FORMAL DESCRIPTION OF DECISION PROCESSES

László Cserny

Institute of Informatics, College of Dunaújváros
H-2400 Dunaújváros, Táncsics M.u.1/a, Hungary
E-mail: cserny@mail.duf.hu

Abstract: The paper presents the mathematical description of the decision process, of one of the most important processes of organizational control. Becoming better acquainted with the decision process, together with the organizational decision-making system, may contribute to the development of usable DSSs considerably.

Keywords: Decision-making process, decision support systems, decision-theory, modelling

1.INTRODUCTION

1.1.Preliminaries

In the competitive and fast changing environment of organizations, the assistance of corporate decisions has become more and more important. The purpose of organizational decision support is to help the decision-making activities

- on every level of organizational hierarchy,
- along the whole decision process.

Prior to developing such a system, it should be known

- the place of support and
- the decision task to be supported; and

it should also be known the relationships

- between the decisions (between the decision points) resulting in organizational decisions and further,
- between the upper level decision-makers and the lower level decision-makers.

The above mentioned conditions make it necessary to learn the decision process in detail so that to develop a usable, structured methodology for the development and implementation of DSSs.

1.2.Decision-making process

How do others see the decision process? One of the main questions is how the authors treat the process: as problem-solving or, without the implementation phase, simply as a decision-making process? The paper does not make such distinction, rather it considers the decision-making process as a problem-solving process.

The most frequently referred description of decision process is from Simon's book(1977) that distinguishes three phases: (1) intelligence, (2) design and (3) choice. This interpretation is used by Holsapple and Whinston(1996), Turban and Meredith(1994), Turban and Aronson(1998). More detailed description can be found in the books written by Rhodes (1994) and by Cooke and Slack(1991). This last one is closest to our conception. All of the above mentioned authors deal with the decision process in itself and do not analyse the relations between the decisions, decision processes. They all give only verbal and not a formal model of decision process.

Several elements of formal description have already been found in (Mesarovic, et al., 1970) and in (Basar and Cruz, 1982). They, however, dealt with the problem of coordination rather than the decision process itself. Though the author also published a rudimentary version of formal description (Cserny, 1988), the main topic of his paper, however, was the organizational decision-making system.

Decision process in the state space. The decision process can be described and explained on the basis of system movement in the system's state space (Fig.1). Any motion of the system can be interpreted as a transition between a starting and an end state determined by the decision taken in the starting state.

The state space of a system can be given by the Cartesian product of the sets $S_i,\ i=1,2,\ldots,n$ of state variable values and one point in the state space can be given in the form[1]

$$\mathbf{s}=(s_1,s_2,\ldots,s_i,\ldots,s_n) \quad and \quad s_i \in S_i \quad for \ \forall i \qquad (1)$$

that is

$$S=\{\mathbf{s}|\mathbf{s}=(s_1,s_2,..,s_i,..,s_n) \ \ and \ \ s_i \in S_i \ \ for \ \ \forall i\}$$
$$= S_1 \times S_2 \times \ldots \times S_i \times \ldots \times S_n \qquad (2)$$

After expanding the sphere of variables with the goal variables(**z**), the state space can be given as follows below. As the quantifiable or non-quantifiable goals of the system have some kind of relations to the input (**x**) and output(**y**) variables of the system, the form of the goal function will be the next:

$$\mathbf{z}=\mathbf{f}(\mathbf{x},\mathbf{y})=(f_1(\mathbf{x},\mathbf{y}),f_2(\mathbf{x},\mathbf{y}),\ldots,f_l(\mathbf{x},\mathbf{y}))$$
$$=(z_1,z_2,\ldots,z_l) \qquad (3)$$

and the enlarged state vector **s**

$$\mathbf{s}=(\mathbf{x},\mathbf{y},\mathbf{z})=(\mathbf{x},\mathbf{y},\mathbf{f}(\mathbf{x},\mathbf{y}))=(s_1,s_2,..,s_n,..,s_{n+l}) \quad (4)$$

and the state space will be

$$S = X \times Y \times Z \qquad (5)$$

2.FORMAL DESCRIPTION

Thus, the decision process will be studied in the state space of the system. As a consequence, the decisions should be treated as the elements of some decision sequence and these decisions should move the system from one state to another state according to the objective of a long-term transition (activity).

2.1.Recognition of decision situation, of decision problem, evaluation of current state

A ***decision situation*** arises, if (1) the decision-maker thinks the current state differs from the planned state; if (2) on the basis of an information set, information constellation, the decision-maker feels to have a chance to make a decision that might be advantageous for him or for his system; if (3) any external effect (e.g. the instruction of an upper level decision-maker within the organization, or the demand of a lower level decision-maker) induces the decision-maker to make a decision.

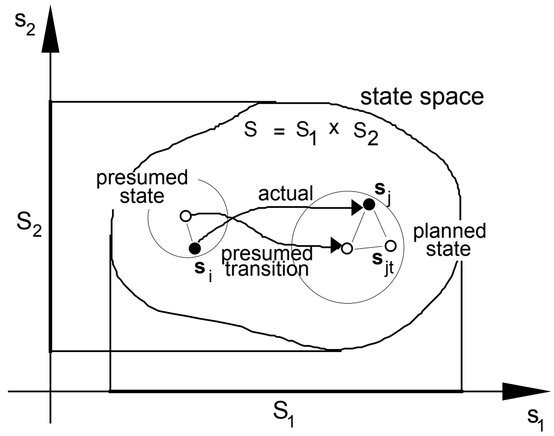

Fig. 1: The movement of a system in the state space

Hence, the first step of the decision process is the determination of the current state of the system and its comparison with the planned state or recognition of a new situation worth to consider. To do this, one should usually collect information with prescribed repetition from the decision-maker's own environment, about the system belonging to the decision-maker's sphere of authority. The extension of the information collection depends on the localization of the decision-maker's system within the organization, on the sphere of task, of authority and that of responsibility.

Thus, there is some set I of information in which, with grouping the information, the decision-maker may find and recognize k pieces of ***decision problems***. The decision-maker has to rank the decision problems by the aid of some kind of methods so that to get a preference order of problems. To these problems non-definitely disjunct subsets of set I may belong, i.e.

$$I=\{i_1,i_2,\ldots,i_j,\ldots i_q\}=I_1 \cup I_2 \cup \ldots \cup I_i \cup \ldots \cup I_k \qquad (6)$$

The subsets that belong to certain decision problems can be given in the following form:

$$I_i=\{i_{i_j}|i_{i_j} \in I, \ \ for \ \ \forall j\}$$
$$=\{i_{i_1},i_{i_2},\ldots,i_{i_j},\ldots,i_{i_m}\} \subseteq I \quad for \ \ \forall i \qquad (7)$$

and we may suppose that their indexes correspond to their ranks at the same time (where the most important problem is denoted by the index of number 1):

$$I_1 \succ I_2 \succ I_3 \succ \ldots \succ I_k \qquad (8)$$

[1] In the paper, there will not be made any distinction, unless it is necessary, between the notation of a vector and its transposed.

Solving a decision problem, means a task such that the decision-maker should collect information of the current state, of criteria influencing the decision, then he should determine a conception of goals to be achieved and that of the alternatives (activities) providing the achievement of those goals.

Among the elements of information set *I*, there are such information that are essential (as decision variables) in the decision-making and there are also such information that only contribute to the development of the ***decision constraint***. After processing this information set, one can get another series of information the elements of which give knowledge of decision variable values to be achieved on the one hand, and give knowledge of alternatives to be applied. During the information processing it may come to light that further information might be required for the decision.

From the point of view of the organizational decision-making, only those decision problems are interesting now that are somehow related to the deviation from the planned goal state. All the more so, because in any decision situation, mentioned previously, any organizational decision can only be made if the decision-maker knows the actual state of the system belonging to the decision-maker's authority and the deviation from the planned state.

The state vector $\mathbf{s}_i$ of the current situation can be determined from the information sets given in (6), (7); while the planned value of the current state comes from the knowledge base. In the majority of practical cases, the deviation between the planned ($\mathbf{s}_{it}$) and the current ($\mathbf{s}_i$) states equals to the deviation between the goal variables; i.e. if

$$\mathbf{s}_{it} = (\mathbf{x}_{it}, \mathbf{y}_{it}, \mathbf{z}_{it}) \in S \qquad and \qquad \mathbf{s}_i = (\mathbf{x}_i, \mathbf{y}_i, \mathbf{z}_i) \in S \tag{9}$$

then the deviation will be:

$$\Delta = \| \mathbf{s}_i - \mathbf{s}_{it} \| \sim \| \mathbf{z}_i - \mathbf{z}_{it} \| \tag{10}$$

Depending on the measure of deviation, the place and the level of the decision may change.

The analysis of deviation in a multi-level decision-making process (according to the short- or long-term decision tasks and not to the hierarchy) requires, of course, the use of several threshold values (ε). The analysis of the causes of deviations helps the decision-maker as well as the organization to make the functioning of the system more efficient, and storing the knowledge obtained in this way, we may say: the system is learning.

2.2.Determination of decision task

The first step is the selection of decision variables. Then, the ***decision state space*** can be constructed by the aid of these variables. It can be proved that the decision state space (in short: decision space) is a subspace of the system state space.

The decision-maker chooses the decision variables of the ***decision task*** from among the system state variables. The selection of variables, and the construction of decision space depend on

- the actual state of the system,
- the decision-maker and his abilities,
- the decision problem,
- the available information,
- the time and
- the processing tools being at his disposal.

The decision-maker selects the ***variables of the decision space*** from the system state variables (4), (5); partly from the input, partly from the output, as well as partly from the goal variables, the possible values of which give the decision space of the task:

$$\begin{aligned} D &= \left\{ \mathbf{d} \,\middle|\, \mathbf{d} = (\mathbf{x}_D, \mathbf{y}_D, \mathbf{z}_D), \quad \mathbf{x}_D \in X_D, \mathbf{y}_D \in Y_D, \mathbf{z}_D \in Z_D \right\} \\ &= X_D \times Y_D \times Z_D \subseteq X \times Y \times Z \end{aligned} \tag{13}$$

or alternatively it can be written in the following form:

$$\begin{aligned} D &= \left\{ \mathbf{d} \,\middle|\, \mathbf{d} = (d_1, d_2, ..d_i, ..d_n), \quad d_i \in D_i, \quad for \ \forall i \right\} \\ &= D_1 \times D_2 \times \ldots D_i \ldots \times D_n \subseteq S \end{aligned} \tag{14}$$

and the decision space will be a subspace of the system state space.

As it has been mentioned earlier, there is a direct relationship between the information set determining the decision problem and the decision variables. Certain part of the information set gives the actual values of the decision variables. The information, being in the information set of the decision task's solution, give the values of the vector to be achieved in the decision space. Thus, it can be written that any decision problem may correspond to a point in the decision space, i.e.

$$I_i \sim \mathbf{d}_i \qquad I_i \subseteq I, \ \mathbf{d}_i \in D \qquad for \ \forall i \tag{15}$$

The selection of goal variables of the decision space as well as the forming of goal function are influenced by the long-term strategy of the system. At the same time, it should be taken into consideration that the decision-maker's personal objectives and the objectives of the system rarely correspond to one another. From this view-point, the goal function worked out by the decision-maker is subjective because it expresses the decision-maker's own interests too.

Thus, the formula of the decision space can be given in the following way on the basis of Formulas (13), (14):

$$D = D_1 \times D_2 \times \ldots D_i \ldots \times D_n = X_D \times Y_D \times Z_D \subseteq S \quad (16)$$

The dimensions of certain subspaces are varying depending on the decision situations and are less than or equal to the dimensions of the corresponding system sets.

2.3. Decision function, courses of action

After constructing the decision space, the development of decision function, the collection of courses of action as well as their assignment to the state transitions of the decision space are the tasks of the next step.

The ***decision function*** evaluating the possible courses of action assigns a value to every point of the decision space, or more precisely to every state transition. In simpler case, this function may be equal to the goal function **z**=**f**(**x**,**y**) determining the values of the goal variables or equal to one of its components. In more complicated cases, the decision function is defined over the entire decision space using also the values of goal variable **z**. The decision function, denoted with δ, is a function(or mapping) usually with a range of real numbers (in certain cases with a range of natural numbers).

As a matter of fact, the decision function evaluates always the transition between two points (e.g. $\mathbf{d}_i, \mathbf{d}_j$) of decision state space but from the point of view of decision itself, the value assigned to the current state can be regarded as zero. In general, the transition between two points is evaluated as

$$\delta\left[(\mathbf{d}_i, \mathbf{d}_j)\right] \qquad \mathbf{d}_i, \mathbf{d}_j \in D \quad (17)$$

or in other form:

$$\delta : D \times D \to R^1 \quad (18)$$

The transition between the points of decision state space is always realized by some ***course of action*** (if any exists). Thus, the set A of courses of action can be assigned to the set DxD of transitions. If h is an assignment function then the set of courses of action can be given in the following way:

$$A = \left\{ a_{ij} \,\middle|\, a_{ij} = h\left[(\mathbf{d}_i, \mathbf{d}_j)\right],\ \mathbf{d}_i, \mathbf{d}_j \in D \quad for \ \ \forall i, j \right\} \quad (19)$$

i.e.

$$h : D \times D \to A \quad (20)$$

Often the reversed assignment is used, as knowing the possible courses of action, one looks for the transitions realized by the aid of those actions i.e. for the achievable goals after all. Thus,

$$h' : A \to D \times D \quad (21)$$

In both cases, the decision function δ evaluates the courses of action because a correpondency can be established between the transitions and the alternatives. This correspondency is usually not an *isomorphic* (one-to-one bijective) assignment, since, for the sake of simpler usage, it is not worth taking all of the possible decision variables into consideration and distinguishing the alternatives from each other. Thus, it may happen that in the case of assignment (20), one transition can correspond to more courses of action; at the same time, in the case of mapping (21), every course of action can be assigned to a transition, but a transition can be realized by more than one course of action.

Thus, in an ideal situation, in connection with the decision function, it can be written that

$$(\delta : D \times D \to R^1) \ \sim \ (\delta : A \to R^1) \quad (22)$$

i.e. the two mappings, from our point of view, are equivalent with one another.

Considering the decision function, it has already been mentioned that it may agree with the goal function in simpler problems.

2.4. Estimation of risk

Besides the evaluation of alternatives, the decision-maker should know the risk of realization too. Interpreting the term *risk*, see Figure 2.

Any state transition in the decision space is always resulted in by the execution of some course of action (a_{ij}). During the execution of an alternative, on the effect of events not to be seen in advance, we can get a result state ($\mathbf{d}_j$) that can deviate from the planned goal state ($\mathbf{d}_{jt}$). This deviation cannot be given at the moment of decision, only the probability can be estimated(even in subjective way) whether the extent of deviation from the planned goal is greater or not than a prescribed value (ε). If the outcome of the course of action is within this domain then the execution of decision will be successful(or 'satisficing').

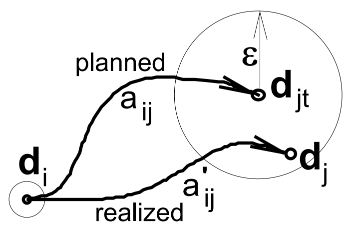

Fig. 2: Risk interpretation in the decision state space

The ***risk of a decision*** is understood as the conditional probability of the event that the outcome of the chosen alternative falls out of a given domain of the planned goal state.

Using the notations of Figure 2, in the *i*-th decision situation the risk(p_i) can be given by the conditional probability below:

$$p_i = P(\left\| \mathbf{d}_j - \mathbf{d}_{jt} \right\| > \varepsilon \mid a_{ij}) \quad \mathbf{d}_j, \mathbf{d}_{jt} \in D,\ a_{ij} \in A,\ \varepsilon > 0 \tag{23}$$

where $\mathbf{d}_j, \mathbf{d}_{jt}$ are the actual and the planned goal states in the decision space,
a_{ij} is the course of action chosen,
ε is the chosen size of deviation.

For the measurement of distance between the planned and the actual decision states, the applied measuring method depends on the actual task, but the Euclidean norm can be seen as common measuring method.

On the basis of previous paragraphs, it can generally be stated that some probability distribution shall belong to each course of action even in the cases when it is not known for the decision-maker.

Thus, a probability field (Ω, $\mathcal{B}$, P) can be given for every alternative, where Ω is the set of elementary probability events, $\mathcal{B}$ is the set of all subsets of Ω, and P is a (probability) measure over $\mathcal{B}$. The events referred to in (23), i.e. the deviations of outcomes from the planned state, belonging to a given value ε, can be given in the following way:

$$\omega_\varepsilon = \left\{ \left(\left\| \mathbf{d}_j - \mathbf{d}_{jt} \right\| > \varepsilon \right),\ \mathbf{d}_j, \mathbf{d}_{jt} \in D,\ \varepsilon > 0 \right\} \subseteq \mathcal{B} \tag{24}$$

The measure of deviation between the planned and the realized outcomes is a probability variable to which a conditional distribution function, the risk distribution function F can be defined:

$$F(\varepsilon) = 1 - P(\left\| \mathbf{d}_j - \mathbf{d}_{jt} \right\| < \varepsilon \mid a_{ij}) \quad a_{ij} \in A,\ for\ \forall i, j\ and\ \varepsilon > 0 \tag{25}$$

The uncertainty of the achievement of goal can be given with the probability field given to every course of action, or another way of saying this, a subset of the events belongs to every course of action and this subset is mapped on the domain of real numbers, more precisely on the interval [0,1] by the probability measure P:

$$\mathsf{P}: A \times \Omega \to R^1 \tag{26}$$

that is

$$\mathsf{P}: A \times \Omega \to V = [0,1] \subset R_0^+ \subset R^1 \tag{27}$$

where R_0^+ is the set of positive real numbers plus the zero.

2.5.Evaluation of alternatives, decision

Now, the essential character of the decision can also be given. So far the *decision* has been understood as a choice from at least two possible alternatives. Hereafter, this definition may be enlarged: knowing and balancing the available result and the probable risk, a ***decision*** is understood as a choice from at least two possible alternatives in order to achieve some objective.

Preparing a decision, the alternatives should be evaluated by the aid of decision function such that the alternatives might at least be ordered on the basis of the assigned values. For ranking the alternatives, it is sufficient to map the alternatives on the set of real numbers(more precisely on the set of positive integers), but it can generally be said that the set of real numbers is usually sufficient to take into consideration.

On the basis of Section 2.4, taking into account the risk, as well as using the equivalence in (22) and completing it with the set Ω of probabilistic events, the uncertainty(as well as the risk of decision) related to the transition between the decision situations and to the execution of the course of action can be given.

$$(\delta : (D \times D) \times \Omega \to R^1) \sim (\delta : A \times \Omega \to R^1) \tag{28}$$

The value of the decision function is determined by the decision space D, by the courses of action A and the assigned set of probabilistic events Ω together. Our purpose is to determine the alternative in the case of which the value of the decision function will take its minimum(or maximum, or 'satisficing' by chance) value and at the same time, the risk (according to (27)) will also take a minimum value. Let δ' denote the decision-maker's balancing function that can evaluate the courses of action and the risks together and by the aid of which the decision-maker can determine the set of his satisficing alternatives. Thus, before the decision, one should look for the set A_O of courses of action such that the ***balancing decision function*** δ' gives minimum or 'satisficing' value over the set A_O, i.e.

$$A_o = \left\{ a \mid a \in A \quad and \quad \delta'(D, A, \Omega) = \min \right\} \subseteq A \tag{29}$$

Thus, the ***decision*** is understood as a mapping γ that determines a certain subset(or element) of the set of courses of action such that the decision-maker's *balancing decision function* δ' gives the most preferable(minimum) value over the subset, i.e.

$$\gamma : A \to A_o = \left\{ a \mid a \in A \quad and \quad \delta'(D, A, \Omega) = \min \right\} \tag{30}$$

If A_O has only one element(course of action), i.e.

$$A_o = A^* = \{a^*\} \tag{31}$$

then the result of decision task is given, the optimal or satisficing solution is achieved. If this is not true, then an additional step, an additional (subjective) criterion is required to assign the only alternative as the solution.

It can be presumed that the decision-maker always finds a *balancing decision function* δ' such that his decision assigns only one alternative at once, i.e.

$$\gamma : A \rightarrow A^* = \{a^*\} \quad (32)$$

2.6.Decision task

Thus, the ***decision task*** is considered as the determination of the best course of action required to achieve the given objective - taking into account the actual decision situation. At the same time, this means the determination as well as the use of decision space, of alternatives, of probability distributions, and of decision function. The decision task is unambiguously given by the quadruplet (D, A, Ω, δ) and therefore, hereafter, a decision task F belonging to a given decision situation is understood as this quadruplet, i.e.

$$F = (D, A, \Omega, \delta) \quad (33)$$

2.7.Founding organizational decision-making

In practice, the decision situations can hardly be imagined solely being independent from earlier or later situations. Therefore, especially in the case of organizational decision-making system, we have to consider the problem of sequential decision-making, too (Cserny, 2001, 2002)

3.CONCLUSIONS

The formal description of decision process can help the developer to design a usable DSS. The main conclusions underlining the importance of mathematical approach of the description are as follows.

- The given process stages meet the demands of decision support. The process is properly structured so as to answer the questions: where and how should the decision-makers be supported and what kind of decision support do they need? *For example*, at the stage of decision situation or decision problem recognition one needs a tool to collect information, to set up relations among them meanwhile taking into consideration the decision-maker's behaviour and customs; hence decision-maker needs the tools of datamining and of AI to discover the decision problems. At the stage of determination of decision variables and courses of action an intelligent DSS might help the decision-maker by electronic brain-storming and other similar group-methods.
- On the basis of mathematical approach, the decision process, the decision support can be automatized without special difficulties. One can develop an intelligent DSS that prepares the decisions and guides the decision-maker in his/her decision-making work. The DSS could help the decision-maker with mathematical models and evaluation processes, with computing power.
- One can study the relationships between the upper and the lower level decision-makers' decisions, the problem of co-ordination, of co-operation in a hierarchical decision-making system (Cserny, 2001, 2002).

5.REFERENCES

Basar, T. and J.B.Jr. Cruz (1982). Concepts and Methods in Multi-person Coordination and Control. In: *Optimisation and Control of Dynamic Operational Research Models* (Tzafestas, S.G.(ed)). North-Holland, Amsterdam

Cooke, S. and N. Slack (1991). *Making Management Decisions*. Prentice-Hall, New York (2nd ed.)

Cserny, L. (1988). The Analysis of Decision-Making Systems. In: *Systems Analysis and Simulation 1988 Vol.I.* (Sydow, A., S.G. Tzafestas and R. Vichnevetsky(eds.)). 53-58, Akademie-Verlag, Berlin

Cserny, L. (2001). Modelling Organizational Co-ordination. In: *Modelling and Control of Economic Systems 2001* (Neck, R.(ed.)). 267-272. Elsevier (Pergamon Press), New York, 2003

Cserny, L. (2002). Organizational Decision-Making as Special Dynamic Game. *Proceeding of The Tenth International Symposium on Dynamic Games and Applications*. Vol.1., 235-239. St.Petersburg

Holsapple, C.W. and A.B. Whinston(1996). *Decision Support Systems, a Knowledge-Based Approach*. West Publ. Co., St. Paul

Mesarovic, M.D., D. Macko and Y. Takahara (1970). *Theory of Hierarchical, Multilevel Systems*. Academic Press, New York-London

Rhodes, P.C. (1994). *Decision Support Systems: Theory and Practice*. Alfred Waller, Henley-on-Thames

Simon,H.A. (1977). *The New Science of Management Decision*. Prentice-Hall, Englewood Cliffs

Turban,E. and J.E.Aronson (1998). *Decision Support Systems and Intelligent Systems*. Prentice Hall, Upper Saddle River (5th ed.)

Turban,E. and J.R.Meredith(1994). *Fundamentals of Management Science*. Irwin, Homewood(6th ed.)