A COLLISION WARNING SYSTEM BASED ON AN INTER-DISTANCE REFERENCE MODEL

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Abstract: In this paper we introduce a new collision warning system which permits to warn the driver with sufficient anticipation about some inter-vehicular distance danger. The warning system is based on the future predictions from a 5-DOF (Degree Of Freedom) vehicle dynamics. The future inter-distance and the future relative velocity are compared with respect to certain safe conditions which are calculated from a dynamic inter-distance reference model. Some simulations with real measure data illustrate the behavior of the proposed driver assistance system.

Keywords: collision warning, trajectory prediction, reference model.

1. INTRODUCTION

During the last decade a wide variety of driver's assistance systems have been developed and described in several works. Some examples of these systems are found in (Vahidi and Eskandarian, 2003) and (Palkovics and Fries, 2001). Most of these systems are based on measures that come from sensors inside or outside of the vehicle. These systems attempt to improve the driver's maneuvers, just to do the maneuvers safer.

In several collision warning systems the *alert space* or *alerting thresholds* are fixed by the calculation of the necessary distances to avoid a collision. Some examples are (Seiler *et al.*, 1998) and (Yi and Chung, 2001). A different approach is found in (Yang and Kuchar, 2002) where a performance-based warning system is developed. This method is applied to automotive warning systems in (Yang *et al.*, 2003) where the alert thresholds are setting in such a way that a stochastic performance metric (using available information of sensor uncertainties) is met.

In general these approaches do not use dynamic vehicle models to calculate the future trajectory and the alert space is calculated from the stationary solutions of the Newtonian motion equations, some times assuming constant braking values. In addition, these alert functions don't give any brake strategy to stop the vehicle.

On the other hand, the behavior of the driver is still not well known and several car following models based on experimental and empirical data, feel more natural, but these ones are not accepted at all due to the larger number of adjustments, some times too contradictory in terms of safety. In addition, a human driver makes decision based on limited sensory tools, thus, imitation of this behavior while using more accurate electronic sensors may not necessarily be optimal. Moreover, statistical accident data show that a considerable portion of accidents are caused by driver's delay in recognizing or judging the "dangerous" situation (Vahidi and Eskandarian, 2003).

A warning system will have to compensate the driver's delay, permitting the driver to make a better decision with sufficient time. Thus, a vehicle model-based predictor may be necessary in this task. Besides, the system will have to evaluate if there still exists a safe manoeuvre, evaluating the conditions to avoid a collision respecting for example the maximal braking capac-

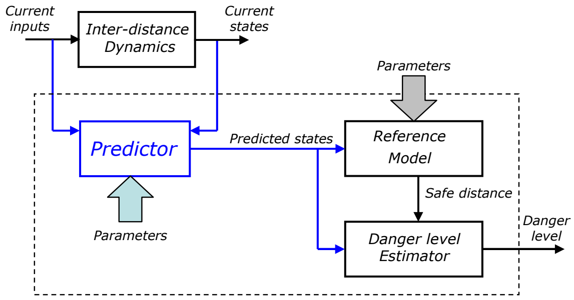

Fig. 1. The warning system scheme.

ity. The above-mentioned suggests to use a reference model to evaluate *on-line* the necessary conditions to avoid a collision.

We propose here an inter-vehicular distance warning system which warns the driver with sufficient anticipation about some inter-vehicular distance danger. The proposed system uses the states predictions obtained from a 5-DOF vehicle dynamics model developed in (Martinez *et al.*, 2004). The future inter-vehicular distance and the future velocity are compared with respect to the expected ones from a dynamic safe inter-distance reference model presented for the first time in (Martinez and Canudas, 2004). The inter-distance reference model is nonlinear, with the particularity that its solutions can be described by explicit integral curves, allowing to explicitly characterize the set of initial condition for which the safety specifications can be met. Some simulations illustrate the behavior of the proposed warning system using real measure data.

2. THE WARNING SYSTEM

The figure 1 illustrates the structure of the proposed warning system. The system is composed by an inter-distance predictor, a reference model and an estimator of the danger level.

In figure 2, an example of the alert space is illustrated. The system states at the instant k, i.e. $z(k)$, are considered as *safe states*, if $z(k)$ belongs to a certain safe set Ω^{safe}, i.e. $z(k) \in \Omega^{safe}$. However, if a projected dynamics shows that the vehicle will penetrates an unsafe region, but by initiating an alert, it is expected that some action exists and it will be performed to alter the course of the state trajectory in order to prevent a danger, then the project states belongs to a pre-crash region, i.e. $z(k+N) \in \Omega^{precrash}$. Otherwise, by the worse one of the cases, if it is not possible to attract the system states to the safe set Ω^{safe}, then the state $z(k+N) \in \Omega^{unsafe}$. That means that there are not exist any safe maneuvers to avoid a collision if the worse case appears at the instant $k+N$.

This classification suggests to define a new discrete state $\sigma(k)$ which indicates the status of the system states, as:

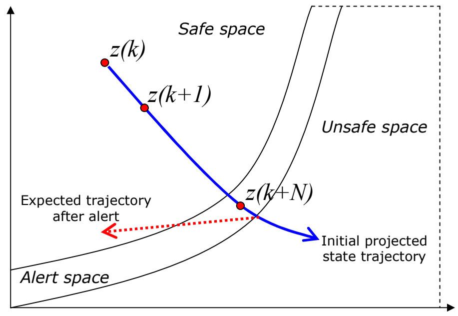

Fig. 2. Warning state space representation.

$$\sigma(k) \triangleq \begin{cases} 1 \equiv \textit{Safe} & \text{if } z(k+N) \in \Omega^{safe} \\ 2 \equiv \textit{Precrash} & \text{if } z(k+N) \in \Omega^{precrash} \\ 3 \equiv \textit{Unsafe} & \text{if } z(k+N) \in \Omega^{unsafe} \end{cases} \quad (1)$$

where $z(k+N)$ corresponds to the predicted system states available at the instant k; N denotes the number of the prediction instants.

Assuming that $z(k) \in \Omega^{safe}$. The question is: Is it possible to start a safe braking maneuver at the instant $k+N$, if we know that a possible danger arrives at this instant, i.e. $z(k+N) \in \Omega^{precrash}$?

To aim this, we will use a dynamic vehicle model in order to calculate the inter-distance predictions, and later, these predictions are compared with a particular safe conditions given by an inter-distance reference model. All this will be described in the next sections.

2.1 The Vehicle Model

The bicycle vehicle model used in this approach consists in a non-linear reduced order one obtained from that developed in (Martinez *et al.*, 2004), where the case pitch motion is not taken into account. The state representation of the vehicle model takes the simple form:

$$\dot{z} = f(z) + g(z)\bar{u} \quad (2)$$

where $z = [x_1, \dot{x}_1, y_1, \dot{y}_1, \psi, \dot{\psi}, \theta_f, \dot{\theta}_f, \theta_r, \dot{\theta}_r]^T$ and $\bar{u} = [\alpha \quad \tau_f \quad \tau_r]^T$. The states x_1 and y_1 define the location of the vehicle with respect to an inertial reference system, ψ represents the vehicle yaw angle (orientation), θ_f and θ_r concern the angular motion of the front and rear tires, respectively. α is the front wheel steering angle, τ_f and τ_r are the front and rear wheel torques. The encouraged readers can consult (Martinez *et al.*, 2004) for more details of the vehicle model. In this study we assume that the vehicle states z are obtained from suitable sensors or estimation.

Equation (2) provides a reduced order model of the vehicle dynamics that will be used to predict the future vehicle states.

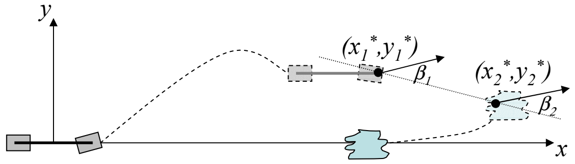

Fig. 3. The inter-distance prediction.

2.2 The Inter-distance Predictor

Consider the general system given by equation (2), and take the Euler numerical integration rule to determine the value of the future state, one step ahead z_{k+1}, from the knowledge of the current state z_k, as

$$z_{k+1} \approx T[f(z_k) + g(z_k)\bar{u}_k] + z_k \tag{3}$$

where T is the prediction step.

The predicted position and velocity of the equipped vehicle are obtained from:

$$\mathcal{P}_{1(k+1)} \triangleq [x^*_{1(k+1)}, y^*_{1(k+1)}]^T \tag{4}$$

$$\mathcal{V}_{1(k+1)} \triangleq [\dot{x}^*_{1(k+1)}, \dot{y}^*_{1(k+1)}]^T \tag{5}$$

while the predicted position and velocity of the leader vehicle are obtained from:

$$\mathcal{P}_{2(k+1)} \triangleq [x^*_{2(k+1)}, y^*_{2(k+1)}]^T \tag{6}$$

$$\mathcal{V}_{2(k+1)} \triangleq [\dot{x}^*_{2(k+1)}, \dot{y}^*_{2(k+1)}]^T \tag{7}$$

where x^*_2 and y^*_2 are the predicted leader position components, and $\dot{x}^*_2$, $\dot{y}^*_2$ its time derivatives. Here, the leader vehicle motion is considered as a massless point motion. A more precise calculation will require a well knowledge of the leader vehicle dynamics, it could be possible using communication between involved vehicles. Figure 3 illustrates the inter-distance predictor.

Therefore, the predicted inter-distance $d^*_{(k+1)}$ will be obtained as the distance between the position vector $\mathcal{P}_{2(k+1)}$ and the position vector $\mathcal{P}_{1(k+1)}$, i.e. [1] :

$$d^*_{(k+1)} = dist\left(\mathcal{P}_{2(k+1)}, \mathcal{P}_{1(k+1)}\right) \tag{8}$$

Now, through successive calculation of the future values from (5) and (8), we can obtain the predicted inter-distance $d^*_{(k+N)}$ and the predicted vehicle velocity $\mathcal{V}_{1(k+N)}$, for a given (nominal) sequence of input $\bar{u}$ between the instant k and $k+N$.

Here we are interested for the velocity components in the direction of the inter-distance vector, that we have denoted as $\mathcal{V}_{1\beta}$ and $\mathcal{V}_{2\beta}$. Based in the figure 3 we can obtain:

$$\mathcal{V}_{1\beta} = \|\mathcal{V}_1\| \cos(\beta_1) \tag{9}$$

$$\mathcal{V}_{2\beta} = \|\mathcal{V}_2\| \cos(\beta_2) \tag{10}$$

[1] Here we can consider an Euclidian distance, but a more precise calculation requires good acknowledge of the global vehicle position and the road geometry.

with β_1 and β_2 calculated as

$$\beta_1 = \arctan\left(\frac{\dot{y}_1}{\dot{x}_1}\right) - \arctan\left(\frac{y_2 - y_1}{x_2 - x_1}\right) \tag{11}$$

$$\beta_2 = \arctan\left(\frac{\dot{y}_2}{\dot{x}_2}\right) - \arctan\left(\frac{y_2 - y_1}{x_2 - x_1}\right) \tag{12}$$

Notice that validation of a possible collision could be a function of the value of β_1, e.g. $\beta_1 \leq \beta_{max}$ and $d^* \leq d_c$. The constants β_{max} and d_c could include the sensor uncertainties.

The vehicle model could feel too complex, but this permits to eliminate most of the importants sources of uncertainties. At least admissibles kinematical and dynamical trajectories are considered when we use a bicycle model instead of a simple massless point.

Here, we are interested in the two following future states:

$$d^*_{(k+N)} = dist\left(\mathcal{P}_{2(k+N)}, \mathcal{P}_{1(k+N)}\right) \tag{13}$$

$$\mathcal{V}_{1\beta(k+N)} = \|\mathcal{V}_{1(k+N)}\| \cos(\beta_{1(k+N)}) \tag{14}$$

obtained from successive calculations of (8) and (9). These predicted states will be used to evaluated the level of danger in the following section.

2.3 The Reference Model

Assume that the *reference* vehicle dynamics is a second order one, i.e.

$$\dot{\mathcal{V}}^r_{1\beta} = u \tag{15}$$

Then the dynamic of the inter-distance could be written as

$$\ddot{d}^r = \dot{\mathcal{V}}_{2\beta} - u \tag{16}$$

Introducing $\tilde{d} \triangleq d^r(0) - d^r(t)$, as being the inter-distance error with respect to the (constant) initial reference inter-distance $d^r(0)$. The dynamics of this error coordinate is

$$\ddot{\tilde{d}} = u - \dot{\mathcal{V}}_{2\beta} \tag{17}$$

The reference model developed in (Martinez and Canudas, 2004) has get inspiration from the nonlinear models resulting from the theory of elasticity and mechanic of the contacts, where the forces are proportional to the penetration of the object into the surface. One of the advantages of this reference model is that in connection with (17), it is possible to compute the integral curves associated to the autonomous nonlinear differential equation. Take,

$$u = -c|\tilde{d}|\dot{\tilde{d}} \tag{18}$$

Due to the necessity of eliminate the excess in kinetic energy when the braking maneuver starts, it is then

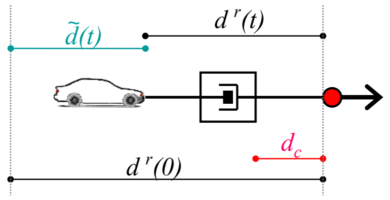

Fig. 4. The inter-distance reference model.

natural to only use a dissipation term to avoid collisions. The figure 4 illustrates the model. Note that the goal of this structure is not to regulate back the reference vehicle to $\tilde{d} = 0$, but to stop the vehicle before it reaches a critical distance d_c (assumed as a constant minimal inter-distance), while respecting the imposed braking constraints.

The penetration distance dynamics will be given by,

$$\ddot{\tilde{d}} = -c|\tilde{d}|\dot{\tilde{d}} - \dot{\mathcal{V}}_{2\beta} \tag{19}$$

For simplicity take the instant $k+N$, the instant that it is possible to start a braking maneuver; without loss of generality, take this instant as the time $t = 0$; So the initial conditions for the reference model will be calculated from the predicted states (13) and (14), as:

$$\begin{aligned} d^r(0) &= d^*_{(k+N)} \\ \mathcal{V}^r_{1\beta}(0) &= \mathcal{V}_{1\beta(k+N)} \end{aligned} \tag{20}$$

The problem here, is to find the necessary conditions to avoid collision, i.e. $d^r(t) > 0$, for all solutions of (19), starting in (20).

Note that equation (19) can be solved analytically. We have,

$$\dot{\tilde{d}}(t) = -\frac{c}{2}\tilde{d}(t)^2 - \mathcal{V}_{2\beta}(t) + c_{int} \tag{21}$$

with $c_{int} = \mathcal{V}^r_{1\beta}(0) + \frac{c}{2}\tilde{d}^2(0) = \mathcal{V}^r_{1\beta}(0)$. Upon substitution of the relation $\mathcal{V}^r_{1\beta}(t) = \dot{\tilde{d}}(t) + \mathcal{V}_{2\beta}(t)$ in (21) one can obtain an explicit relation between the reference vehicle velocity and the “penetration” distance $\tilde{d}$, i.e.

$$\mathcal{V}^r_{1\beta}(t) = -\frac{c}{2}\tilde{d}(t)^2 + \mathcal{V}^r_{1\beta}(0) \tag{22}$$

From this expression, we can find a parameter c such that for all $0 \le \mathcal{V}^r_{1\beta}(0) \le V_{max}$, the critical distance d_c is not attained (V_{max} is a given maximal velocity). From:

$$\tilde{d}(t) = \sqrt{\frac{2(\mathcal{V}^r_{1\beta}(0) - \mathcal{V}^r_{1\beta}(t))}{c}} \tag{23}$$

the maximum penetration distance $\tilde{d}_{max}$ can be computed as $\tilde{d}_{max} = \sqrt{\frac{2\bar{c}_{int}}{c}}$; ($\bar{c}_{int} \triangleq \max_{\forall t}\{\mathcal{V}^r_{1\beta}(0) - \mathcal{V}^r_{1\beta}(t)\} = \mathcal{V}^r_{1\beta}(0)$). Making $\tilde{d}_{max} \le d^r(0) - d_c$, we have,

$$\tilde{d}_{max} = \sqrt{\frac{2\mathcal{V}^r_{1\beta}(0)}{c}} \le d^r(0) - d_c \tag{24}$$

which provides a first inequality for c, i.e.

$$\mathcal{C}_1 : \quad c \ge \frac{2\mathcal{V}^r_{1\beta}(0)}{(d^r(0) - d_c)^2} \tag{25}$$

By taking time-derivatives from (22), and proceeding in the same way, by imposing an associated braking constraint, we have:

$$\begin{aligned} \dot{\mathcal{V}}^r_{1\beta}(t) &\ge -\frac{2}{3}\mathcal{V}^r_{1\beta}(0)\sqrt{\frac{2\mathcal{V}^r_{1\beta}(0)c}{3}} \\ &\ge -B_{max} \end{aligned} \tag{26}$$

where B_{max} is a positive constant.

The relation (26) yields one more inequality providing an upper bound for c, i.e.

$$\mathcal{C}_2 : \quad c \le (\frac{27}{8})\frac{B^2_{max}}{\mathcal{V}^r_{1\beta}(0)^3} \tag{27}$$

Therefore, a sufficient condition for c to exist is that $\mathcal{C}_1$ and $\mathcal{C}_2$ hold, i.e.

$$d^r(0) \ge \sqrt{\frac{16}{27}\frac{(\mathcal{V}^r_{1\beta}(0))^2}{B_{max}}} + d_c \tag{28}$$

If (28) holds, then we can calculate c from $\mathcal{C}_2$, as:

$$c = \frac{27B^2_{max}}{8(\mathcal{V}^r_{1\beta}(0))^3} \tag{29}$$

That means that there still exists an inter-distance dynamics to avoid collision, which respects the maximum braking capacity.

Now, using the safe condition described by (28), the following regions in the state-space can be defined as:

$$\begin{aligned} \Omega^{safe} &\triangleq \{d, \mathcal{V}_1 : d > d_s + d_c\} \\ \Omega^{precrash} &\triangleq \{d, \mathcal{V}_1 : d_s + d_c \ge d \ge d_s\} \\ \Omega^{unsafe} &\triangleq \{d, \mathcal{V}_1 : d < d_s\} \end{aligned} \tag{30}$$

where $d_s \triangleq \sqrt{\frac{16}{27}\frac{(\mathcal{V}_1)^2}{B_{max}}}$.

Thus, we can re-define the discrete variable σ, initially defined in (1), to indicate at the instant k, the level of the danger at the instant $k+N$, as

$$\sigma(k) \triangleq \begin{cases} 1 \text{ if } [d^*_{(k+N)}, \mathcal{V}^*_{1\beta(k+N)}] \in \Omega^{safe} \\ 2 \text{ if } [d^*_{(k+N)}, \mathcal{V}^*_{1\beta(k+N)}] \in \Omega^{precrash} \\ 3 \text{ if } [d^*_{(k+N)}, \mathcal{V}^*_{1\beta(k+N)}] \in \Omega^{unsafe} \end{cases} \tag{31}$$

Notice that $\sigma(k)$ could be used to warn the driver about some future danger N instants of time in advance. In addition, we can use $\sigma(k)$ to determine when an automatic braking system must be activated. The Figure 5 illustrates the different state-space regions for a given critical inter-distance d_c and a given braking capacity B_{max}.

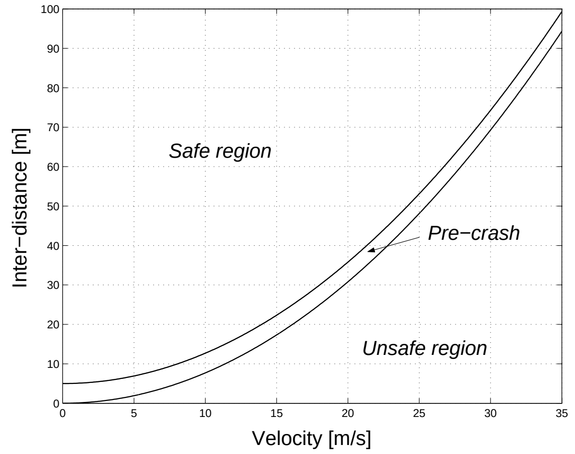

Fig. 5. Safe, unsafe and pre-crash state space regions with $d_c = 5m$ and $B_{max} = 10m/s^2$.

3. SIMULATIONS

3.1 Description of the Test-bed

To test the proposed system, we have used real measured data obtained from a Stop-and-Go scenario. The scenario was produced in collaboration with the LIVIC [2] Laboratory.

The follower vehicle is a Renault Scénic $1.6\ l$, 4 cylinders, and ABS brakes. This vehicle is well provided of sensors, but in this study only odometry measurements, an inertial sensor and a radio-modem are used.

The leader vehicle is also equipped of an odometry and a radio-modem. The radio modem transmits the vehicle position and velocity to the computer into the follower vehicle which stores the measurements related to the vehicle speed, vehicle acceleration and the inter-distance. The latter calculated as the difference between the absolute positions.

The track is a straight line and assumptions about the road geometry and the driver intentions permit to assume that the steering angle, the lateral speed and the yaw angle are equal to zero during the test. In addition, the wheel torques are assumed to produce a constant wheel speed during the prediction task.

3.2 Results

We have designed a scenario which permits to illustrate the useful of the proposed warning system. A leader car driver is demanded to accelerated and decelerate with elevated values, while the follower car driver tries to maintain a constant distance. The effect is that the distance is difficult to maintain constant and the dangerous inter-distance is reached. Here, we have tested predictions for $N = 10$, with sample time $T = 0.1s$, that gives a prediction time of $1s$.

[2] LIVIC is a French Laboratory, where the principal research field concerns the Vehicles-Infrastructure-Driver Interactions; See http://www.inrets.fr/ur/livic

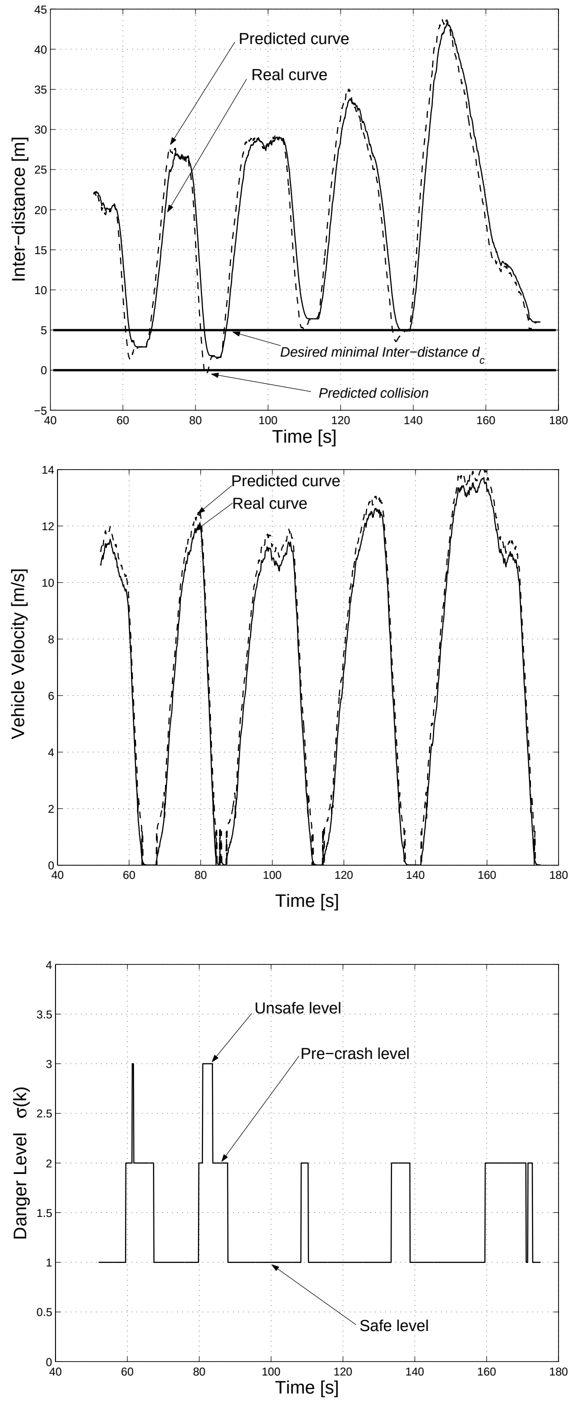

Fig. 6. Inter-distance, velocities and danger level for a given scenario.

The figure 6 shows the inter-distance, the vehicle velocity, its corresponding predictions and the respective danger level. We can see that the unsafe levels are reached at the instants when the vehicle penetrates the minimal distance d_c. In the figure 7, we have plotted a zoom of the same scenario. Notice that the prediction (i.e. the dotted curve), is obtained almost one second in advance. So, the *pre-crash* level and the *unsafe* level are activated almost $4s$ and $3s$ in advance, before that the vehicle stops completely. This means that the driver delay is perfectly compensated and the driver is able to start a safe braking maneuver.

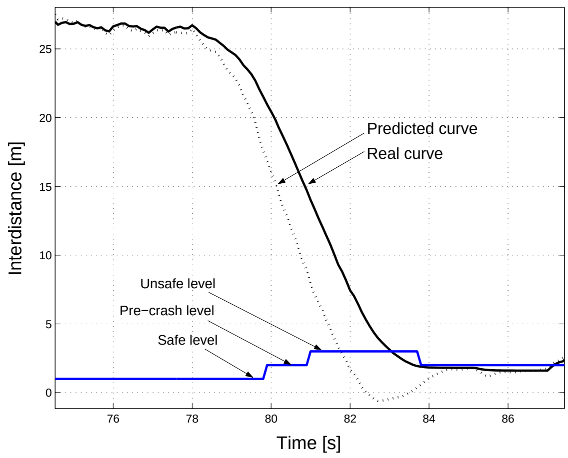

Fig. 7. Zoom of the inter-distance curves and the danger levels.

4. COMPARISON AND DISCUSSION

The proposed system has two important differences with respect to the classic warning systems. First, the proposed system project the vehicle position and the inter-distance based on a dynamic vehicle model. In the other hand, the alert space is defined from a dynamic reference model instead of the typical stationary newtonian motion equations. Thus, the reference inter-distance model permits to analytically calculate the necessary conditions to avoid a collision and provides a suitable brake manoeuver to stop the vehicle.

The vehicle model used here could feel too complex, but in situations where state trajectories are very predictable, such as when projecting only few seconds into the future, this model may be quite accurate, permitting additionally to discriminate the admissible trajectories. In addition, the uncertainties due to the sensors measures could be managed by introducing a safety buffer in the reference model, e.g. the distance d_c. The distance d_c acts as a buffer to account for possible deviations or sources of error. In general, the robustness of any warning system are highly sensible to the sensor quality. In this study we are assumed that the physical metrics are available through suitable sensors, either directly or through some additional filtering or estimation. Thus, the boundary between alerting and not alerting is then defined also by the knowledge of the sensor uncertainties.

5. CONCLUSION

A new inter-distance collision warning system has been presented. The proposed system warns the driver about an inter-distance danger. The prediction of the future states permit to compensate the human delay (of almost one second), that the drivers take to recognize a danger. In addition, we can use the same system to determine when an automatic braking or pre-crash action must be activated.

The proposed system uses a reference model to obtain the conditions with which a collision could be avoided, and gives a possible inter-distance dynamics (a braking strategy) that respects the maximal braking capacity.

In this study, we have tested the system using odometry measurements. The odometry may be not a practical method to calculate the inter-distance, but this first experience permits to evaluate the performance of the proposed system. As a future work, tests using commercial sensors would be considerate.

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[3] ARCOS is a French program on safety vehicle and secure roads. For details, see http://www.arcos2004.com