# DYNAMIC OUTPUT FEEDBACK STABILIZATION OF A CLASS OF NONHOLONOMIC HAMILTONIAN SYSTEMS

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Abstract: This paper is concerned with discontinuous output feedback stabilization of a class of nonholonomic systems in a port-controlled Hamiltonian form. First, in order to obtain a dynamic feedback, an integrator is added to the system via a generalized canonical transformation. Second, we clarify an equivalence between asymptotic stability of a state feedback system and that of the corresponding output feedback system. An output feedback stabilization method is derived based on this equivalence. Furthermore, some numerical examples show the effectiveness of our technique. 

Keywords: output feedback systems, nonholonomic systems, Hamiltonian systems

## 1. INTRODUCTION

Hamiltonian control systems (van der Schaft 1986, Nijmeijer and van der Schaft 1990) are the systems described by Hamilton's canonical equations which represent general physical systems. Recently port-controlled Hamiltonian systems are introduced as a generalization of Hamiltonian systems (Maschke and van der Schaft 1992). They can represent not only ordinary mechanical, electrical and electro-mechanical systems, but also a class of nonholonomic systems (Maschke and van der Schaft 1994, Khennouf *et al.* 1995) which can not be stabilized by any continuous time-invariant controllers. The special structure of physical systems allows us to utilize the passivity which they innately possess and a lot of fruitful results were obtained so far. These methods are so called passivity based control.

One of the advantages of passivity based control is output feedback control, see e.g. (Ortega *et al.* 1998) for Euler-Lagrange systems and extended for Hamiltonian systems in (Stramigioli *et al.* 1998), also a more general result can be found in (Ortega *et al.* 1999). It is usually difficult to stabilize a nonlinear system using output feedback because there is no efficient way of designing a state observer of nonlinear systems. Utilizing the intrinsic passive property of physical systems, however, it is easy to stabilize the system by only using the information of output.

On the other hand, when we control a mechanical systems in practice, the velocity signals are not always measured. This means that the performance of the feedback system can be poor when the velocity signals are given by difference approximations of the position signals. In particular, in the case where discontinuous feedbacks are employed, even stability can be lost because of the the approximation error. In order to avoid these problems, the framework of output feedback control is needed. In addition, in the case where initial cost of the mechanical systems should be emphasized, the framework is also important with respect to the number of sensors.

This paper is devoted to output feedback stabilization of a class of nonholonomic Hamiltonian systems. We derive a discontinuous output feedback compensator for nonholonomic Hamiltonian systems. First, we refer to a state feedback stabilization of the systems (Fujimoto and Sugie 2001). Second, a framework of dynamic extension in order to obtain a dynamic compensator for nonholonomic Hamiltonian systems is derived. Third, an equivalence between stability of a state feedback system and that of the corresponding output feedback system is clarified. Furthermore, a gain-tuning guideline is proposed to improve transient behavior. Finally the effectiveness of the proposed method is demonstrated via some numerical examples.

## 2. NONHOLONOMIC HAMILTONIAN SYSTEMS

A port-controlled Hamiltonian system with nonholonomic velocity constraint is a system with the following state-space realization (van der Schaft 2000)

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & J_{12}(q) \\ -J_{12}(q)^T & J_{22}(q,p) \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q}^T \\ \frac{\partial H}{\partial p}^T \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u \\ y = G(q)^T \frac{\partial H}{\partial p}(q,p)^T \end{cases} \quad (1)$$

where $q \in \mathbf{R}^n$ is position, $p \in \mathbf{R}^m$ is momentum and $u, y \in \mathbf{R}^m$ are input and output, respectively. $J_{12}$ is a full column rank matrix, $J_{22}$ is a skew-symmetric matrix, $H = (1/2)p^T M(q)^{-1} p$, $M(q) > 0$ is a symmetric matrix, $G(q)$ is a nonsingular matrix, and $J_{22}(q, 0) \equiv 0$ holds. Note that this $n + m$-order system is a reduced system of ordinary $2n$-order Hamiltonian system.

Here, we refer to a state feedback stabilization of (1) using generalized canonical transformations (Fujimoto and Sugie 2001) . These transformations are natural generalization of the canonical transformations which are well-known in classical mechanics and preserve the structure of the port-controlled Hamiltonian systems.

**Theorem 1** (Fujimoto and Sugie 2001) *Consider the port-controlled Hamiltonian system (1) which is converted by a generalized canonical transformation so as to have $J_{12}$ matrix as*

$$J_{12} = \begin{bmatrix} I_2 \\ q_{12}^T S_2 \end{bmatrix} S_1, \ \det S_2 > (\mathrm{tr} S_2/2)^2, \ \det S_1 \neq 0 \quad (2)$$

*where $q = (q_{12}^T, q_3)^T$, $q_{12} = (q_1, q_2)^T$. Choose a positive definite function $U(\|q_{12}\|, q_3)$ which is smooth for $q_{12} \neq 0$ and satisfies* [1]

$$\left.\frac{\partial U(s, q_3)}{\partial s}\right|_{s=0} = \lim_{s \to +0} \frac{\partial U}{\partial s} \ (q_3 \neq 0) \quad (3)$$

$$\left.\frac{\partial U(s, q_3)}{\partial s}\right|_{s=0} = 0 \ (q_3 = 0) \quad (4)$$

$$-\infty < \lim_{s \to 0} \frac{\partial U(s, q_3)}{\partial s} < 0 \ (q_3 \neq 0) \quad (5)$$

$$q_{12} \neq 0 \Rightarrow \frac{\partial U}{\partial q} \neq 0. \quad (6)$$

*Then the following feedback with $C > 0$ renders the state converge to the origin.*

$$u = -Cy - G^{-1} J_{12}^T \frac{\partial U}{\partial q}^T - G^{-1} J_{22} M^{-1} p \quad (7)$$

The examples of the function $U$ which stabilizes (1) are already given in the literature (Fujimoto and Sugie 2001) as follows,

$$U = \frac{1}{2} q^T q + \frac{|q_3|^3}{(\|q_{12}\| + |q_3|)^2} \quad (8)$$

and (Nakamura *et al.* 2003). Fig.1 depicts the shape of the function (8).

In the right-hand side of (7), the first term and the third term depend on the momentum $p$. However, in practice, the velocity signal $\dot{q}$ , which is required to compute $p$, is not always measured. In addition, as the system scale becomes larger, it becomes more difficult to identify the parameters of inertial matrix $M(q)$ accurately. In the next chapter, we consider the stabilization without measuring the momentum $p$.

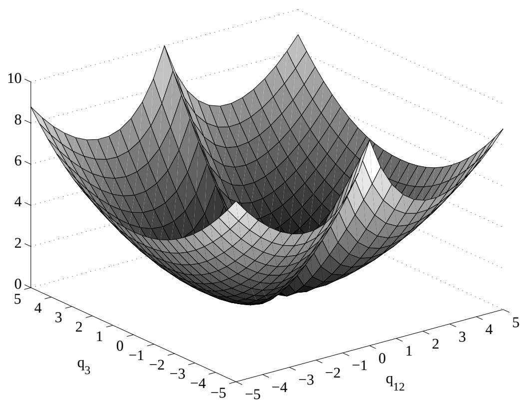


Fig. 1. Shape of function $U$

## 3. OUTPUT FEEDBACK STABILIZATION

This section is devoted to the main result of output feedback stabilization of nonholonomic Hamiltonian systems. First, it is shown how to derive a dynamic compensator using generalized canonical transformations. Second, the equivalence between

[1] Th derivatives of $U$ is design parameters, that is, defined to be used to controller.

the asymptotic stability of the state feedback system and that of the corresponding output feedback system is clarified for nonholonomic Hamiltonian systems with the derived compensator. Finally, an output feedback compensator is derived based on the equivalence and Theorem 1.

### *3.1 Dynamic extension*

Consider the system (1) again and suppose that we can only measure the position $q$. We add the integrator $r \in \mathbf{R}^n$ to the system:

$$\begin{cases} \begin{bmatrix} \dot{q} \\ \dot{p} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & 0 \\ -J_{12}^T & J_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q}^T \\ \frac{\partial H}{\partial p}^T \\ \frac{\partial H}{\partial r}^T \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ G & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} u \\ u_o \end{bmatrix} \\ \begin{bmatrix} y \\ y_o \end{bmatrix} = \begin{bmatrix} G^T \frac{\partial H}{\partial p}^T \\ \frac{\partial H}{\partial r}^T \end{bmatrix} \end{cases} \tag{9}$$

whose Hamiltonian is

$$H = \frac{1}{2} p^T M^{-1} p. \tag{10}$$

Here $r$ is the state of the compensator, however is not connected to the system (1) yet. The following lemma connects the dynamics of the original system and that of $r$-integrator via a generalized canonical transformation.

**Lemma 1** *Consider the system (9) with the Hamiltonian (10) without loss of generality. Then the generalized canonical transformation*

$$\begin{aligned} \bar{H} &= H(q,p,r) + \frac{1}{2}(q-r)^T R (q-r) \\ \begin{bmatrix} \bar{q} \\ \bar{p} \\ \bar{r} \end{bmatrix} &= \begin{bmatrix} q \\ p \\ q-r \end{bmatrix} \\ \begin{bmatrix} \bar{y} \\ \bar{y}_o \end{bmatrix} &= \begin{bmatrix} G^{-T} & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} y \\ y_o \end{bmatrix} + \begin{bmatrix} 0 \\ R\bar{r} \end{bmatrix} \\ \begin{bmatrix} \bar{u} \\ \bar{u}_o \end{bmatrix} &= \begin{bmatrix} G & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} u \\ u_o \end{bmatrix} + \begin{bmatrix} -J_{12}^T R\bar{r} \\ 0 \end{bmatrix} \end{aligned} \tag{11}$$

*transforms the system (9) into the following port-controlled Hamiltonian system*

$$\begin{cases} \begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{\bar{r}} \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & 0 \\ -J_{12}^T & J_{22} & -J_{12}^T \\ 0 & J_{12} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{q}}^T \\ \frac{\partial \bar{H}}{\partial \bar{p}}^T \\ \frac{\partial \bar{H}}{\partial \bar{r}}^T \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{u}_o \end{bmatrix} \\ \begin{bmatrix} \bar{y} \\ \bar{y}_o \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{p}}^T \\ \frac{\partial \bar{H}}{\partial \bar{r}}^T \end{bmatrix} \end{cases} \tag{12}$$

*whose Hamiltonian is*

$$\bar{H} = \frac{1}{2}\bar{p}^T M^{-1} \bar{p} + \frac{1}{2}\bar{r}^T R \bar{r} \tag{13}$$

*where $R > 0$ is any positive definite matrix.*

**Proof of Lemma 1.** The proof is straightforwardly obtained from a direct calculation.

### *3.2 Output feedback stabilization*

We will prove the equivalence between the asymptotic stability of the state feedback system in Theorem 1 and that of the corresponding output feedback system constructed in Lemma 1.

**Theorem 2** *Consider the systems $\Sigma_s$ and $\Sigma_o$:*

$$\Sigma_s \begin{cases} \begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \end{bmatrix} = \begin{bmatrix} 0 & J_{12} \\ -J_{12}^T & J_{22} \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{q}}^T \\ \frac{\partial \bar{H}}{\partial \bar{p}}^T \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} \bar{u}_s \\ \bar{y}_s = G^T \frac{\partial \bar{H}}{\partial \bar{p}}^T \end{cases} \tag{14}$$

$$\Sigma_o \begin{cases} \begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{\bar{r}} \end{bmatrix} = \begin{bmatrix} 0 & J_{12} & 0 \\ -J_{12}^T & J_{22} & -J_{12}^T \\ 0 & J_{12} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{q}}^T \\ \frac{\partial \bar{H}}{\partial \bar{p}}^T \\ \frac{\partial \bar{H}}{\partial \bar{r}}^T \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \bar{u} \\ \bar{u}_o \end{bmatrix} \\ \begin{bmatrix} \bar{y} \\ \bar{y}_o \end{bmatrix} = \begin{bmatrix} \frac{\partial \bar{H}}{\partial \bar{p}}^T \\ \frac{\partial \bar{H}}{\partial \bar{r}}^T \end{bmatrix} \end{cases} \tag{15}$$

*where $\bar{H} = (1/2)\bar{p}^T M^{-1} \bar{p} + (1/2)\bar{r}^T R \bar{r}$.*

*Then the following two conditions are equivalent under the assumption of the existence and uniqueness of the trajectories of both systems.*
*(i)The equilibrium set of the closed-loop system of $\Sigma_s$ and the compensator*

$$\bar{u}_s = -C_s \bar{y}_s - J_{12}^T \frac{\partial U}{\partial \bar{q}}^T \tag{16}$$

*is the only origin and asymptotically stable, where $C_s > 0$ is any positive definite matrix.*
*(ii)The equilibrium set of the closed-loop system of $\Sigma_o$ and the compensator*

$$\begin{bmatrix} \bar{u} \\ \bar{u}_c \end{bmatrix} = \begin{bmatrix} -J_{12}^T \frac{\partial U}{\partial \bar{q}}^T \\ -C_o \bar{y}_c \end{bmatrix} \tag{17}$$

*is the only origin and asymptotically stable, where $C_o > 0$ is any positive definite matrix.*

Note that $\bar{y}$ of $\Sigma_o$ depends on $p$ but is not used for the compensator (17).

**Proof of Theorem 2.** (i)⇒(ii) First of all, we clarify equilibrium sets of the both closed-loop systems. The equilibrium set of the closed-loop system of $\Sigma_s$ and (16) is

$$\Omega_s = \{(\bar{q}, \bar{p}) \big| \bar{p} = 0,\ J_{12}^T \frac{\partial U}{\partial \bar{q}}^T = 0\}. \tag{18}$$

The equilibrium set of the closed-loop system of $\Sigma_o$ and (17) is

$$\Omega_o = \{(\bar{q}, \bar{p}, \bar{r}) \big| \bar{p} = 0,\ \bar{r} = 0,\ J_{12}^T \frac{\partial U}{\partial \bar{q}}^T = 0\}. \qquad (19)$$

It is trivial that $\Omega_o$ is the origin if and only if $\Omega_s$ is the origin.

In addition, suppose that the origin of the closed-loop system of $\Sigma_s$ and (16) is asymptotically stable. As for the closed-loop system of $\Sigma_o$ and (17), let $\tilde{H}(t) = \bar{H}(t) + U(t)$, then $\dot{\tilde{H}}(t) = -\bar{r}^T R^T C_o R \bar{r} \leq 0$ holds.

Here, we focus on a set $\Omega_o^0 = \{(\bar{q}, \bar{p}, \bar{r}) | \bar{r} = 0\}$ which has the following vector field at each point:

$$\begin{bmatrix} \dot{\bar{q}} \\ \dot{\bar{p}} \\ \dot{\bar{r}} \end{bmatrix} = \begin{bmatrix} J_{12} M^{-1} \bar{p} \\ -J_{12}^T \frac{\partial U}{\partial \bar{q}}^T - \left( J_{12}^T \frac{\partial (M^{-1} \bar{p})}{\partial \bar{q}}^T - J_{22} M^{-1} \right) \bar{p} \\ J_{12} M^{-1} \bar{p} \end{bmatrix}. \qquad (20)$$

In the case of $\bar{p} \neq 0$, $\dot{\bar{r}} \neq 0$ holds because $J_{12}$ is a full column matrix. In the case of $\bar{p} = 0$, $\dot{\bar{p}} \neq 0$ holds except the origin because $\Omega_o$ is the origin. This means that $\Omega_o^0 \backslash \{0\}$ is not an invariant set.

Thus, the semi-definiteness of $\dot{\tilde{H}}$ and the definiteness of $\tilde{H}$ imply that the origin of the closed-loop system of $\Sigma_o$ and (17) is asymptotically stable.

(i)⇐(ii) This can be proved in a similar way to the case (i)⇒(ii). (Q.E.D.)

Theorem 2 implies that we can derive output feedback stabilizing compensators using the state feedback ones in Theorem 1. This means that $\Sigma_o$ is stabilized by using any function $U$ which stabilizes $\Sigma_s$ in Theorem 1.

The explicit expression of the output feedback stabilizing compensator is

$$\begin{cases} \dot{r} = C_o R(q - r) \\ u = -G^{-1} J_{12}^T \left( R(q - r) + \frac{\partial U}{\partial q}^T \right) \end{cases}. \qquad (21)$$

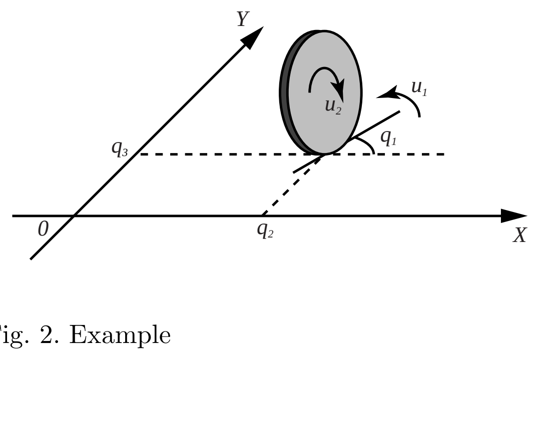


Fig. 2. Example

Note that this compensator does not depend on any dynamical parameters of (1).

## 4. NUMERICAL EXAMPLES

### 4.1 Stability

The well known "rolling coin" example is considered here. Let $X$-$Y$ denote the orthogonal coordinates of the point of contact of the coin and the horizontal plane. Let $q_1$ denote the heading angle of the coin, and $(q_2, q_3)$ the position of the coin in $X$-$Y$ plane. Furthermore let $p_1$ be the angular velocity with respect to the heading angle $q_1$, $p_2$ be the rolling angular velocity of the coin, $u_1$ and $u_2$ be the acceleration with respect to $p_1$ and $p_2$, respectively. See (Fujimoto and Sugie 2001) about the details of the coin system.

The state feedback stabilization of this example is reported in (Fujimoto and Sugie 2001). We now apply the proposed output feedback stabilization method based on Theorem 1 and 2. We choose the function $U$ of (21) as (8) and all the parameters as unity for simplicity.

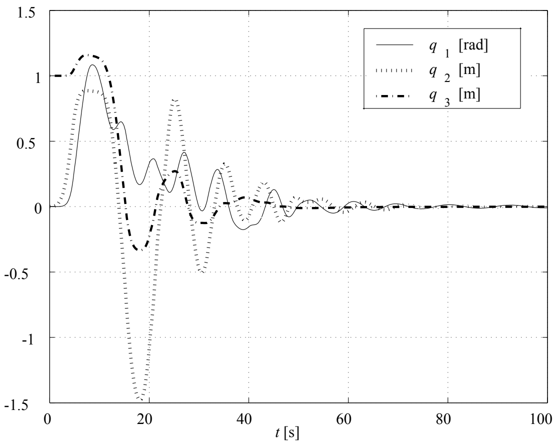


Fig. 3. Response of $q$, $q(0) = (0\ 0\ 1)^T$

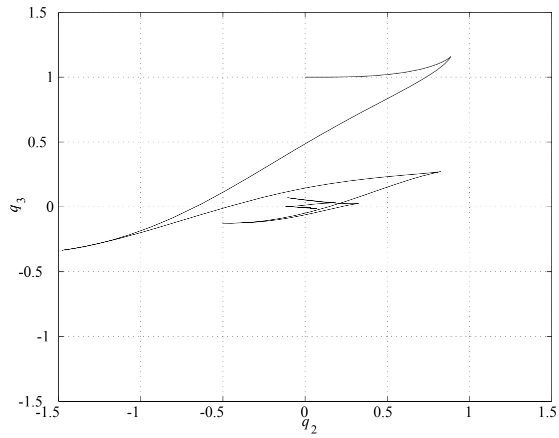


Fig. 4. Response in $X$-$Y$ plane, $q(0) = (0\ 0\ 1)^T$

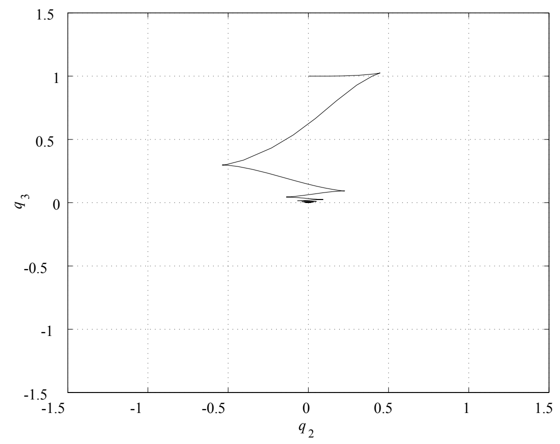


Fig. 5. Response in $X$-$Y$ plane ($\Sigma_s$)

Fig.3 shows the response of $q$ from the initial conditions $q(0) = (0, 0, 1)$, $p(0) = 0$. In this figure, solid, dashed and dashed-dotted lines denote $q_1$, $q_2$ and $q_3$, respectively. Fig.4 shows the response in $X$-$Y$ plane from the same initial condition. In this case, $C_o = 3I_3$, $R = I_3$. The turns of the coin are automatically generated and every state converges to the origin. These figures show the effectiveness of the proposed method.

### *4.2 Transient behavior*

For comparison, we show results of the state feedback stabilization method in Theorem 1. We choose the same function $U$ and $C_s = 3I_2$. Fig.5 shows the response in $X$-$Y$ plane from the initial conditions $q(0) = (0, 0, 1)$, $p(0) = 0$. This figure and Fig.4 point out the importance of improvement of the transient behavior.

However a gain-tuning guideline is not clear so far. Thus, we start from the discussion in linear systems case.

*4.2.1. Gain-tuning guideline* Consider an ordinary second-order linear system:

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K & 0 \end{bmatrix} \begin{bmatrix} q \\ p \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad (22)$$

and the output feedback stabilizing compensator by the proposed method

$$\begin{cases} \dot{r} = -C_o Rr + C_o Rq \\ u = Rr - Rq \end{cases} . \qquad (23)$$

Note that the function $U$ is included in the spring constant $K$ in this case.

Let us regard $C_o$ as a controller, then a loop transfer function

$$\frac{C_o R(s^2 + K)}{s(s^2 + K + R)} \qquad (24)$$

has the poles and zeros on the imaginary axis. This means the existence of the poles whose real part converges to zero even if we make $C_o$ too large or too small. Even though let us regard $R$ as a controller, a loop transfer function does not have the zeros on the imaginary axis.

It is shown that dominant poles have non-zero imaginary part and that the optimal gain $C_o$ which minimizes the real part of dominant poles is given as

$$C_{linear} = \frac{\sqrt{K}}{R} + \frac{1}{2\sqrt{K}} . \qquad (25)$$

which corresponds to the least oscillation. Here, we propose a gain-tuning guideline that (25) is chosen as the initial gain.

*4.2.2. Numerical examples and discussion* In the case of the initial values $q(0) = (0, 0, 1)^T$, $p(0) = (0, 0)^T$, we show the result of gain-tuning of $R$, $C_o$. First, the settling time becomes smaller as $R$ is chosen larger, however, does not change so when $R$ is more than 5. Second, the first term of (8) corresponds to $K = 1$ and the second term converges to zero as the state leaves the set $q_{12} = 0$. Thus, we choose $(K, R) = (1, 5)$ which gives an initial gain $C_{linear} = 0.7I_3$.

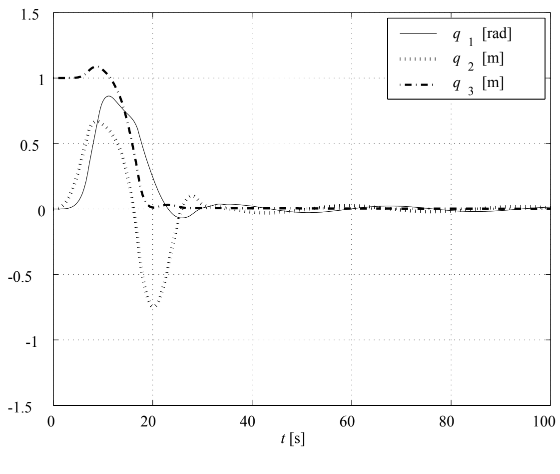


Fig. 6. Response of $q$ ($C_o = 1, R = 5$)

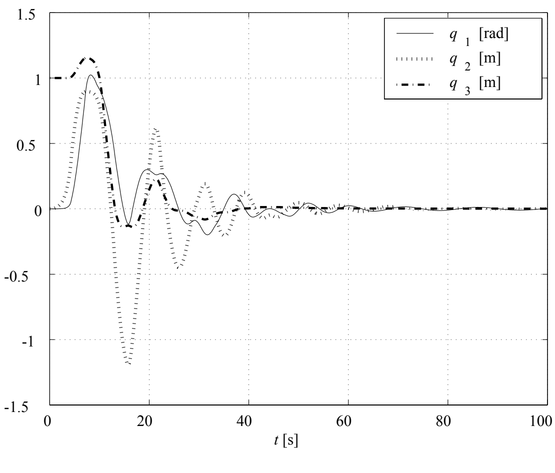


Fig. 7. Response of $q$ ($C_o = 3, R = 5$)

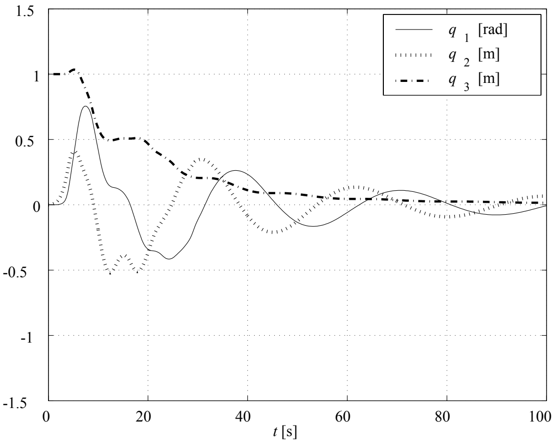


Fig. 8. Response of $q$ ($C_o = 1/3, R = 5$)

Figs.6,7 and 8 show the response of $q$ in $C_o = I_3,\ 3I_3,\ (1/3)I_3$, respectively. The highest transient performance is achieved in the case of $C_o = I_3$ which is the closest to $0.7I_3$. This result qualitatively corresponds to that of the above linear system. This shows that our initial choise of $C_o$ based on (25) is reasonable. Fig.9 shows the response in $X$-$Y$ plane from the same initial conditions and also from $q(0) = (1, 1, 1)^T$, $p(0) = (0, 0)^T$. In comparison with Fig.4, the validity of the proposed guideline is confirmed.

## 5. CONCLUSION

This paper was devoted to output feedback asymptotic stabilization of a class of nonholonomic systems in a port-controlled Hamiltonian form. First, it was shown how to derive the dynamic compensator using generalized canonical transformation. Second, the equivalence between the stability of the state feedback system and that of the corresponding output feedback system was clarified. Third, the discontinuous output feedback stabilizing compensators for the nonholonomic Hamiltonian systems were derived based on the equivalence. Furthermore, the gain-tuning guideline was proposed to improve the transient behavior. Finally, some numerical examples show the effectiveness of our technique. The authors believe that this is the first result of output feedback stabilization of Hamiltonian systems with nonholonomic constrains using discontinuous feedbacks. Applications of the proposed method to more general Hamiltonian systems will be investigated in the future work.

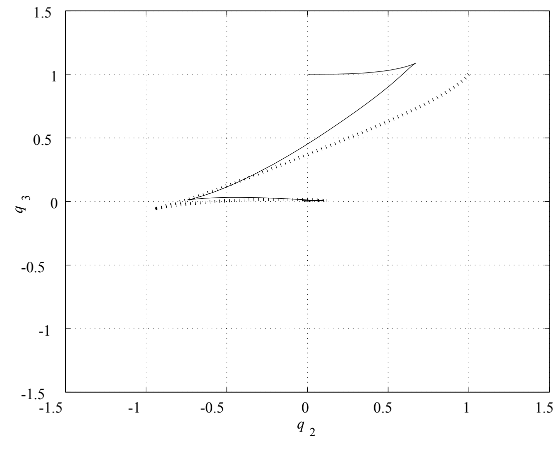


Fig. 9. Response in $X$-$Y$ plane ($C_o = 1,\ R = 5$)

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