OUTPUT TRACKING FOR CHUA'S CIRCUIT IN PRESENCE OF DISTURBANCES

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Abstract: In this paper we address the problem of output tracking for a Chua's circuit. The reference waveform is generated by another Chua's circuit. This problem is of interest in chaos communications. The tracking problem can be formulated in the well–known frame of output regulation, allowing also the rejection of a modelled disturbance acting on the system.

Keywords: Chua's circuit, output regulation, chaos communication, chaotic behaviour, nonlinear control systems.

1. INTRODUCTION

In this paper we study the problem of reducing to zero the tracking error for a Chua's circuit (Matsumoto and Komuro, 1985), (Chua and Matsumoto, 1986), in presence of a deterministic disturbance due to a magnetic coupling through the circuit inductor. It is a practical problem whenever resonant circuits are used, as in today high integrated systems where a lot of signal sources work together in close proximity. In fact, from an applicative point of view, the Chua's circuit has assumed a big interest in the field of chaos communications (Dedieu and Ogorzalek, 1997), (Fradkov, 1997), (Suykens and Chua, 1997), (Boutat-Baddas and Tauleigne, 2004) where such circuits are used in modulation and demodulation schemes, along with other circuits capable of generating chaos (Cuomo and Strogatz, 1993). Consequently, any practical implementation would require to share the same available circuit area, resulting in close proximity operation and high probability of unwanted signal coupling among circuits. Due to the simplicity of the electrical network which realizes the Chua's circuit, this is the preferred network used to check the effectiveness of any control theory applied to chaos. In fact, the simplest form of Chua's circuit only requires an operational amplifier for the active circuit, plus some few passive components.

The problem of chaos synchronization plays an important role in chaos communication research area, as well as the chaos tracking problem we address in this paper, since it translates in a synchronization problem. We apply the regulator theory for nonlinear systems (Isidori, 1995) in order to synchronize two chaotic Chua's circuits, which is one of the fundamental request in chaos communication. Among the others, two simple methods are used in chaos communication: the "Masking Technique" and the "Parameter Modulation" (Frey, 1993), (Cuomo and Strogatz, 1993). In the "Masking Technique" the information, with a small amplitude, is added to the chaotic signal; the small amplitude is required to not alter the chaotic signal spectrum. This signal is then transmitted to the receiver. Here, a similar chaotic circuit is located, which synchronizes to the chaotic

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regime of the incoming signal. At this point, a difference is made between the synchronized receiver chaotic signal and the incoming signal, and in such a way the information is extracted from the received signal and available to the user. In the "Parameter Modulation" a parameter of the transmitter, for example a resistor, is modulated in binary mode, and the whole signal is transmitted to the receiver. Here a similar chaotic circuit has the same parameter used in the transmitter (the resistor) fixed to one of the two possible values: the two chaotic circuits then synchronize when the received signal comes with the same information of the parameter value (the resistor value) used in the receiver, and stay unsynchronized when such parameters have different values. This synchronous and asynchronous states are used to recover the digital information transmitted.

Various control techniques have been implemented for Chua's circuit. Among the others, we recall the adaptive–robust control via fuzzy approach (Chang, 2001) in order to face system uncertainties and external disturbances, the linear control (Puebla and Cervantes, 2003) for designing a simple tracking controller for Chua's circuit which takes into account plant uncertainties and external disturbances, digital control (Xu and Shieh, 1996) in order to apply an analog controller via a digital microprocessor, the sliding mode approach (Boutat-Baddas and Tauleigne, 2004) for secured data transmission based on chaotic synchronization and observability singularity.

In this work we consider a Chua's circuit to be controlled and a second Chua's circuit which generates the reference to be tracked. Moreover, we consider on the Chua's circuit to be controlled the presence of a deterministic disturbance due to a magnetic coupling through the circuit inductor. By solving an output regulation problem, we determine a control law, based only on the knowledge of the circuit's state, which allows the tracking of the desired reference output while rejecting the disturbance.

The paper is organized as follows. In Section 2 the mathematical model of the Chua's circuit is presented and the regulation problem is formulated. In Section 3 the output regulation problem for the Chua's circuit is solved. In Section 4 some simulation results are shown. Brief comments conclude the paper.

2. MATHEMATICAL MODEL OF CHUA'S CIRCUIT AND PROBLEM SETTING

The Chua's circuit is shown in Figure 1. It consists of a parallel resonant circuit L–C_2 coupled through a resistor R to a parallel of a capacitor C_1 and an active nonlinear element R_{nl}. Such a nonlinear element, often named Chua's diode, can be characterized by the cubic function $i(v_1) = \bar{c}_1 v_1 + \bar{c}_3 v_1^3$ (Zhong, 1994), where v_1 is the voltage at the ends of the C_1 capacitor and i the current passing inside the nonlinear element, and $\bar{c}_1 < 0$, $\bar{c}_3 > 0$ are constants. Alternatively, one could use for the nonlinear element a piece–wise linear function. Here we considered a third degree polynomial since it is more realistic, because it corresponds to an element which is eventually passive. So, this function does not introduce in the model unrealistic behaviors due to the presence of initial states from which the system trajectories would tend to infinity.

The driven Chua's circuit is analytically represented by the following system

$$\begin{aligned} \dot{x}_1 &= \mu_1\Big(-(1+c_1)x_1 + x_2 - c_3 x_1^3\Big) - \mu_2 u \\ \dot{x}_2 &= \mu_3(x_1 - x_2) - \mu_4 x_3 \\ \dot{x}_3 &= \mu_5(x_2 - v_d) \\ e &= x_2 - v_r \end{aligned} \tag{1}$$

where $x_1 = v_1$, $x_2 = v_2$ is the voltage at the ends of C_2, $x_3 = i_L$ is the current in the inductance L, and where we have set

$$\bar{c}_1 = c_1\frac{\mu_1}{\mu_2}, \qquad \bar{c}_3 = c_3\frac{\mu_1}{\mu_2}.$$

Note the presence of the current generator u. In (1) e denotes the tracking error between v_2 and an appropriate reference voltage v_r. Finally, note the presence of a disturbance signal v_d acting on the inductance current dynamics. In (1) the parameters are defined as follows

$$\mu_1 = \frac{1}{RC_1}, \qquad \mu_2 = \frac{1}{C_1}, \qquad \mu_3 = \frac{1}{RC_2}$$
$$\mu_4 = \frac{1}{C_2}, \qquad \mu_5 = \frac{1}{L}.$$

The control problem is to track the prescribed signal v_r while rejecting the disturbance v_d. Both these signals are supposed to be modelled. More precisely, we want that the output x_2 of the controlled Chua's circuit tracks a signal v_r generated by another (autonomous) Chua's circuit, shown in Figure 2 and with the following dynamics

$$\begin{aligned} \dot{w}_1 &= \theta_1\Big(-(1+g_1)w_1 + w_2 - g_3 w_1^3\Big) \\ \dot{w}_2 &= \theta_3(w_1 - w_2) - \theta_4 w_3 \\ \dot{w}_3 &= \theta_5 w_2 \\ \dot{w}_4 &= \omega w_5 \\ \dot{w}_5 &= -\omega w_4 \\ v_r &= w_2 \\ v_d &= w_4 \end{aligned} \tag{2}$$

where $g_1 < 0, g_3 > 0$,

$$\theta_1 = \frac{1}{R_e C_{e1}}, \qquad \theta_2 = \frac{1}{C_{e1}}, \qquad \theta_3 = \frac{1}{R_e C_{e2}}$$

$$\theta_4 = \frac{1}{C_{e2}}, \qquad \theta_5 = \frac{1}{L_e}.$$

Note that (2) is nonlinear, autonomous, and can be put in the form

$$\dot{w} = s(w). \tag{3}$$

These dynamics generate the disturbance v_d as well. For this reason the exogenous input w is also called the extended disturbance, meaning that it contains the desired output function to be tracked and the undesired disturbance to be rejected. Both determine an error to be asymptotically zeroed. The disturbance v_d is supposed here simply sinusoidal and of fixed frequency. More complex cases than that considered with the exosystem (2) can be considered in the same way, at the expense of more complex dynamics.

An important characteristic that (2) has to meet, in view of the study of the steady state of the plant (1), is that the input which corresponds to this steady state and which is determined on the basis of the exogenous variable w must be persistent in time. In fact in this way we do not consider inputs decaying asymptotically to zero and we can speak of steady state response, depending on the specific system characteristics and not on its particular initial state. Another obvious requirement is that w be bounded. In order to have persistent inputs it is usual to require the property of Poisson stability. A point w_0 is Poisson stable if the solution $w(t)$ of (3) is defined for all time t and, for each neighborhood U_0 of w_0 and for each real $T > 0$, there exist $t_1 > T$ and $t_2 < -T$ such that $w(t_1), w(t_2) \in U_0$. This means that the solution $w(t)$ passes arbitrarily close to w_0 for arbitrarily large times in the future and in the past. Clearly the condition of Poisson stability required for the points of a neighborhood U_0 implies that any control law $u = \gamma(w)$, with $\gamma(0) = 0$, can not converge asymptotically to zero.

The formulation of the considered control problem perfectly fits the regulation theory (Isidori, 1995), (Byrnes and Isidori, 1997). In this setting (1) is the plant to be controlled, and (2) is the so–called exosystem which generates the references and the disturbances acting on the plant.

The formulation of the output regulation problem is the following (Isidori, 1995).

Output Regulation Problem. Given a nonlinear system $\dot{x} = f(x, w, u)$ with output $e = h(x, w)$, and a Poisson stable exosystem $\dot{w} = s(w)$ with bounded trajectories, find a mapping $\alpha(x, w)$ satisfying

(S) the origin of $\dot{x} = f(x, 0, \alpha(x, 0))$ is asymptotically stable in the first approximation;

(R) the solution $(x(t), w(t))$ of

$$\dot{x} = f(x, w, \alpha(x, w))$$
$$\dot{w} = s(w)$$

for any initial condition (x_0, w_0) in a neighborhood U of $(x, w) = (0, 0)$, is such that $\lim_{t\to\infty} h(x(t), w(t)) = 0$.

In the next Section we will solve the Regulation Problem for the Chua's circuit.

3. REGULATION OF THE CHUA'S CIRCUIT

The following result gives sufficient conditions for the existence of a solution to the regulation problem (Isidori, 1995), (Byrnes and Isidori, 1997).

Proposition 1. Let

$$A = \left.\frac{\partial f}{\partial x}\right|_{(0,0,0)} \qquad B = \left.\frac{\partial f}{\partial u}\right|_{(0,0,0)}.$$

The output regulation problem is solvable if and only if the pair (A, B) is stabilizable and there exist mappings $x_{ss} = \pi(w)$ and $u_{ss} = \gamma(w)$, with $\pi(0) = 0$ and $\gamma(0) = 0$, defined in a neighborhood W_0 of the origin, satisfying the so–called *regulator equations*

$$\begin{aligned} \frac{\partial \pi(w)}{\partial w} s(w) &= f\Big(\pi(w), w, \gamma(w)\Big) \\ 0 &= h\Big(\pi(w), w\Big) \end{aligned} \tag{4}$$

for all $w \in W_0$.

The mapping $x_{ss} = \pi(w)$ represents the steady state zero output submanifold and $u_{ss} = \gamma(w)$ is the steady state input which makes invariant the steady state zero output submanifold.

First, it is easy to check the controllability of the pair

$$A = \begin{pmatrix} -\mu_1(1+c_1) & \mu_1 & 0 \\ \mu_3 & -\mu_3 & -\mu_4 \\ 0 & \mu_5 & 0 \end{pmatrix}, \; B = \begin{pmatrix} -\mu_2 \\ 0 \\ 0 \end{pmatrix}.$$

Then, we determine the solution of the regulator equations, which in our case are written as follows

$$\frac{\partial \pi_1(w)}{\partial w} s(w) = \mu_1\Big(-(1+c_1)\pi_1(w) + \pi_2(w) - c_3\pi_1^3(w)\Big) - \mu_2\gamma(w)$$

$$\frac{\partial \pi_2(w)}{\partial w} s(w) = \mu_3\Big(\pi_1(w) - \pi_2(w)\Big) - \mu_4\pi_3(w)$$

$$\frac{\partial \pi_3(w)}{\partial w} s(w) = \mu_5\Big(\pi_2(w) - d(w)\Big)$$

$$0 = \pi_2(w) - r(w)$$

with $r(w) = w_2$, $d(w) = w_4$, and

$$s(w) = \begin{pmatrix} \theta_1\Big(-(1+g_1)w_1 + w_2 - g_3 w_1^3\Big) \\ \theta_3(w_1 - w_2) - \theta_4 w_3 \\ \theta_5 w_2 \\ \omega w_5 \\ -\omega w_4 \end{pmatrix}.$$

First, one considers that the tracking error is zero for

$$\pi_2(w) = w_2.$$

Then, from the third equation one determines

$$\pi_3(w) = \frac{\mu_5}{\theta_5} w_3 + \frac{\mu_5}{\omega} w_5.$$

Moreover, from the second equation one works out

$$\pi_1(w) = \frac{\theta_3}{\mu_3} w_1 + \left(1 - \frac{\theta_3}{\mu_3}\right) w_2 - \frac{\theta_4}{\mu_3}\left(1 - \frac{\mu_4}{\theta_4}\frac{\mu_5}{\theta_5}\right) w_3 + \frac{\mu_4\mu_5}{\omega\mu_3} w_5.$$

Therefore, the steady state zero output manifold $\pi(w) = \Big(\pi_1(w) \quad \pi_2(w) \quad \pi_3(w)\Big)^T$ remains determined.

Finally, from the first equation one gets the steady state input which makes $\pi(w)$ invariant

$$\gamma(w) = a_1 w_1 + a_2 w_2 + a_3 w_3 + a_4 w_4 + a_5 w_5 + b_1 w_1^3 - \mu_3 c_3 \pi_1^3(w)$$

where

$$a_1 = \frac{\theta_3}{\mu_3}(\theta_3 - \mu_3 - \mu_1 + \theta_1 - \mu_2 c_1 + \theta_2 g_1)$$

$$a_2 = \frac{-\theta_3^2 + (\mu_2 c_1 + \mu_3 + \mu_1 - \theta_1)\theta_3}{\mu_3} + \frac{-\mu_4\mu_5 + \theta_4\theta_5 - \mu_2 c_1 \mu_3}{\mu_3}$$

$$a_3 = \frac{(\mu_3 - \theta_3)\theta_5\theta_4 - (\mu_2 c_1 + \mu_1)(\mu_4\mu_5 + \theta_5\theta_4)}{\mu_3\theta_5}$$

$$a_4 = \frac{\mu_4\mu_5}{\mu_3}$$

$$a_5 = -\mu_4\mu_5 \frac{\mu_1 + \mu_2 c_1}{\omega\mu_3}$$

$$b_1 = \frac{\theta_3 g_3 \theta_2}{\mu_3}.$$

Finally, the control law which solves the output tracking problem is

$$u = \gamma(w) + K\Big(x - \pi(w)\Big) \qquad (5)$$

which contains a term proportional to $x - \pi(w)$. When this term vanishes since the plant trajectories approach to the steady state manifold, the control tends to the steady state input $\gamma(w)$.

4. A SIMPLE OBSERVER FOR THE DISTURBANCE

The control law (5) needs the knowledge of the components w_4, w_5 of the exosystem, corresponding to the disturbance $v_d = w_4$ and its derivative. In practical applications even the measurability of the disturbance is an unrealistic hypothesis. All the more reason this is valid for its derivative. In order to bypass this problem two possibilities can be considered: the first is to solve the so–called output regulation problem from the error, namely to build a dynamic controller based only on the signal e; the second is to construct an observer for the disturbance components w_4, w_5. The obstacle in the first possibility consists of solving the immersion of the control (5) with a nonlinear exosystem. This is still an open problem, in general cases, and in the present case it does not seem easily solvable.

Therefore, we built a simple reduced–order observer considering the following system, deduced from (1), (2)

$$\begin{aligned} \dot{x}_3 &= -\mu_5 w_4 + \mu_5 x_2 \\ \dot{w}_4 &= \omega w_5 \\ \dot{w}_5 &= -\omega w_4 \\ z &= x_3 \end{aligned}$$

where $z = x_3$ is seen as the measured output and x_2 the input to the output's dynamics.

Considering as desired eigenvalues to assign to the reduced–order observer the solutions of

$$p^*(\lambda) = \lambda^2 + k_1\lambda + k_0$$

$k_0, k_1 > 0$, standard passages lead to the following reduced–order observer

$$\dot{\xi} = \begin{pmatrix} -k_1 & \omega \\ -\frac{k_0}{\omega} & 0 \end{pmatrix} \xi + \begin{pmatrix} \frac{-k_0 + k_1^2 + \omega^2}{\mu_5} \\ \frac{k_0 k_1}{\omega\mu_5} \end{pmatrix} x_3 + \begin{pmatrix} -k_1 \\ \frac{-k_0 + \omega^2}{\omega} \end{pmatrix} x_2$$

$$\begin{pmatrix} \hat{w}_4 \\ \hat{w}_5 \end{pmatrix} = \xi + \begin{pmatrix} -\frac{k_1}{\mu_5} \\ \frac{-k_0 + \omega^2}{\omega\mu_5} \end{pmatrix} x_3$$

where $\hat{w}_4$, $\hat{w}_5$ converge exponentially to w_4, w_5. Hence, the control (5) can be substituted with the following

$$u = \gamma(w_1, w_2, w_3, \hat{w}_4, \hat{w}_5) + K\Big(x - \pi(w_1, w_2, w_3, \hat{w}_5)\Big).$$
$$= \gamma(w) + a_4(\hat{w}_4 - w_4) + a_5(\hat{w}_5 - w_5) - K \begin{pmatrix} \frac{\mu_4\mu_5}{\omega\mu_3} \\ 0 \\ \frac{\mu_5}{\omega} \end{pmatrix} (\hat{w}_5 - w_5).$$

5. SIMULATIONS

The simulations refer to the case of a Chua's system (1) is in chaotic regime when $u = 0$. This happens for instance with the following parameter values

$$\mu_1 = 5.128\ 10^4, \quad \mu_2 = 6.667\ 10^7, \quad \mu_3 = 6.575\ 10^3$$
$$\mu_4 = 8.547\ 10^6, \quad \mu_5 = 58.666$$

and with $c_1 = -9.769\ 10^{-4}$, $c_3 = 2.801\ 10^{-6}$. The plant initial conditions have been set equal to

$$x_1(0) = 10^{-3}, \quad x_2(0) = 0, \quad x_3(0) = -10^{-3}.$$

As far as the exosystem is concerned, in the first case we suppose to consider a reference behavior of the Chua's circuit corresponding to the so–called period–2 orbit. Hence the parameters of the exosystem (2) have been fixed equal to

$$\theta_1 = \mu_1, \qquad \theta_2 = \mu_2, \qquad \theta_3 = 5.233\ 10^3$$
$$\theta_4 = 6.803\ 10^6, \qquad \theta_5 = 74.468$$

and $g_1 = -9.231\ 10^{-4}$, $g_3 = 1.603\ 10^{-4}$. For the disturbance we considered $v_d = D \sin \omega t$, with $\omega = 21.99\ 10^3$ rad/s and $D = 0.05$ V. Finally, the exosystem initial conditions have been set equal to

$$w_1(0) = x_1(0), \quad w_2(0) = x_2(0), \quad w_3(0) = x_3(0)$$
$$w_4(0) = 0.05, \quad w_5(0) = 0.05.$$

The matrix $A + BK$ has been rendered Hurwitz by means of the gain matrix $K = \begin{pmatrix} 0.031 & 2.368 & 4.468 \end{pmatrix}$, which sets the eigenvalues in $\lambda_1 = -1.1283\ 10^6$, $\lambda_2 = -9.301\ 10^5$, $\lambda_3 = -1.090\ 10^3$.

In Figure 3 it is reported the voltages x_2 and the reference w_2, respectively in solid and dotted lines. The tracking is accurate, as shown in Figure 4 where the tracking error e is shown, and the solid line covers the dotted one. It is possible to note that the tracking error has a value less than $5.0\ 10^{-4}$ V after only 2 ms.

The second case simulated refers to a situation of interest in chaos communication synchronizing problems. In this case both the plant and the exosystem are in chaotic regime, with the same disturbance considered in the first case. The plant's parameters and initial conditions are those of the first case. Also the exosystem's parameters and initial conditions are equal to the plant ones, namely $\theta_i = \mu_i$, $w_i(0) = x_i(0)$, $i = 1, \cdots, 5$, but $c_1 = -9.769\ 10^{-}4$, $c_3 = 2.801\ 10^{-6}$, $g_1 = -9.409\ 10^{-4}$, $g_3 = 2.701\ 10^{-6}$. The gain matrix is now $K = \begin{pmatrix} 0.315 & 23.684 & 44.682 \end{pmatrix}$ and sets the eigenvalues in $\lambda_1 = -2.048\ 10^7$, $\lambda_2 = -5.125\ 10^5$, $\lambda_3 = -1.112\ 10^3$. In Figures 5, 6 we report x_2, w_2, and the tracking error e, respectively. Also in this case the tracking error quickly converges to zero while the disturbance is rejected.

6. CONCLUSIONS

In this paper we present a solution to the tracking problem for a Chua's circuit, in presence of a disturbance due to a magnetic coupling in the circuit inductor. We solve the problem applying the regulation theory, finding a solution to the regulator equations and building a simple reduced–order observer in order to avoid the need of the disturbance measurements. The simulation results show the effectiveness of the proposed control. This result can be applied in the field of chaos communications, where such a tracking problem is of relevant interest. The result presented in this paper is a first step towards a complete solution of the tracking problem for such a circuit. In fact, other issues important in practical applications, such as the robustness of the proposed solution, should be addressed, and constitute the subject of future work.

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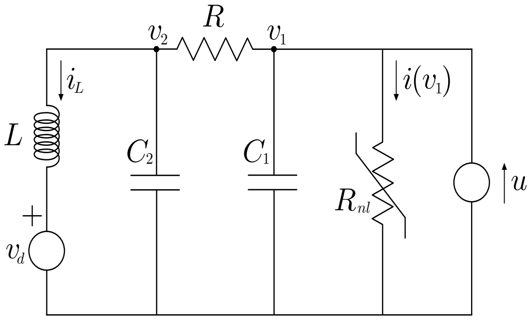

Fig. 1. Chua's circuit

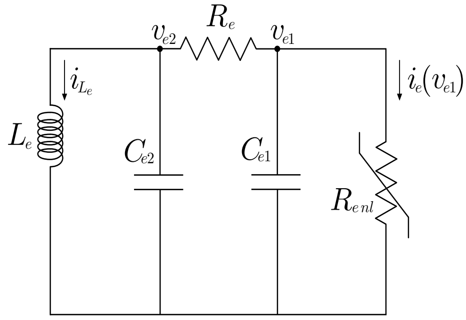

Fig. 2. Generator of the reference voltage

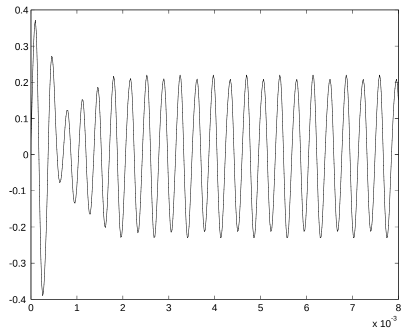

Fig. 3. x_2, $v_r(w)$ – First case

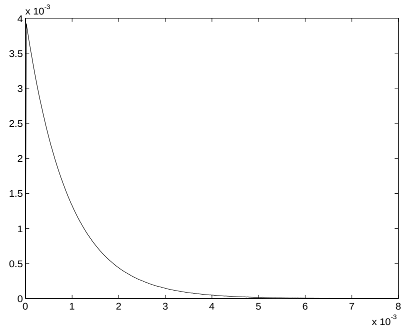

Fig. 4. Tracking error $e = x_2 - w_2$ – First case

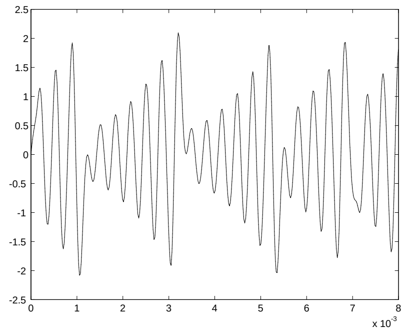

Fig. 5. x_2, $v_r(w)$ – Second case

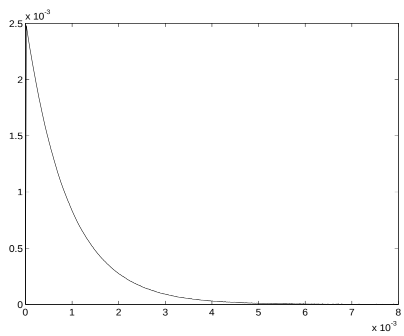

Fig. 6. Tracking error $e = x_2 - w_2$ – Second case