STABILIZATION OF AN UNDERWATER VEHICLE

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Abstract: We present a sliding mode based control law for stabilizing an underactuated underwater vehicle (UUV) moving in a horizontal plane. The novelty of the proposed control law is that we exploit the dynamics of the vehicle to define three sliding surfaces, at whose intersection the system is asymptotically stable. The task of the available two actuators is then to steer the system to this intersection set.

Keywords: Finite-time stable, underactuated systems, sliding mode

1. INTRODUCTION

Control of underactuated systems: systems with fewer control inputs than the degrees of freedom has drawn widespread attention in the last one decade. Underactuation arises out of the need to reduce the actuator cost and weight or to increase the reliability of the system in case of actuator failure. Typical applications include space robotics, mobile robots and underwater/surface vehicles. Controller design for underactuated systems are more complex than the fully actuate systems.

In this paper we consider the problem of controlling a underwater vehicle moving in a horizontal plane with only two actuators. Underwater rigid vehicles are inherently underactuated in a sense that motion along certain degrees-of-freedom which are usually not encountered are unactuated or they could be realized by a combination of the existing controls forces. Control of underwater vehicles has been dealt in (Astolfi *et al.* 2002, Leonard 1991, Woolsey and Leonard 1997, Reyhanoglu 1997, Do *et al.* 2002). In (Astolfi *et al.* 2002), the problem of asymptotic stabilization of an underactuated underwater vehicle moving in an ideal fluid with partial actuation has been addressed. A control methodology known as IDA (Interconnection and Damping Assignment) is used to stabilize the vehicle in selected equilibria. Control of the planar position and orientation of an autonomous surface vessel using two independent thrusters has been addressed by (Reyhanoglu 1997). The stabilization of bottom heavy underwater vehicle has been addressed by (Leonard 1991). Global exponential stabilization of an underwater vehicle using internal rotors has been addressed by (Woolsey and Leonard 1997).

A single controller has been designed to achieve stabilization and tracking simultaneously in (Do *et al.* 2002). Backstepping and Lyapunov's direct methods have been used to design the controller. In this paper we design a sliding mode based controller for controlling position and orientation of an UUV . In this methodology we first define a surface in which the closed-loop system dynamics are asymptotically stable, then the control task is to move from any initial condition to the surface in some finite-time and maintain it there.

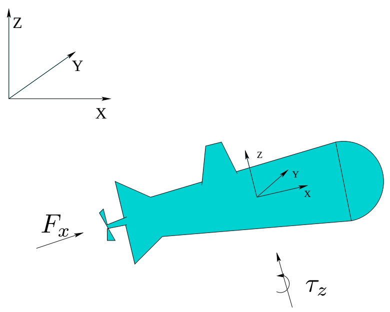

Fig. 1. Underwater vehicle

2. DYNAMIC MODEL

The modeling of an underwater vehicle has been carried out by many authors (Yuh 1990, Goheen 1991). The dynamic model of the vehicle is described by a set of six nonlinear differential equations in (Yuh 1990). All the forces including Coriolis, drag, currents, gravity and buoyancy forces are considered as external forces in this technique. Goheen's article describes techniques which can be used to derive the underwater vehicle dynamics. (Fossen and Fjellstad 1995) have developed a unified framework for vectorial parameterization of inertia, Coriolis, centrifugal and hydrodynamic added mass force for a marine vehicle with six degrees-of-freedom. All the hydrodynamic forces are considered in the model. The dynamics are derived using both Newtonian and the Lagrangian method. It has been proved that the nonlinear equation in vectorial form satisfies certain matrix properties like symmetry, skew-symmetry and positive definiteness. Some empirical formulation is presented to calculate the added mass co-efficient and the drag forces for a body with cylindrical geometry . A few assumptions like the fluid is irrotational and unbounded, added mass and drag forces are constant have been made while deriving the equations of motion. In this paper we consider a system consisting of an underwater rigid body moving in a horizontal plane (neutrally buoyant). The configuration space is $Q \triangleq I\!R^2 \times S^1$ and is parametrized by the co-ordinates (x, y, θ) represented in inertial frame. The triple (x, y, θ) represents the position of the center of mass and orientation of the body in the inertial frame. The corresponding linear and angular velocities *in the body frame* are denoted by (v_x, v_y, ω_z). The inertial velocities and the body velocities are related by the equations

$$\begin{aligned}
\dot{x} &= v_x \cos\theta - v_y \sin\theta \\
\dot{y} &= v_x \sin\theta + v_y \cos\theta \\
\dot{\theta} &= \omega_z
\end{aligned}$$

The equations of motion are developed by (Petterson and Egeland 1996) and are given by

$$\begin{aligned}
m_{11}\dot{v}_x - m_{22}v_y\omega_z + d_{11}v_x &= F_x \\
m_{22}\dot{v}_y + m_{11}v_x\omega_z + d_{22}v_y &= F_y \\
m_{33}\dot{\omega}_z + (m_{22} - m_{11})v_x v_y + d_{33}\omega_z &= \tau_z
\end{aligned} \qquad (1)$$

where $m_{ii}, d_{ii}, i = 1, 2, 3,$ are positive constants that represent the elements of the inertia matrix including added masses and the elements of the damping matrix respectively. Typically the vehicles are actuated by only two control forces. In the following section we analyze the properties of the underwater vehicle with two actuators.

3. UNDERACTUATED CONFIGURATION

In this configuration, the control inputs are the yaw (τ_z) and forward thrust F_x. A schematic of UUV is illustrated in Figure 1. The unactuated dynamics constitutes a second-order nonholonomic constraint on the system. This renders the system unsuitable to apply full-state feedback linearization.

We define the state vector $X = (x_1, x_2, x_3, x_4, x_5, x_6)^T$ as (Petterson and Egeland 1996)

$$X \triangleq \begin{pmatrix} \theta \\ x\cos\theta + y\sin\theta \\ -x\sin\theta + y\cos\theta \\ v_y \\ \omega_z \\ v_x \end{pmatrix}$$

The ordering of the states is motivated by a desire to split the state-vector into actuated and unactuated subsystems. The control inputs after a partial feedback linearization are defined as

$$u_1 \triangleq (\tau_z - d_{33}\omega_z + (m_{11} - m_{22})v_x v_y)/m_{33}$$
$$u_2 \triangleq (F_x + m_{22}v_y\omega_z - d_{11}v_x)/m_{11}.$$

which yields the state-space representation of (1) with $F_y = 0$, defined on the manifold $\mathcal{M} \triangleq S^1 \times I\!R^5$ as

$$\dot{X} = f(X) + g_1(X)u_1 + g_2(X)u_2 \qquad (2)$$

where

$$f(X) = \begin{pmatrix} x_5 \\ x_6 + x_3x_5 \\ x_4 - x_2x_5 \\ -\alpha x_4 - \beta x_5 x_6 \\ 0 \\ 0 \end{pmatrix};$$

$$g_1(X) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}; g_2(X) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix};$$

and $\alpha = d_{22}/m_{22}, \beta = m_{11}/m_{22}$. The equilibrium configuration denoted by X^e is all of Q since the motion of the vehicle is in the horizontal plane. The absence of potential terms in the unactuated dynamics renders the linearized model of the nonlinear system uncontrollable. This system does not admit time-invariant continuous state-feedback control that can locally asymptotically stabilize the equilibrium (Brockett 1983). A much weaker notion of controllability called small-time local controllability (STLC) holds for this system (Reyhanoglu 1997). The existence of time-varying/periodic or discontinuous control that can asymptotically stabilize the equilibrium is guaranteed in view of the STLC property (Sussmann 1979). This configuration has been studied by (Petterson and Egeland 1996, Reyhanoglu 1997), a discontinuous control law has been presented (Reyhanoglu 1997) to achieve asymptotic stabilization to an equilibrium configuration with exponential convergence rates. The discontinuous nature of the control law is due to the rational transformation introduced for the states of the system. The control law has at most one switching. We propose a sliding mode control strategy to asymptotically stabilize the origin of the system. (2).

4. CONTROLLER DESIGN

With out loss of generality , the problem of stabilizing the system to a given equilibrium point X^e can be reduced to the problem of stabilizing the system (2) to the origin (Reyhanoglu 1997).

Proposition 1. The origin of (2) is AS (asymptotically stable) under the following control law

$$u_1 = \begin{cases} C & \{\|X\|_2 \neq 0 \text{ and } x_1 = x_5 = 0\} \forall t \in [0, T) \\ & -(x_1 + x_5)^{1/3} - x_5 \quad \text{elsewhere} \end{cases}$$

$$u_2 = \begin{cases} \mathcal{H}(X) & X \in \mathcal{K} \\ 0 & \text{elsewhere} \end{cases}$$

where
T - a finite time.
C - any nonzero constant.
$\mathcal{H}(X) \triangleq \frac{\eta(Z_1,Z_2)+u_1(\alpha x_2+\beta x_6)}{-\beta x_5} - \frac{\alpha}{\beta}(x_6 + x_3x_5)$
$\eta(Z_1, Z_2) \triangleq -\text{sign}(Z_1)|Z_1|^{1/3} - \text{sign}(Z_2)|Z_2|^{1/3}$
$Z_1(X) \triangleq (\alpha x_3 + x_4)$
$Z_2(X) \triangleq -x_5(\alpha x_2 + \beta x_6)$
$\mathcal{A}(X) \triangleq (x_1 + x_5)$
$\mathcal{K} \triangleq \{X \in \mathcal{M} | x_5 < 0 \;\; and \;\; \mathcal{A}(X) \geq 0 \;\; or \;\; x_5 > 0 \;\; and \;\; \mathcal{A}(X) \leq 0\}$

Proof: The proof is split into three lemmas. Let us define the following surfaces

$$\mathcal{A}(X) \triangleq (x_1 + x_5) = 0$$
$$\mathcal{B}(X) \triangleq (\alpha x_3 + x_4) = 0$$
$$\mathcal{C}(X) \triangleq (kx_2 + x_6) = 0$$

where $k \triangleq \frac{\alpha}{\beta}$.
The intersection of the surfaces can be written as the set

$$\mathcal{O} = \{X \in \mathcal{M} | x_5 = -x_1, x_4 = -\alpha x_3, x_6 = -kx_2\}$$

Lemma 2. There exists a finite-time $T_1 \geq 0$ such that all the trajectories starting from any arbi-

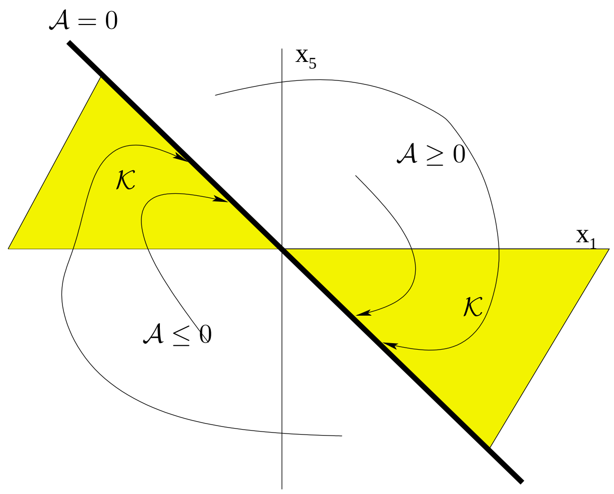

Fig. 2. Phase plot between x_1 and x_5

trary point of $\mathcal{M}$ will enter $\mathcal{K}$ and stay there for all time $t \geq T_1$.

Proof: Consider the following equations

$$\dot{x}_1 = x_5$$
$$\dot{x}_5 = u_1$$

Under the control law u_1 given by Proposition (1), the dynamics of $\mathcal{A}(X)$ becomes

$$\dot{\mathcal{A}}(X) = -[\mathcal{A}(X)]^{1/3}$$

The surface $\mathcal{A}(X) = 0$ is finite-time stable (Haimo 1986). So there exists a time $T_1 \geq 0$ such that the trajectories will enter $\mathcal{K}$ and stay there. The phase plot between x_1 and x_5 alone shown in fig (2). $\square$

Lemma 3. There exists a time $T_2 \geq T_1 \geq 0$ such that trajectories starting from $\mathcal{K}$ will enter the set $\mathcal{O}$ and stay there for all time $t \geq T_2$.

Proof: The control law u_2 is non-zero in $\mathcal{K}$ from Proposition (1). Our objective now is to reach the intersection of the surfaces $\mathcal{B}(X) = 0$ and $\mathcal{C}(X) = 0$. Note that $Z_1(X) = \mathcal{B}(X)$ and $Z_2(X) = -\beta x_5 \mathcal{C}(X)$. Let us consider the dynamics of Z_1 and Z_2 , which under the control law u_2, becomes

$$\dot{Z}_1 = Z_2$$
$$\dot{Z}_2 = -\text{sign}(Z_1)|Z_1|^{1/3} - \text{sign}(Z_2)|Z_2|^{1/3}$$

The variables Z_1 and Z_2 go to zero in finite-time (Haimo 1986). Since x_5 goes to zero asymptotically, this implies that the control law u_2 as result becomes

$$u_2 = -k(x_6 + x_3 x_5)$$

Plugging this into the dynamics of $\dot{Z}_2$ yields $\dot{Z}_2 = 0$, which implies $Z_2 \equiv 0 \Rightarrow Z_1 \equiv 0$. $Z_1 = 0$ implies $\mathcal{B}(X) = 0$. Since x_5 goes asymptotically to zero in the set $\mathcal{K}$, $Z_2 = 0$ implies $\mathcal{C}(X) = 0$. Recall that under the control law u_1, the surface $\mathcal{A}(X) = 0$ is finite-time stable. So there exists a time $T_2 \geq T_1 \geq 0$ such that the trajectories reach the set $\mathcal{O}$ and stay there for $t \geq T_2$. $\square$.

Lemma 4. The largest positively invariant set in $\mathcal{O}$ is the origin.

In set $\mathcal{O}$ the closed-loop system dynamics becomes

$$\begin{aligned}
\dot{x}_1 &= -x_1 \\
\dot{x}_2 &= -kx_2 - x_3 x_1 \\
\dot{x}_3 &= -\alpha x_3 + x_2 x_1 \\
\dot{x}_4 &= -\alpha x_4 - \alpha x_1 x_2 \\
\dot{x}_5 &= -x_5 \\
\dot{x}_6 &= -kx_6 + kx_3 x_1
\end{aligned}$$

consider a Lyapunov candidate function

$$V(X) = \frac{x_1^2}{2} + \frac{x_2^2}{2} + \frac{x_3^2}{2} + \frac{x_4^2}{2} + \frac{x_5^2}{2} + \beta^2 \frac{x_6^2}{2}$$

Take $\dot{V}$ along the closed loop solution

$$\begin{aligned}
\dot{V} &= -x_1^2 - kx_2^2 - x_1x_2x_3 - \alpha x_3^2 + x_1x_2x_3 \\
&- \alpha x_4^2 - \alpha x_1 x_2 x_4 - x_5^2 - k\beta^2 x_6^2 + k\beta^2 x_1 x_3 x_6 \\
&= -x_1^2 - kx_2^2 - \alpha x_3^2 - \alpha x_4^2 \\
&- \alpha x_1 x_2(-\alpha x_3) - x_5^2 - k\beta^2 x_6^2 + k\beta^2 x_1 x_3(-kx_2) \\
&= -x_1^2 - kx_2^2 - \alpha x_3^2 - \alpha x_4^2 - x_5^2 - k\beta^2 x_6^2 \\
&\leq 0
\end{aligned}$$

and $\dot{V} = 0$ only when $X = 0$. $\square$.

Now we verbally summarize the control strategy. Starting from any initial condition in $\mathcal{M}$, the system reach in $\mathcal{K}$ in some finite time T_1 under the control law u_1. Once in $\mathcal{K}$, control law u_2 will drive the states to $\mathcal{O}$. In set $\mathcal{O}$, by LaSalle's theorem, the largest positively invariant set is the origin. The closed-loop trajectory is shown in figure (3)

5. SIMULATION

We simulate this control law for the following model parameters.

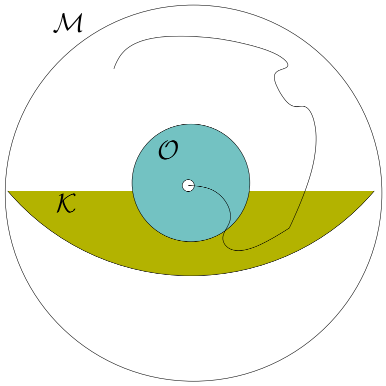

Fig. 3. Convergence to the origin

$$m_{11} = 100kg, \quad m_{22} = 125kg, \quad m_{33} = 40kg.m^2$$
$$d_{11} = 35kgs^{-1},\ d_{22} = 100kgs^{-1},\ d_{33} = 50kgm^2s^{-1}$$

and the initial conditions are given by

$$\theta(0) = 2rad, \quad x(0) = 1m, \quad y(0) = 2.5m$$
$$x_4(0) = 1m/s,\ x_5(0) = 0.5r/s,\ x_6(0) = 1m/s$$

The simulation results are shown in figures (4-11).

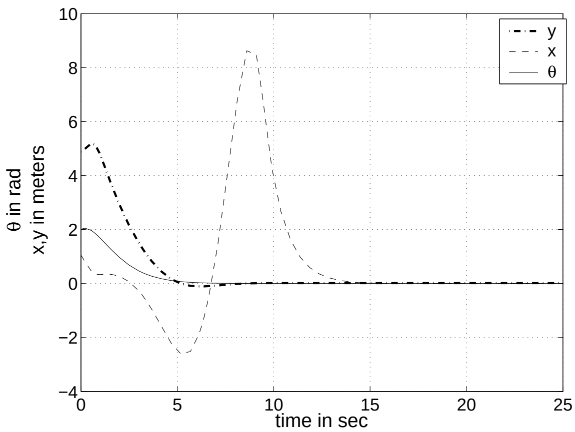

Fig. 4. Inertial coordinate of the vehicle

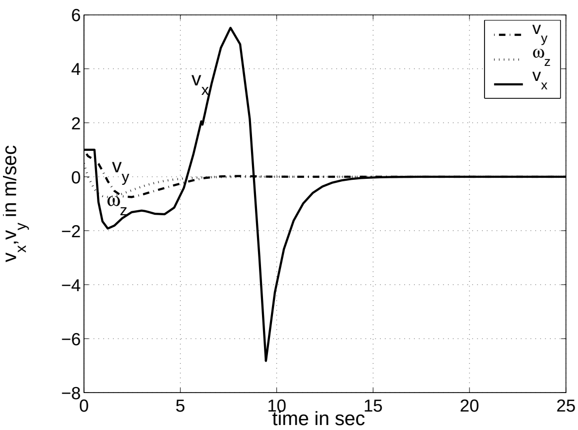

Fig. 5. Body Velocities

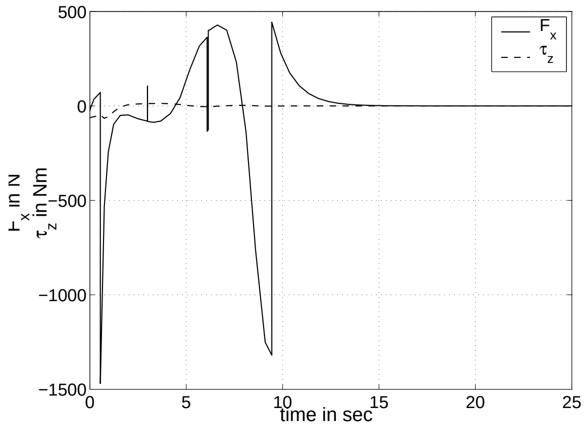

Fig. 6. External Force and Torque

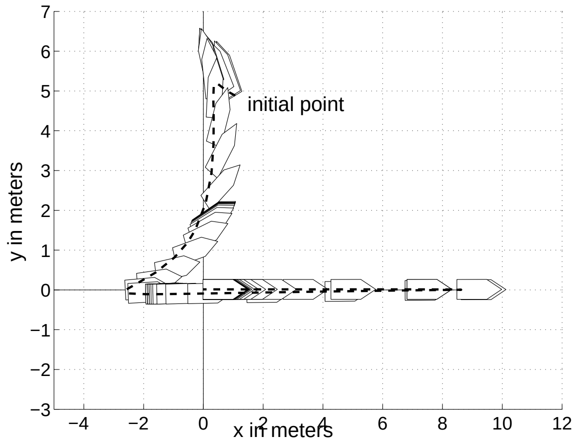

Fig. 7. Vehicle path

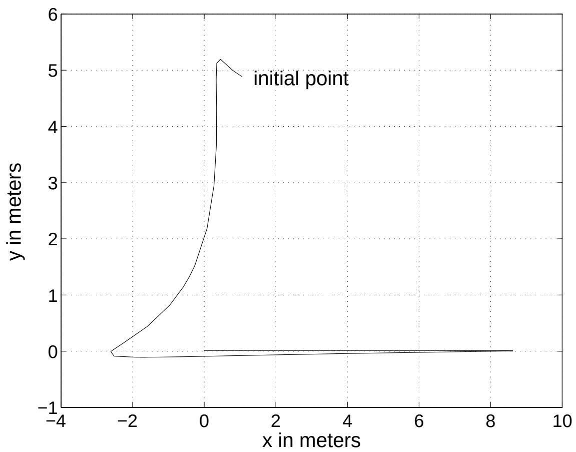

Fig. 8. Path traced by the vehicle

6. CONCLUSION

We present a control law for asymptotic stabilization of an underwater underactuated vehicle moving in a horizontal plane. The task of the two actuators is to bring the system in a finite time to a set $\mathcal{O}$ on which the system is asymptotically

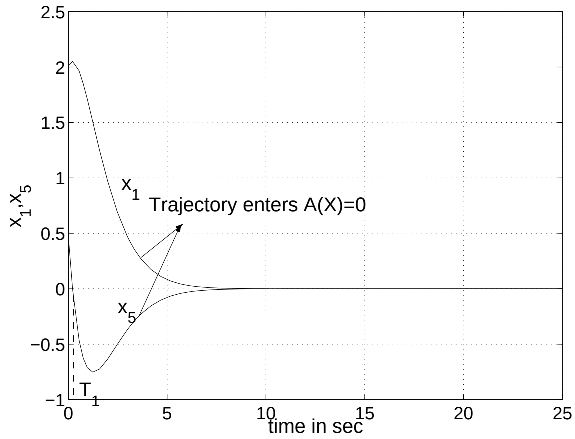

Fig. 9. Reaching the surface $\mathcal{A}(X) = 0$

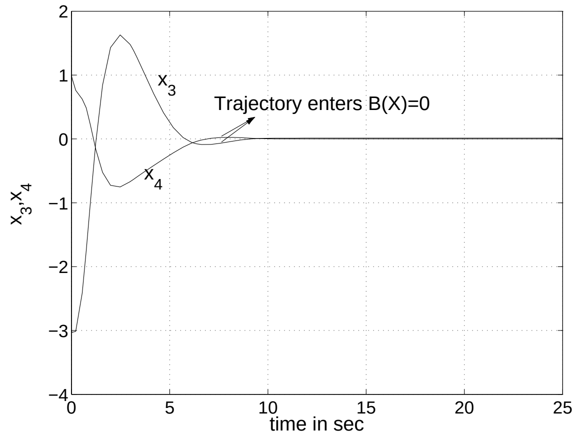

Fig. 10. Reaching the surface $\mathcal{B}(X) = 0$

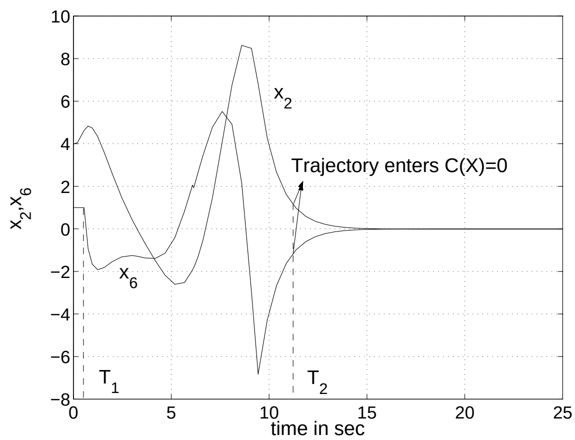

Fig. 11. Reaching the surface $\mathcal{C}(X) = 0$

stable. The set $\mathcal{O}$ is defined by the intersection of three sliding surfaces.

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