A NOVEL TWO-DEGREE-OF-FREEDOM CONTROL STRATEGY FOR UNSTABLE PROCESSES WITH DELAY

Danying Gu, Linlin Ou, Peiying Chen and Weidong Zhang

Dept. of Automation, Shanghai Jiaotong University, Shanghai, 200030, P. R. China
Tel/Fax:+86.21.62826946; Email: gudanying@sjtu.edu.cn

Abstract: A simple and practical two-degree-of-freedom control strategy is proposed for unstable processes with time delay. It can decouple load disturbance from setpoint response. By combining the theory on stabilizing PID controller proposed by Silva *et al.* with IMC method, the process in which the ratio of time delay to unstable time constant is smaller than 2 can be controlled effectively. Simulation examples are given to show the performance of the proposed method.

Keywords: PID controllers；time delay; unstable; stabilizing controllers; robustness.

1． INTRODUCTION

Besides many open-loop stable processes, there still exist some unstable ones in the chemical industries. Unstable processes are more difficult to control than stable ones because of simultaneous existence of the time delay and the right-half-plane (RHP) poles. It is even more difficult to control the unstable processes with long time delay.

Since the PID controller remains the most popular approach for industrial process control, a great deal of effort has been directed at finding effective tuning methods of PID controller for unstable processes, such as the extended internal model control (IMC) method (Morari and Zafiriou, 1989), gain-phase margin method (De Paor and O'Malley, 1989; Venkatashankar and Chidambaram, 1994), graphical method (Shafiei and Shenton, 1994) and other advanced methods (Rotstein and Lewin, 1991; Huang and Lin, 1995). However, all these methods only apply to the case where the ratio of time delay to unstable time constant ($|L/T|$) is smaller than unity. Huang and Chen (1997) propose a three-element structure to control unstable processes where $|L/T|$ is smaller than 2.

It is interesting to note that even though numerous methods have been developed for setting the parameters of PID controllers, the set of all stabilizing PID parameters remains unknown. In other words, it is significant to provide the complete solution to the problem of characterizing the set of all PID parameters that can stabilize a given first-order process with time delay. Silva, *et al.* (2002) have proposed a complete solution to this problem on the basis of a version of the Hermite-Biehler Theorem. The set of stabilizing PID parameters is determined for both open-loop stable and unstable processes.

In this paper a two-degree-of-freedom control structure is proposed based on the theory of stabilizing PID controller for unstable processes with time delay (Silva, *et al*., 2002) and on the IMC method (Morari and Zafiriou, 1989). Simple and straightforward tuning rules are suggested. The load disturbance response is decoupled from the setpoint response. Furthermore, the proposed method is also applicable to the case where $|L/T|$ is smaller than 2.

The paper is organized as follows. In Section 2, the two-degree-of-freedom control structure is introduced and analyzed briefly. In Section 3, both the necessary and sufficient condition for existence of the stabilizing PID controller for unstable processes and the algorithm for determining PID parameters is presented in detail, and the extended IMC method is proposed. Examples are given in Section 4 and the conclusions are given in Section 5.

2． TWO-DEGREE-OF-FREEDOM CONTROL SYSTEM FOR UNSTABLE PROCESSES WITH TIME DELAY

This paper considers that the dynamic behavior of the unstable processes with time delay can be described by the following transfer function:

$$P_u(s) = \frac{k}{1+Ts}e^{-Ls} = G(s)e^{-Ls} \qquad (1)$$

where k represents the static gain of the plant, T represents the time constant of the plant, and L represents the time delay. In this case, $T<0, k>0, L>0$.

Kaya and Atherton proposed a new PI-PD Smith predictor for control of processes with long dead time (1999). In this paper, a new two-degree-of-freedom control structure for unstable processes with time delay is proposed, which is shown in Fig. 1. In this control structure, Ge^{-Ls} is the model of the real process $G_m e^{-L_m s}$. G_d is a compensator which can stabilize unstable processes and reject disturbance. The filter F and the controller G_c are used to control the performance of setpoint response. When $F=0$ and $G_d=0$, the structure is equivalent to the standard Smith predictor.

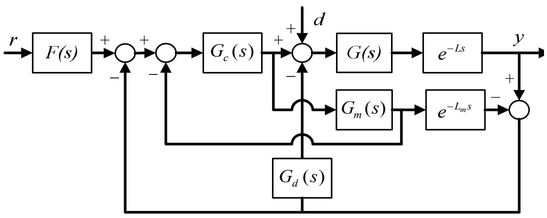

Fig.1. Two-degree-of-freedom control structure for unstable processes

Suppose that the model describes dynamic characteristic of real process, i.e. it is in nominal situation: $G=G_m$, $L=L_m$. The response of the closed-loop system for setpoint changes and load disturbance is derived from Fig. 1 as:

$$Y(s)=Y_r(s)R(s)+Y_d(s)D(s) \qquad (2)$$

where $Y_r(s)$ and $Y_d(s)$ are closed-loop transfer functions for setpoint changes and load changes, respectively, with:

$$Y_r(s)=F\frac{G_m G_c e^{-L_m s}}{1+G_m G_c} \qquad (3)$$

$$Y_d(s)=\frac{G_m[1+G_m G_c(1-e^{-L_m s})]e^{-L_m s}}{(1+G_m G_c)(1+G_d G_m e^{-L_m s})} \qquad (4)$$

Eqs. (3) and (4) show that $Y_r(s)$ is determined only by the setpoint controller G_c in nominal situation, while $Y_d(s)$ is determined by G_c and the compensator G_d . Furthermore, since the denominators of $Y_r(s)$ and $Y_d(s)$ have the same factor $(1+G_m G_c)$, $Y_d(s)$ will be dependent only on G_d when G_c is designed independently at first. This makes it possible to improve the performances of both setpoint tracking and disturbance rejections by separately tuning the PID controllers G_c and G_d . Hence the setpoint response and the disturbance response are decoupled from each other. It can also be observed that the setpoint response does not contain the delay term in its characteristic equation, and therefore G_c will be designed by employing the IMC method to guarantee that $Y_r(s)$ only has left-half-plane poles. As to the disturbance response, its stability performance is dependent only on the factor $(1+G_d G_m e^{-L_m s})$, which is equivalent to that of a closed-loop system, with the process $G_m e^{-L_m s}$ and the controller G_d . So we apply the method of characterizing the set of stabilizing PID parameters proposed by [6] to designing the PID-type compensator G_d .

3. STABILIZATION PROCEDURE OF UNSTABLE PROCESSES WITH TIME DELAY

3.1 Stabilization of unstable processes with time delay by using PID controller

Suppose that G_d is of the PID type as

$$G_d(s)=k_p+\frac{k_i}{s}+k_d s \qquad (5)$$

where the derivative factor is an ideal one. In practice the pure derivative term $k_d s$ is always replaced by $k_d s/(0.01s+1)$. The first order unstable process with time delay is given in the expression (1).

Theorem: A necessary and sufficient condition for the existence of a stabilizing PID controller for the open-loop unstable process is $|L/T|<2$. If this condition is satisfied, then the range of k_p for which a given open-loop unstable process can be stabilized using a PID controller is given by

$$k_u=\frac{1}{k}[\frac{T}{L}\alpha_1\sin(\alpha_1)-\cos(\alpha_1)]<k_p<-\frac{1}{k} \qquad (6)$$

where α_1 is the solution of the equation

$$\tan(\alpha)=-\frac{T}{T+L}\alpha, \quad \alpha\in(0,\pi) \qquad (7)$$

For k_p values outside this range, there don't exist stabilizing PID controllers. For each k_p which satisfies the expression (6), the cross section of the stabilizing region in the (k_i,k_d) space is the quadrilateral shown in Fig. 2.

Hence, the algorithm for determining stabilizing PID parameters for unstable processes is:

Step 1: Find the roots in the interval $(0,\pi)$ of Eq. (7);

Step 2: Determine the range of k_p from expression (6) and let k_p equal to the median of this range;

Step 3: Solve the equation

$$kk_p+\cos(z)-\frac{T}{L}z\sin(z)=0 \qquad (8)$$

and denote the positive-real roots by $z_j, j=1,2$,……, arranged in ascending order of magnitude;

Step 4: Calculate m_j, b_j, w_j from

$$m_j=m(z_j), b_j=b(z_j) \qquad (9)$$

$$m(z)=\frac{L^2}{z^2}, b(z)=-\frac{L}{kz}[\sin(z)+\frac{T}{L}z\cos(z)] \qquad (10)$$

$$w_j = \frac{z_j}{kL}[\sin(z_j) + \frac{T}{L} z_j(\cos(z_j) + 1)] \quad (11)$$

Step 5: Determine the stabilizing region in the k_i - k_d space using Fig. 2.

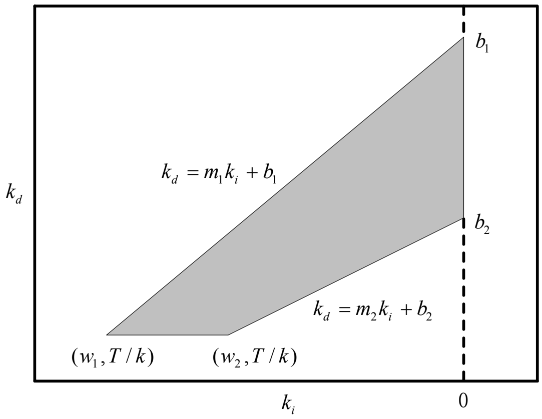

Fig. 2. Stabilizing region of (k_i, k_d) when $k_u < k_p < -(1/k)$

3.2 Design of the controller G_c

IMC method is powerful for control-system synthesis, and it is well-known that the control system cannot be implemented by the IMC structure if the process is an unstable one [1]. However, as suggested by Morari and Zafiriou, one can still design the controllers for processes without delay using the IMC method, and then, implement the controllers in an equivalent-feedback structure. Since the setpoint response does not contain the delay term in its characteristic equation, G_c can be designed as an IMC feedback controller for the unstable process $G_m = k/(1+Ts)$. The standard IMC controller is chosen to meet the requirements of internal stability and pole zero excess:

$$Q(s) = \widetilde{Q}(s) f(s) = \frac{Ts+1}{k} \frac{\lambda(\lambda/T+2)s+1}{(\lambda s+1)^2} \quad (12)$$

where λ is an adjustable parameter. Then the equivalent feedback controller is derived from

$$G_c(s) = \frac{Q(s)}{1 - G_m(s)Q(s)} \quad (13)$$

Thus we have

$$G_c(s) = k_c(1 + \frac{1}{T_i s}) \quad (14)$$

where

$$k_c = \frac{-(\lambda + 2T)}{k\lambda}, \; T_i = \lambda(\frac{\lambda}{T} + 2) \quad (15)$$

Hence, the setpoint response is rewritten as

$$Y_r(s) = F \frac{G_m G_c e^{-L_m s}}{1 + G_m G_c} = F \frac{\lambda(\lambda/T+2)s+1}{(\lambda s+1)^2} e^{-L_m s} \quad (16)$$

Let the filter function $F(s)$ be

$$F(s) = \frac{(\lambda s+1)^2}{[\lambda(\lambda/T+2)s+1](\lambda_1 s+1)} \quad (17)$$

where λ_1 is another adjustable parameter. When the uncertainty of processes is unknown, we often choose $\lambda = L/2$ and $\lambda_1 = |T|$. Substituting it into (16), we have:

$$Y_r(s) = \frac{1}{\lambda_1 s+1} e^{-L_m s} \quad (18)$$

The parameter λ_1 relates directly to the nominal performance and robustness of the system. The smaller λ_1 is, the faster the input response is, while the robustness is worse.

4. SIMULATION EXAMPLES AND PERFORMANCE ANALYSIS

Consider the simulation example used in Huang and Chen' paper [7], i.e. $G_p(s) = e^{-1.2s}/(1-s)$. By carrying out the new control method, $\lambda = 0.60$, $\lambda_1 = 1.00$, $k_c = -4.33$, $T_i = 1.56$ are firstly obtained. Then we get $-1.32 < k_p < -1.00$ and let k_p equals to the median of this range, i.e. $k_p = -1.16$. So the stabilizing region of (k_i, k_d) is calculated and shown in Fig. 3.

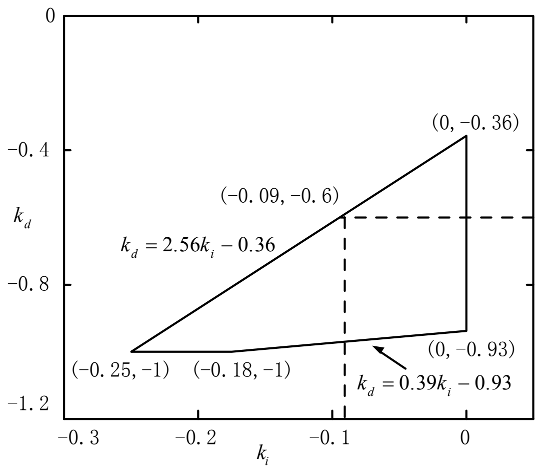

Fig. 3. Stabilizing region of (k_i, k_d) when $-1.16 < k_p < -1$

Note that the system is unstable when $k_i > 0$. By simulating the closed-loop response of the system in the case of $k_i < 0$, we find that the smaller $|k_i|$ is, the better performance of disturbance rejection and robustness can be obtained. This rule is the same to k_p. As to k_d, the value corresponding to the best performance of disturbance rejection is near the median of stabilizing range. Finally, we choose $k_i = 0$, $k_p = -1.05$ and $k_d = -0.5$, which means that the PID controller becomes a PD controller. Fig. 4. shows the nominal closed-loop response of the proposed method and also for comparison, of Huang and Chen's method [7].

The new method gives better results for setpoint response and the disturbance rejection is also good. The closed-loop response for model mismatch in L (+10%) is shown in Fig. 5.

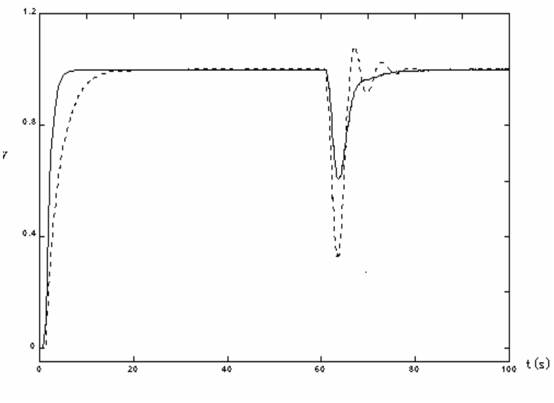

Fig.4 The closed-loop response of the nominal process
(Solid line: proposed method; dotted line: Huang and Chen's method)

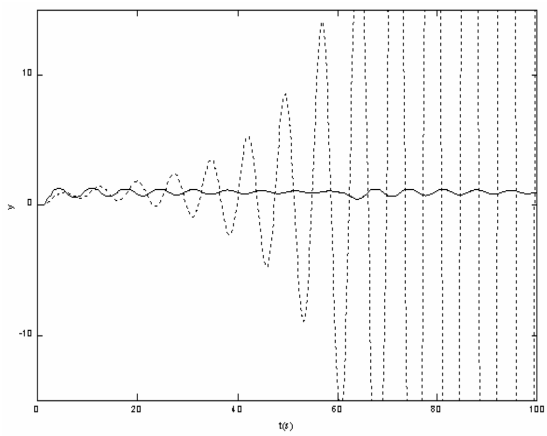

Fig.5 The closed-loop response of the process with L increased by 10% (Solid line: proposed method; dotted line: Huang and Chen's method)

From the results, we observe that the proposed control method is extremely robust. In practice, we can make λ_1 be even smaller so as to expedite the speed of input response. The robustness can be tuned by parameters of PID controller.

5. CONCLUSIONS

In this paper a two-degree-of-freedom control strategy is proposed based on the theory on stabilizing PID controller for unstable processes with time delay and the IMC method. The load disturbance is decoupled from the setpoint response. As shown in the simulation example, the proposed method has better nominal performance and robustness than Huang and Chen's method. Moreover, it is also applicable to the case where $|L/T|$ is smaller than 2.

ACKNOWLEDGMENT

This paper is supported by National Science Foundation of China (60274032), STPPS (04QMH1405), and partly by the Alexander von Humboldt Research Fellowship.

REFERENCES

De Paor, A. M. and O'Malley, M. (1989), Controllers of Ziegler-Nichols type for unstable process with time delay, *Int. J. Control*, **49(4),** pp.1273-1284.

Huang, C. T. and Lin, Y. S. (1995), Tuning PID controller for open-loop unstable process with time delay. *Chem. Eng. Comm.*, **133**, pp.11-30.

Huang, H. P. and Chen, C. C. (1997), Control-system synthesis for open-loop unstable process with time delay, *IEE Proc. Control Theory Appl.*, **144**(4), pp.334-346.

Kaya, I. and Atherton, D. P. (1999), A new PI-PD Smith predictor for control of processes with long dead time, *14th IFAC World Congress*, Beijing, China, C-2a-08-06, pp.283-288.

Morari, M. and Zafiriou, Z. (1989), *Robust Process Control*, Englewood Cliffs, NJ: Prentice Hall.

Rotstein, G. E. and Lewin, D. R.(1991), Simple PI and PID tuning for open-loop unstable systems. *Ind. Eng. Chem. Res.*, **30**, pp.1864-1869.

Shafiei, Z. and Shenton, A. T. (1994), Tuning of PID-type controllers for stable and unstable systems with time delay. *Automatica*, **30**(10), pp.1609-1615.

Silva, G. J., Datta, A. and Bhattacharyya, S. P. (2002), New Results on the Synthesis of PID Controllers, *IEEE Trans. Auto. Control*, **47**(2), pp.241-252.

Venkatashankar, V. and Chidambaram, M. (1994), Design of P and PI controllers for unstable first-order plus time delay systems, *Int. J. Control*, **60**(1), pp.137-144.