A NEW POSITION-BASED METHOD FOR GPS SIGNALS FDI

P. Castaldi[*], M. Crisci[§], M. Zanzi[*]

[*]*Department of Electronics, Computer Science and Systems, University of Bologna.*
Via Fontanelle 40, 47100 Forlì, Italy.
Ph. +39-0543-786934/ email: pcastaldi@deis.unibo. it
[§]*European Space Agency/ESTEC Noordwijk NL*

Abstract: Integrity is a critical requirement for aerospace navigation systems. In this paper integrity issues are coped with a new FDI technique implemented by a snapshot RAIM algorithm based on position domain tests. The approach consists of the joint exploitation of all the possible Least Squares (LS) solutions of the GPS linear estimation problem under a single fault on a pseudorange measurement. The loci described by the different LS position solutions by varying the fault size and the faulty satellite are investigated. The exact non linear dependence of the loci from the fault size is considered. No linear approximation is used. This results in a new criterion and algorithm for the faulty satellite isolation. The effectiveness of the proposed method in the isolation of faulty satellite has been compared with a classic FDI method and demonstrated by Montecarlo simulations based on real data.

Keywords: RAIM, GPS, Identification Scheme, Fault Detection and Isolation, LS.

1. INTRODUCTION

The use of GPS positioning in an aerospace navigation system depends on its capability in fulfilling the Required Navigation Performance (RNP), as stated by the International Civil Aviation Organization (ICAO), in terms of accuracy, continuity, integrity, availability. Integrity is the RNP parameter that has a direct impact on safety because it refers to the level of confidence that can be relied on the information of position (ICAO, 1995; RTCA/DO-236A, 2000). As it is well known, GPS has the advantage of an high accuracy positioning but suffers the drawback of high disturbance vulnerability (e.g. jamming, multipath) as well as sometimes inadequate time to alarm to manage possible failure situations. Integrity can be studied with methods developed for Fault Detection and Isolation (FDI). A large literature exists where integrity monitoring schemes and FDI algorithms for GPS are proposed and analyzed (Van Graas, 1996; Brenner, 1995; Lee, 1995). The purpose of FDI algorithms is to detect and isolate, eventually exclude, a possible fault with a prescribed probability of missed detection and without exceeding a maximum false alarm rate. FDI for GPS is essentially based on the availability of redundancy in the information used to compute the navigation solution. The redundancy in GPS positioning can result from considering a number of satellite signals greater than that strictly necessary and/or can be obtained from other sensors (e.g. inertial navigation systems). Hence, the FDI algorithms can be classified into two main groups: 1) snapshot algorithms, 2) history-based algorithms. The formers are generally based on least-square methods that process set of data sampled at the same time (Van Graas, 1996), while the latter make use of multiple Kalman filters fed with different innovations (Brenner, 1995). Moreover, the FDI algorithms can be classified between range-based and position-based methods, depending on the kind of test statistic they adopt for the detection function. Range-based methods work in the pseudorange domain, that is the measurement space: they are able to detect the occurrence of a fault in the signal-in-space according to the above mentioned probabilities. Range-based methods follow the procedures developed for parity-space or residuals FDI (Brown, 1988; Van Graas, 1996; Patton 2000) Alternatively, the position-based methods use the spread of the position solutions compatible with the available measurements in order to detect a possible failure situation. The most widely known position-based test statistic function is based on the so-called “Solution Separation Method” (Brenner, 1995; Young, 2002). While the equivalence between range and position-based methods has been demonstrated (Lee, 1995; Young 2002), nevertheless a subtle difference holds between them: a range-based algorithm is capable to detect a fault in GPS signals. A position-based algorithm can detect a failure in the navigation solution: this seems to better agree to the ICAO requirements in the sense that they are primarily concerned on the impact of a fault on the navigation solution (the navigation failure and its detection) rather than on the fault itself (and its detection).. The concept, the solution separation method is based on, is the definition of a threshold for the spread of the position sub-solutions computed by removing a satellite signal one at a time from the overall measurement set. In case of a faulty measurement, the culprit affects all the computed positions but one and then the position spread serves as a test statistic for detection. In this paper, a different perspective from the classical snapshot position-based FDI algorithm is proposed. The algorithm is based on the observation that LS method can be thought as an identification scheme suitable to find a set of parameters (i.e. the three position coordinates) that relates the pseudorange vector to the geometric satellite-user unit vectors by means of linear equations. As will be clear in the following, five different LS schemes for each hypothesis on the faulty satellite can be considered in order to find the linear relationship parameters. These five different LS schemes, in case of a faulty measurement, provide different sets of parameters going away one from the other with a non linear growing law depending on the magnitude of the fault and on the GPS linearization point. Moreover, in the noiseless case, only if the hypothesis on the faulty satellite is correct, the corresponding set of five loci present a common point. A new algorithm is devised by assuming that this property approximately holds in the noisy case. Hence, the faulty satellite is chosen corresponding to the set of loci characterized by the minimal distance. The good results of the algorithm have been highlighted by comparisons with the classic FDI range-based method performed by means of Montecarlo simulations based on real GPS data.

2. THE GPS MEASUREMENT MODEL

The mathematical model of the GPS relates the measured user/receiver-to-satellite ranges, known as pseudoranges, to the user position coordinates. It is described by a set of non linear equations that can be easily linearized around a reference point (Parkinson 1988). The influence of various type of disturbances and the presence of possible faulty measurements can be easily taken into account by the following algebraic, linear model:

$$\boldsymbol{\rho} = \mathbf{H}\mathbf{p} + \tilde{\boldsymbol{\rho}} + \boldsymbol{\rho}_f \qquad (1)$$

Where $\mathbf{p}$ *is* 4-dimension vector made of the three position coordinates respect to the reference position and the receiver clock bias; $\boldsymbol{\rho}$ is a *N*-dimension vector of the measured pseudorange for each in view GPS satellite; $\mathbf{H}$ is a *N*x4 matrix whose rows are the cosine directors between each GPS satellite and the user receiver together with a 1 in the fourth position for the clock bias, that is

$$\mathbf{H} = \begin{bmatrix} h_1^x & h_1^y & h_1^z & 1 \\ \vdots & \vdots & \vdots & \vdots \\ h_N^x & h_N^y & h_N^z & 1 \end{bmatrix};$$

$\tilde{\boldsymbol{\rho}}$ is a *N*-dimension vector of the disturbances (noise, atmospheric and multipath errors) affecting the measurement; $\boldsymbol{\rho}_f$ is a *N*-dimension vector of possible faults affecting the measurements.

1.1 Hypothesis on fault $\boldsymbol{\rho}_f$ and matrix $\mathbf{H}$

In this paper an only-one per time satellite measurement fault hypothesis is assumed, in fact multiple GPS signal faults are extremely improbable.

Assumption 1 The fault can be modeled as an error that adds up to the fault-free pseudorange measurement related to a certain satellite, that is $\boldsymbol{\rho}_f = \bar{\alpha}\mathbf{e}_k$ where $\bar{\alpha}$ is a real scalar representing the amplitude of the fault; *k* is an index of the faulty signal. *k* ranges from 1 to *N;* $\mathbf{e}_k$ is a *N*-dimension vector with a 1 in the k^{th} position and 0 otherwise

Of course, both *k* and $\bar{\alpha}$ have to be estimated by the FDI process. Moreover when dealing with real data, matrix $\mathbf{H}$ has usually full rank and in order to have isolation of the faulty satellite $N > 5$, so that the following hypothesis is introduced.

Assumption 2 Matrix $\mathbf{H}$ has a number of rows $N > 5$ and satisfies $rank(\mathbf{H}) = 4$

In order to avoid, as it will be seen in the following, rather pathological cases corresponding to multiple solutions, a further assumption it is introduced.

Assumption 3 Matrix $\mathbf{H}$ and vector $\mathbf{p}$ fulfill the relation $\mathbf{H}\mathbf{p} \neq \lambda(\bar{\alpha}\mathbf{e}_k - \alpha\mathbf{e}_i), \forall \lambda, \alpha, i :$ $\lambda(\bar{\alpha}\mathbf{e}_k - \alpha\mathbf{e}_i) \neq 0_{N\times 1}$

1.2 Classic residual-based FDI process.

In order to have an estimation of the fault by way of classical FDI method, the vector $\alpha\mathbf{e}_i$, representing an arbitrary assumed fault on satellite i^{th}, can be subtracted by both the members of equation (1) leading, by *Assumption* 1, to

$$\boldsymbol{\rho} - \alpha\mathbf{e}_i = \mathbf{H}\mathbf{p} + \tilde{\boldsymbol{\rho}} + \bar{\alpha}\mathbf{e}_k - \alpha\mathbf{e}_i \qquad (2)$$

By defining $\boldsymbol{\rho}_c^i(\alpha) := \boldsymbol{\rho} - \alpha\mathbf{e}_i$, with the further assumption that $\tilde{\boldsymbol{\rho}}$ is a vector white noise characterized, without loss of generality, by a covariance matrix $\boldsymbol{\Sigma}_\rho = \sigma_\rho^2\mathbf{I}_N$, a statistically unbiased estimation $\mathbf{p}_i^{LS}(\alpha)$ of $\mathbf{p}$ can be obtained by the least square solution of equation (2), i.e.:

$$\mathbf{p}_i^{LS}(\alpha) = \mathbf{H}^+\boldsymbol{\rho}_c^i = \mathbf{p} + \mathbf{H}^+\tilde{\boldsymbol{\rho}} + \mathbf{H}^+(\bar{\alpha}\mathbf{e}_k - \alpha\mathbf{e}_i) \qquad (3)$$

where $^+$ denotes the pseudo-inverse matrix operator. Note that (3) is the LS solution corresponding to the hypothesis that a fault of size α is present on the i^{th}

satellite. The residual vector $\mathbf{res}_i^{LS}(\alpha)$ associated to (3) is

$$\begin{aligned}\mathbf{res}_i^{LS}(\alpha) &= \boldsymbol{\rho}_c^i - \mathbf{H}\mathbf{p}_i^{LS} = \\ &= (\mathbf{I} - \mathbf{H}\mathbf{H}^{+})(\tilde{\boldsymbol{\rho}} + \bar{\alpha}\mathbf{e}_k - \alpha\mathbf{e}_i)\end{aligned} \tag{4}$$

Equation (4) shows that if the assumed faulty satellite coincides with the true faulty one, i.e. $i = k$, and α equates the amount of true fault $\bar{\alpha}$, in case of $\tilde{\boldsymbol{\rho}} = 0$, the residue $\left\|\mathbf{res}_i^{LS}(\alpha)\right\|^2$ is equal to zero. Moreover, in the noisy case, $\tilde{\boldsymbol{\rho}} \neq 0$, the residues have a non central chi-square pdf with (N-4) degrees of freedom. Their mean value is given by

$$\begin{aligned}&\mathrm{E}\left\{\left\|\mathbf{res}_i^{LS}(\alpha)\right\|^2\right\} = \\ &= (N-4)\sigma_\rho^2 + \left\|(\mathbf{I} - \mathbf{H}\mathbf{H}^{+})(\bar{\alpha}\mathbf{e}_k - \alpha\mathbf{e}_i)^T\right\|^2\end{aligned} \tag{5}$$

where E is the expected value operator. It is worth observing that the expected value of the residue reaches its minimum when $i = k$ and $\alpha = \bar{\alpha}$. For these reasons, the pair i and α minimizing the actual residue is the one candidate to identify the faulty measurement and gives also an estimate of the size α and of the position **p**. Equation (2) can be rewritten in a manner that will be useful in the subsequent paragraph $\bar{\mathbf{H}}\begin{bmatrix}\mathbf{p} & 1\end{bmatrix}^T = -\tilde{\boldsymbol{\rho}} - \bar{\alpha}\mathbf{e}_k + \alpha\mathbf{e}_i$. The structure of $\bar{\mathbf{H}}$ is as follows

$$\begin{aligned}\bar{\mathbf{H}} &= \begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_2 & \mathbf{H}_3 & \mathbf{H}_4 & \mathbf{H}_5\end{bmatrix} = \\ &= \begin{bmatrix} h_1^x & h_1^y & h_1^z & 1 & -\rho_{c1}^i(\alpha) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_N^x & h_N^y & h_N^z & 1 & -\rho_{cN}^i(\alpha)\end{bmatrix} = \\ &= \begin{bmatrix} h_1^x & h_1^y & h_1^z & h_1^b & h_1^\rho \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_N^x & h_N^y & h_N^z & h_N^b & h_N^\rho\end{bmatrix}\end{aligned} \tag{6}$$

3. EIV ESTIMATION FOR THE CASE WITHOUT FAULT

With reference to (6), the case with $\bar{\alpha} = \alpha = 0$ is considered, that is only the noise affects the data and no fault is present. For this case, the EIV framework is defined by the following assumption on the noises affecting the columns of $\bar{\mathbf{H}}$.

Assumption 4 $h_i^{\S} = \hat{h}_i^{\S} + \tilde{h}_i^{\S}, \quad (i = 1,\dots,N)$. Where § can be either x, y, z, b, ρ

Assumption 5 The noises $\tilde{h}_i^{\S}$ have zero expected value and are mutually uncorrelated, $\forall i$, § and uncorrelated from noiseless data $\hat{h}_j^{\aleph}$, $\forall j, \aleph$.

With the previous assumptions our data model is often called in statistics literature as *Errors-in Variables* (EIV) model. In this way, all the variables are treated symmetrically.

Remark 1 The errors $\tilde{h}_i^{\S}$ reflect the influence which may come from stochastic disturbances, errors of measurement, nonlinear effects and more generally from anything that is not analyzed or analyzable.

Remark 2 According to model (7), $\tilde{h}_i^{\S} = 0$ with § equal to x, y, z, b, while $\tilde{h}_i^{\S}$ with § equal to $\boldsymbol{\rho}$ is the pseudorange noise. It follows that it seems useless to suppose that the whole set of columns of $\bar{\mathbf{H}}$ is affected by errors. The reason is that in the EIV framework it is possible to compute five LS solutions by projecting a column of $\bar{\mathbf{H}}$ on the iperplane spanned by the remaining columns. In the following it will be shown how to use these further solutions in order to isolate the faulty satellite.

If *Assumption* 4 and *Assumption* 5 hold and no fault is present on pseudoranges, matrix $\bar{\mathbf{H}}$ can be decomposed as follows $\bar{\mathbf{H}} = \hat{\bar{\mathbf{H}}} + \tilde{\bar{\mathbf{H}}}$, where $\hat{\bar{\mathbf{H}}}$, $\tilde{\bar{\mathbf{H}}}$ are the noiseless and the noise matrices defined analogously to $\bar{\mathbf{H}}$. The noisy data do not fulfill a linear relation and a solution, can be found only by an identification scheme. Kalman has shown that in the EIV framework previously described, it exists a whole family of solutions corresponding to a simplex in the space of the solutions. The vertices of this simplex are the 5 different LS solutions $\mathbf{x}^{LSn}$ $(n = 1,\dots,5)$, given by the following relations by assuming $\bar{\alpha} = \alpha = 0$ in $\mathbf{H}_5$

$$\mathbf{X}_{jk\ell m} = \begin{bmatrix}\mathbf{H}_j & \mathbf{H}_k & \mathbf{H}_\ell & \mathbf{H}_m\end{bmatrix} \tag{7}$$

$$\mathbf{P}_{jk\ell m} = \mathbf{X}_{jk\ell m}\left(\mathbf{X}_{jk\ell m}{}^T\mathbf{X}_{jk\ell m}\right)^{-1}\mathbf{X}_{jk\ell m}{}^T \tag{8}$$

$$\mathbf{z}^{LSn} = \ker\left(\begin{bmatrix}\mathbf{X}_{jk\ell m} & \mathbf{P}_{jklm}\mathbf{H}_n\end{bmatrix}\right) \tag{9}$$

where $\mathbf{H}_n$ denotes the n^{th} column of $\bar{\mathbf{H}}$, $(n = 1,\dots,5)$, while j, k, ℓ m are the remaining vectors. $\mathbf{x}^{LSn}$ (the n^{th} LS solution obtained by projecting the n^{th} column on the remaining ones) can be obtained by normalizing $\mathbf{z}^{LSn}$ (note that the coefficient of $\mathbf{H}_5$ is 1). For example $\mathbf{x}^{LS5}$, is given $\mathbf{z}^{LS5} = \ker\left(\begin{bmatrix}\mathbf{X}_{1234} & \mathbf{P}_{1234}\mathbf{H}_5\end{bmatrix}\right)$, $\mathbf{x}^{LS5} = \mathbf{z}^{LS5} / \mathbf{z}^{LS5}\big|_5$. Where $\mathbf{z}^{LS5}\big|_5$ is the 5^{th} element of $\mathbf{z}^{LS5}$. See (Guidorzi, 1992; Kalman, 1982) for further details. The other infinite solutions belonging to the simplex are called the Frisch solutions, and constitute the entire set of solutions fulfilling Assumptions 1 and 2. Among them, a well known solution is the Total Least Squares one. So the LS solutions are only a subset of a larger family of solutions corresponding to different assumptions on the errors in the variables. These assumptions have been called by Kalman *prejudices.*

Remark 3 The EIV framework is useful when all the variables can be affected by errors. This can be the case of the presence of ephemeris errors. In the GPS problem usually only $\mathbf{x}^{LS5}$ is computed because ephemeris errors are negligible with respect to pseudorange noise. Therefore, among Frisch solutions this is the only one which is unbiased. For this reason, the computation of other Frisch solutions could seem surprising or useless. The answer is that, under faulty conditions, as demonstrated in the next paragraph, the vertices of the simplex move highlighting the presence of a fault.

4. EIV FDI

In this paragraph a procedure for the isolation of the faulty satellite is described. This procedure can also be used to determine a position and a fault size estimate. In this work only the isolation capability of the algorithm is considered. In order to determine a consistent criteria to isolate the faulty satellite, the noiseless but faulty case is previously considered. Next the noisy case will be considered. This two step methodology to analyze EIV model is used analogously to the identification of dynamic model (Castaldi, 1996,1999).

1.3 Noiseless case

Consider now the faulty but noiseless case characterized by the presence of $\boldsymbol{\rho}_f = \bar{\alpha}\mathbf{e}_k$ but with $\tilde{\boldsymbol{\rho}} = 0$. For this case under the assumption previously stated, it is possible to define the following procedure in order to isolate the faulty satellite and to exactly determine the position and the size of the fault.

Remark 4 It is worth observing that by means of relation (6) and the presence of the fault, a parameter α, in general different from zero, is introduced. So that, with the exception of the classical least square LS5, $\mathbf{X}_{jk\ell m} = \mathbf{X}_{jk\ell m}(\alpha)$, $\mathbf{P}_{jk\ell m} = \mathbf{P}_{jk\ell m}(\alpha)$ and $\mathbf{H}_5 = \mathbf{H}_5(\alpha)$. In order to shorten notation this dependence will be omitted in the following *Procedure 1* and *Theorem 1* .

Procedure 1

1. For a given value of α, if $rank(\mathbf{X}_{jk\ell m}) = 4$, compute the n^{th} LS solution with the hypothesis of fault of size α on the i^{th} satellite by mean of the following relation

$$\mathbf{z}_i^{LSn}(\alpha) = \ker\left(\begin{bmatrix}\mathbf{X}_{jk\ell m} & \mathbf{P}_{jklm}\mathbf{H}_n\end{bmatrix}\right) \quad (10)$$

$$\mathbf{x}_i^{LSn}(\alpha) = \mathbf{z}_i^{LSn}(\alpha)\cdot\left(\frac{1}{\mathbf{z}_i^{LSn}(\alpha)\big|_5}\right) \quad (11)$$

2. If $rank(\mathbf{X}_{jk\ell m}) = 3$ compute the *n* scheme solution by augmenting vector $\ker(\mathbf{X}_{jk\ell m})$ with a zero in the n^{th} position

3. For every $n \neq s$, $(n, s = 1,\ldots,5)$, compute the distance $d_i^{LSnLSs}(\alpha)$ in the position space between the position estimates given by n^{th} and s^{th} LS schemes

$$d_i^{LSnLSs}(\alpha) = \left\|\mathbf{x}_i^{LSn}(\alpha) - \mathbf{x}_i^{LSs}(\alpha)\right\|_2 \quad (12)$$

4. Compute the total distance

$$d_i(\alpha) = \sum_{n\neq s} d_i^{LSnLSs}, (n, s = 1,\ldots,5) \quad (13)$$

5. Compute the satellite i^{th} and the fault size α corresponding to the minimal total distance, i.e. *i* and α satisfying

$$\min_{i,\alpha}(d_i(\alpha)) \quad (14)$$

The following *Theorem 1* shows that, when dealing with the noiseless case, the minimal total distance is equal to zero if and only if $i = k$ and $\alpha = \bar{\alpha}$, hence allowing the isolation of the faulty satellite.

Theorem 1 With reference to the noiseless case, i.e. $\tilde{\boldsymbol{\rho}} = 0$, the total distance $d_i(\alpha)$, defined in (13), is equal to zero if and only if $i = k$ and $\alpha = \bar{\alpha}$.

Proof . If: in the noiseless case with $i = k$ and $\alpha = \bar{\alpha}$, vectors in (6) are collinear and by means of (11), if $rank(\mathbf{X}_{jk\ell m}) = 4$, or by step 2 in Proc. 1, in case of $rank(\mathbf{X}_{jk\ell m}) = 3$, it is possible to compute the solution corresponding to each scheme. This solution coincides with the actual kernel of $\mathbf{H}$, $\begin{bmatrix}\mathbf{p} & 1\end{bmatrix}^T$, hence $\min_{i,\alpha}(d_i(\alpha)) = d_k(\bar{\alpha}) = 0$

Only if: it is straightforward to verify that if by assuming $i \neq k$ and/or $\alpha \neq \bar{\alpha}$ and by means of *Assumption* 3, it is $rank(\mathbf{X}_{jk\ell m}) = 4$ and $\min_{i,\alpha}(d_i(\alpha)) \neq 0$.

Remark 5 Note that by means of *Assumption* 2 *Assumption* 3, the minimal rank of $\mathbf{X}_{jk\ell m}$ is 3. This is the case in which one or more entries of $\mathbf{p}$ are exactly equal to zero and $i = k$ and $\alpha = \bar{\alpha}$. In this case, the solution corresponding to *n* scheme is given by augmenting vector $\ker(\mathbf{X}_{jk\ell m})$ with a zero in the n^{th} position.

Remark 6 Note that the distance defined in (12) can be restricted to the tree dimensional position domain by using the first three elements of every solution $\mathbf{x}^{LSn}(\alpha)$. The results given by *Theorem 1*, obviously, still hold.

The situation corresponding to the noiseless case with $i = k$, $\alpha = \bar{\alpha}$ and a vector $\mathbf{p}$ with every entries different from zero, is depicted in Figure 1. In this figure, in the position domain, the five loci, described by the first three elements of $\mathbf{x}^{LSn}(\alpha)\big|_{1,2,3}$, $(n = 1,\ldots,5)$ are reported. As demonstrated in (Castaldi, 2004) the loci $\mathbf{x}^{LSn}(\alpha)\big|_{1,2,3}$ $(n = 1,\ldots,4)$ are segment of iperbolas and $\mathbf{x}^{LS5}(\alpha)\big|_{1,2,3}$ is a straight line. By means of *Theorem 1*, they present a common point corresponding to the actual position $\mathbf{p}|_{1,2,3}$

Remark 7 In Figure 1, the straight line corresponds to the classical least square solution LS5 as α varies. If $i = k$ and $\alpha = \bar{\alpha}$ and $\tilde{\boldsymbol{\rho}} = 0$, if the linearization point is characterized by some components equal to zero, some iperbolas may degenerate in straight line. In fact, it can be easily shown that if the j^{th} element of $\mathbf{p}$ is equal to zero, then $\mathbf{x}^{LSj}(\alpha)\big|_{1,2,3}$ becomes a straight line.

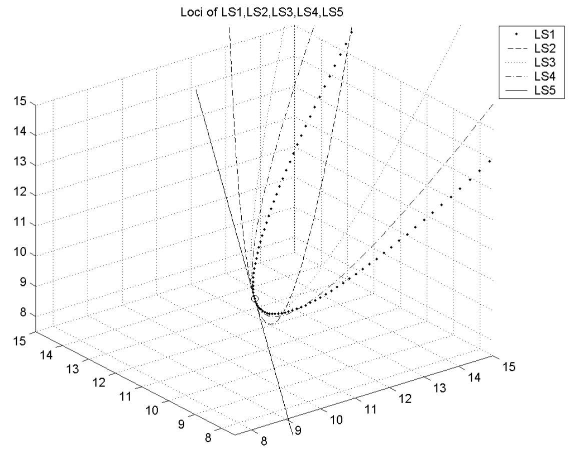

Figure 1. Noiseless case with $i = k$ and $\alpha = \bar{\alpha}$. Loci present a common point corresponding to actual position $\mathbf{p}|_{1,2,3}$ (denoted with a circle).

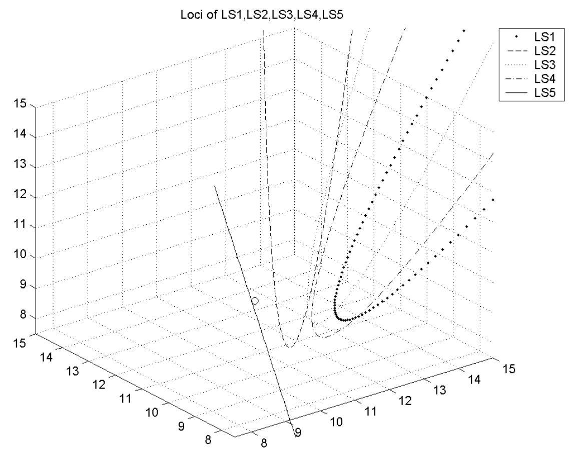

Figure 2. Noiseless case with $i \neq k$ and/or $\alpha \neq \bar{\alpha}$ The loci do not have common a point.

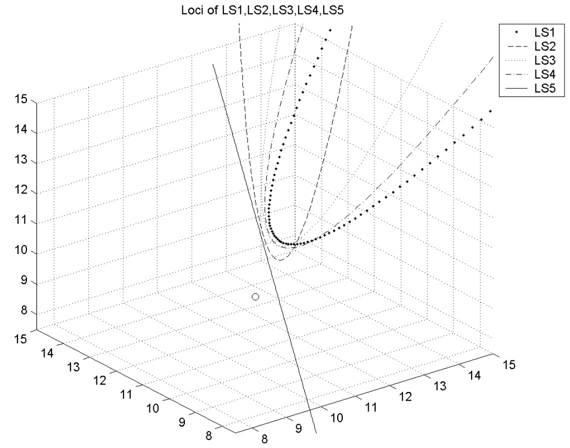

Figure 3. Noisy case with $i = k$ and $\alpha = \bar{\alpha}$. Loci approximately present a common point.

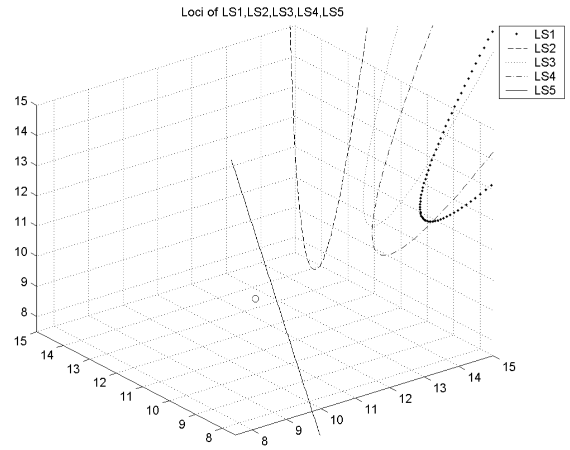

Figure 4. Noisy case with $i \neq k$ and/or $\alpha \neq \bar{\alpha}$. Loci present do not have a common point, even approximately.

5.2 Noisy case

For the noisy case, *Procedure 1* can still be used in order to isolate the faulty satellite (and to estimate the fault size and the actual position). In this case $\min_{i,\alpha}(d_i(\alpha)) \neq 0$ even by choosing $i = k$ and $\alpha = \bar{\alpha}$, due to the presence of $\tilde{\boldsymbol{\rho}} \neq 0$. The statistical properties of this consistent criterion will be investigated by montecarlo simulation in the next section. In particular, in this work, will be investigated the isolation properties of the procedure. Fig. 2,3,4 show the other possible cases.

Remark 8 Investigating real time capabilities of procedure 1 is beyond the aim of this methodological work and will be investigated in a further work. However, MATLAB tests performed in next section require a low computational burden: 0.093 sec CPU time/run on mobile P4 3.2GHz.

5. RESULTS

In this section, the results of some tests carried on in order to assess the performance of the described FDI algorithm are presented. The aim of the tests is to evaluate how the algorithm behaves in presence of different levels of noise and faults. The goal of the algorithm is to detect and isolate the faulty satellite from measurements corrupted by noise. As a consequence, the tests have been run by using both real and simulated data. In fact, real satellite ephemeredes have been collected by a ground fixed NOVATEL® GPS geo-referenced receiver sited near Forlì airport: these data, together with the GPS receiver coordinates, have been used in order to form the *H* observation matrix. Moreover, exact pseudorange values have been computed from the satellite-receiver distance and simulated noise with a known level of standard deviation has been added to them. In addition, simulated values of fault over a single pseudorange have been injected. The observation matrix *H* is obtained by means of a linearization around a point far 10 meters, in each direction, from the actual point where the receiver is. That is, the noiseless part of the measurements have been generated by multiplying *H* for the coordinate vector $\mathbf{p} = \begin{bmatrix} 10 & 10 & 10 & 10 \end{bmatrix}^T$. The fourth element of **p**, the clock bias, is irrelevant respect to the linearization process and adds the same way to all the measurements: therefore it does not affect the detection and exclusion process whichever value is chosen for it. The linearization point has to be a near, previously computed fault-free position. In this case the non-linearity errors are negligible and no fault affects the **H** matrix. The latter requirement is not stringent because the linearization point can be chosen simply as the last computed position that has successfully passed the previous FDI test. For each pair of fault/noise sigma level, 100 Montecarlo simulations have been run. The numbers of times the classic FDI algorithm as well as the proposed one have correctly detected and isolated the fault and the responsible satellite have been counted for each set of 100 simulations. As expected, the simulations show that the best results are achieved when the fault is greater then the noise level. In fact, in this cases, the FDI algorithm easily finds the culprit satellite because the fault stands out over the noise. Similarly, worst results come out when the fault is covered and confused below the noise level.

Moreover, as table 1 shows, the simulations indicate that the performances of the FDI proposed algorithm always surpass or equal to those of the classic FDI method.

Table 1 Successful FDI for classic/proposed methods over 100 trials in different fault and noise conditions

Fault [m]	Noise sigma levels [m]		
	2.5	5	10
2.5	22/34	16/31	14/23
5	34/56	22/36	16/26
10	82/99	34/58	22/29
20	100/100	82/93	34/44
30	100/100	100/100	58/63

In particular, the best improvements of the proposed FDI algorithm emerge when the fault value is smaller than the noise sigma and the detection is, therefore, harder (see upper-right cells of table 1).

6. CONCLUSIONS

In this paper, the enhanced level of information provided by the joint exploitation of all the LS solutions of the position estimation problem under faulty conditions has been used. This has allowed for the design of a robust new snapshot position-based FDI algorithm. The performances of the proposed algorithm have demonstrated better robustness property in isolating the faulty satellite respect to the classical FDI approach.

REFERENCES

Brenner M., (1995). Integrated GPS/Inertial Fault Detection Availability, *ION GPS-95*. Palm Springs, CA, USA

Brown R.G., (1988) A Baseline GPS RAIM Scheme and a Note on the Equivalence of Three RAIM Methods, *Navigation*, **Vol. 39, No. 3**, pg. 301-316.

Castaldi P. and U. Soverini (1996). Identification of dynamic errors-in-variables models. *Automatica*, **vol. 32, n. 4**, pp. 631—636.

Castaldi P., R. Diversi, R. Guidorzi, U. Soverini (1999). Identification of multivariable errors-in-variables models. *Proc. of 5th ECC*, Karlsruhe, Germany.

Castaldi P., M. Crisci, M. Zanzi. (2004) FDI Algorithms for GPS Signals: a Different Perspective of a Known Problem. *Proc. of 16th IFAC Symposium on Automatic Control in Aerospace.* S. Petesburg, Russia.

Guidorzi R., (1992). Certain models from uncertain data: the algebraic case, *Systems and Control Letters*, **Vol. 17, Issue 6**, pg. 415-424

ICAO, (1995). Report on the item 4 of the Special Communications \ Operations *15th Divisional Meeting on the application of the (RNP) concept to approach, landing and departure operations* - SP COM/OPS/95-WP/151, ICAO, Montreal.

Kalman R.E., (1982). Identification from Real Data,. *Current development in the Intereface : Economics Econometrics, Mathematics*, (M. Hazewinkel and H.G Rinnoykan (Ed.)), pg. 161-196, Dordrecht, Reidel.

Lee Y. (1995). New Techniques Relating Fault Detection and Exclusion Performance to GPS Primary Means Integrity Requirements, *ION GPS-95*, Palm Springs, CA.

RTCA, (2000). DO-236A: MASPS RNP for Area Navigation,.RTCA Inc., Washington.

Van Graas F., (1996). Signals Integrity, *AGARD Lecture Series 207*, pg. 7.1-7.12, AGARD, Neully-Sur-Seine, France.

Young S. Y. Ryan and McGraw A. Gary (2002): Fault Detection and Exclusion Using Normalized Solution Separation Methods, *ION GPS-2002*, Portland, OR, USA.