence of the algorithm is found to depend on the selection of the initial trajectories of state and control vectors and of the initial trajectory of the conjugate variable vector. The adaptive selection of the gradient procedure step sizes, allows to make the convergence faster. Different types of gradient procedures can be used in the above method.

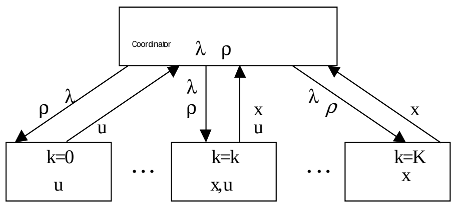


Fig.1. Two level calculating structure

4.MAXIMUM PRODUCTION OF XANTHAN GUM IN A BATCH FERMENTATION PROCESS

Xanthan gum, produced by Xanthomonas campestris -- ITS-342, is considered The main problem in producing xanthan is the enormous increase of viscosity as the product accumulates because of changes in oxygen transfer in the fermentor. The influence of the oxygen supply is reflected in the mathematical model. The process is described by the following three difference equations

$$x(k+1)=x(k)+$$
$$+b_{x1}\Delta t\{1-[x(k)/b_{x2}]^{b_{x3}}\}x(k)=f_x(k),$$
$$s(k+1)=s(k)-$$
$$-b_{s1}\Delta t\{1-p(k)/[b_{s2}u(k)+b_{s22}]\}x(k)=f_s(k),$$
$$p(k+1)=p(k)+$$
$$+b_{p1}\Delta t\{1-p(k)/[b_{p2}u(k)+b_{p22}]\}x(k)=f_p(k),$$
$$x(0)=x_0,\ s(0)=s_0,\ p(0)=p_0=0 \qquad (24)$$

where *x,s,p,u* [g/l] are respectively the concentrations of biomass, substrate, xanthan gum and dissolved oxygen, *b* are the parameters. The oxygen concentration is considered as a control input.

The optimal control problem is to find the control trajectory $u(k), k=\overline{0,K-1}$, and the state trajectories $v(k)=x(k), s(k), p(k),\ k=\overline{0,K}$, which maximize the end concentration of the xanthan gum:

$$J=p(K)\rightarrow \max, \qquad (25)$$

under the model equations (24) and satisfy constraints of the type (4)

The optimal control problem is solved using the above decomposition method. An augmented Lagrange's functional is introduced:

$$L_a=p(K)+\sum_{k=0}^{K-1}\sum_{v=x,s,p}\{\lambda_v(k)[-v(k+1)+f_v(k)]+$$
$$+(1/2)\mu_v[-v(k+1)+f_v(k)]^2\}, \qquad (26)$$

The selection of the coordinating variables is:

$$\lambda_v(k)=\lambda_v^j(k), \rho_v(k)=\rho_v^j(k), v=x,s,p, \qquad (27)$$
$$\rho_v(k)=v(k+1), k=\overline{0,K-1}, v=x,s,p, \qquad (28)$$

The coordinating sub-problem is obtained from the necessary conditions for optimality and is the same as the sub-problem (11)-(14). The optimal control sub-problems, defined by the sub-functionals $L_a(k)$ are solved by gradient procedures according to the necessary conditions for optimality:

$$\partial L_a(k)/\partial v(k)=0, v=x,s,p,u, k=\overline{0,K-1}. (29)$$

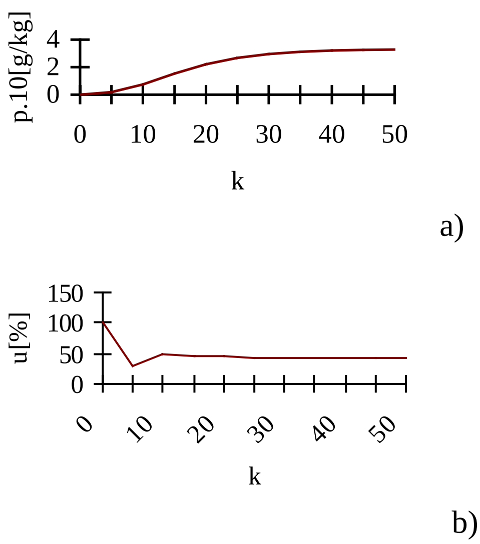


Fig.2.Optimal product (a) and control (b) trajectories

The first level sub-problems for are:

$$v^{j,l+1}(k) = v^{j,l}(k) + \alpha_v e_v^{j,l}(k), \qquad (30)$$

where the gradients are obtained at each moment k and have the form

$$e_v^{j,l}(k) = \sum_{z=x,s,p} [\partial f_z^{j,l}(k)/\partial v(k)][\lambda_z^{j,l}(k) - \mu_z^j e_{\lambda z}^{j,l}(k)],$$

$$v = x,s,p,u, k = \overline{0,K-1} \qquad (31)$$

and $e_{\lambda z}^{j,l}(k), z = x,s,p,\ k = \overline{0,K-1}$ are the values of the gradients of the Lagrange's functional according to the conjugate variables. The results are given in Fig.2.

5.OPTIMAL START-UP OF CONTINUOUS FERMENTATION PROCESS

The problem for minimizing start-up time of continuous fermentation process is considered (Tzoneva and Patarinska, 1995). Dynamic behaviour of these processes is nonlinear one with time delays in states. The process for growth of *Saccharomyces cerevisiae* is taken under study.

Mathematically, discrete minimum time control problem for the continuous fermentation processes is:
- find the control trajectory $D(k)$, $k = \overline{0,K-1}$, which in minimum time

$$J = K\Delta t \qquad (32)$$

leads the system

$$x(k+1) = \{1 + \Delta t[[\mu_m s(k-\tau)/k_s + s(k-\tau)] - k_D]\}x(k) - \Delta t D(k)[x(k) - x^0] = f_x(k), \qquad (33)$$

$$s(k+1) = \{1 - \Delta t \mu_m x(k)/Y[k_s + s(k)]\}s(k) + +\Delta t D(k)[s^0(k) - s(k)] = f_s(k), \qquad (34)$$

from the initial state

$$x(0) = x_0,\ s(0) = s_0,\ s(k) = \varphi_s(k),\ k \in [-\tau,\ 0], \qquad (35)$$

to the optimal steady state

$$x(K) = \bar{x},\ \ s(K) = \bar{s}, \qquad (36)$$

while satisfying constraints of the type (4) for the state and control variables and a constrains for the sampling interval

$$0 < \Delta t \le 0.01[k_s + s_{\max}]/ /[(\mu_m - k_D - D_{\max})s_{\max} - k_s(k_D + D_{\max})], \qquad (37)$$

determined on the basis of stability of the discrete time model behaviour. In the above problem x, s, x^0, s^0 are the biomass, the limiting substrate, the inlet biomass and the inlet substrate concentrations *[g/l]*, D is the dilution rate $[h^{-1}]$, μ_m is the maximum growth rate $[h^{-1}]$,Y is the yield coefficient *[g/g]*, k_s is the Michaelis-Menten parameter *[g/l]*, τ is the time delay *[h]*, k_D is the dead constant $[h^{-1}]$, x_0, s_0 are the initial concentrations *[g/l]*, $\bar{x}, \bar{s}$ are the steady state values *[g/l]*, $v_{\min}, v_{\max}, v = x,s,D$, are the bound values of variables. The dilution rate is the control signal, the biomass and the limiting substrate are the process states.

The minimum time problem is solved when the number of the steps in the optimization horizon is known. The minimum time problem is transformed into a problem for minimizing the value of the sampling interval in the limits (37) and it is solved using the following augmented functional of Lagrange:

$$L_a = K\Delta t + + \sum_{v=x,s}\{\lambda_v(K)[v(K) - \bar{v}] + (1/2)\mu_v[v(K) - \bar{v}]^2\} + + \sum_{k=0}^{K-1}\sum_{v=x,s}\{\lambda_v(k)[-v(k+1) + f_v(k)] + + (1/2)\mu_v[-v(k+1) + f_v(k)]^2\}, \qquad (38)$$

The coordinating variables are

$$\lambda(k) = \lambda^j(k), k = \overline{0,K}, \qquad \Delta t = \Delta t^j$$

$$\rho(k) = v(k+1),\ \ \rho(k) = \rho^j(k),,\ k = \overline{0,K-1}, \qquad (39)$$

The coordinating sub-problem for λ, ρ is given by the equations (11)-(14), but with a positive step of the gradient procedure, where

$$e_{\lambda x}^{(k)} = -\rho_x^j(k) + \{1 + \Delta t^j[\frac{\mu_m s^j(k-\tau)}{k_s + s^j(k-\tau)} - k_D]\}x^j(k) - - \Delta t^j D^j(k)[x^j(k) - x^0] + \Delta t^j w_1(k), k = \overline{0,K-1},$$

$$e_{\lambda s}^{(k)} = -\rho_s^j(k) + \{1 - \Delta t^j \frac{\mu_m x^j(k)}{Y[k_s + s^j(k)]}\}s^j(k) + + \Delta t^j D^j(k)[s^0 - s^j(k)] + \Delta t^j w_{21}(k), k = \overline{0,K-1}, \qquad (40)$$

and for Δt is given by the analytical solution of the necessary condition for optimality of (38) according to Δt

$$\Delta t^{j+1} = \{\frac{\mu_m s^j(K)x^j(K)}{Y[k_s + s^j(K)]}[\lambda_s^j(K) + \mu_s^j s^j(K)] - K - -[\frac{\mu_m s^j(K-\tau)}{k_s + s^j(K-\tau)} - k_D]x^j(K)[\lambda_x^j(K) + \mu_x^j x^j(K)]\}/ /\{\mu_x^j\{[\frac{\mu_m s^j(K-\tau)}{k_s + s^j(K-\tau)} - k_D]\}x^j(K)\}^2\} + + \mu_s^j\{\frac{\mu_m s^j(K)x^j(K)}{Y[k_s + s^j(K-\tau)]}\}^2\}, \qquad (41)$$

The sub-problems on the first level are calculated by gradient procedures

$$v^{j,l+1}(k) = v^{j,l}(k) - \alpha_v e_v^{j,l}(k),\ v = x,s,D, \qquad (42)$$

where l is an index of the procedures and the gradients are

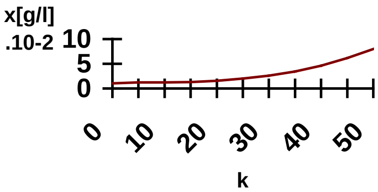
$$e_x^{j,l}(k) = \{1 + \Delta t^j[\frac{\mu_m s^{j,l}(k-\tau)}{k_s + s^{j,l}(k-\tau)} - k_D] - \Delta t^j D^{j,l}(k)\}^T \times \times[\lambda_x^j(k) - \mu_x^j e_{\lambda x}(k)] -$$

$$-\{\frac{\mu_m s^{j,l}(k)}{Y[k_s+s^{j,l}(k)]}\}^T[\lambda_s^j(k)-\mu_s^j e_{\lambda s}(k)], \qquad (43)$$

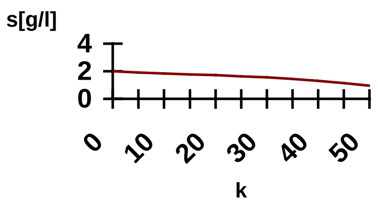
$$e_s^{j,l}(k)=\Delta t^j[\frac{\mu_m k_s x^{j,l}(k+\tau)}{[k_s+s^{j,l}(k-\tau)]^2}]^T\times$$
$$\times[\lambda_x^j(k+\tau)-\mu_x^j e_{\lambda x}^j(k)]+$$
$$+\{1-\Delta t^j[\frac{\mu_m k_s x^{j,l}(k)}{Y[k_s+s^{j,l}(k)]^2}-\Delta t^j D^{j,l}(k)\}^T\times$$
$$\times[\lambda_s^j(k)-\mu_s^j e_{\lambda s}^j(k)],$$

$$e_D^{j,l}(k)=\Delta t^j[x^{j,l}(k)-x^0]^T\ [\lambda_x^j(k)-\mu_x^j e_{\lambda x}^j(k)]+$$
$$+\Delta t^j[s^0-s^{j,l}(k)]^T\ [\lambda_s^j(k)-\mu_s^j e_{\lambda s}^j(k)], \qquad (44)$$

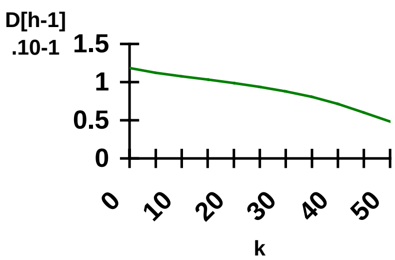
$k=\overline{0,K-1}$, and α is the gradient procedure step. The optimal trajectories are given in Fig.3. The minimum time is 8.065h.



a)



b)



c)

Fig.3. Optimal biomass (a), substrate (b) and control (c) trajectories of the continuous fermentation process

6. CONCLUSION

A decomposition method is proposed to solve an optimal control problem for processes with high dimension, time delays, nonlinearities and singular control. The method is based on the augmented Lagrange's functional and on a new coordinating vector for its decomposition in time domain.

This method overcomes the difficulties encountered in solving the nonlinear two point boundary value problem with state and control delays and reduces the number of calculations. It is not necessary to calculate a singular kind of control trajectory by means of boundary layers, because an augmented Lagrange's functional is used. At the same time the new coordinating vector allows the dual optimal control problem to become separable and the time domain decomposition to be obtained based on a conjugate variable prediction. The method convexifies and transforms a non convex problem into one that preserves the separability of the dual problem which is necessary for applying the of the decomposition approach. In this way the applicability of time domain decomposition is extended to non-convex problems.

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