

# MANAGING SENSOR HARDWARE REDUNDANCY ON A SMALL COMMERCIAL AIRCRAFT WITH $H_\infty$ FDI OBSERVERS

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Abstract: In this paper the problem of managing Hardware redundancy on the Attitude Heading System and the Air Data System of a small commercial aircraft is considered. The proposed approach is based on a bank of extended  $H_\infty$  observers guaranteeing sensor fault detection and isolation in the presence of external disturbances and model uncertainties. Each observer is composed of an open loop nonlinear part replicating the system dynamics and a linear feedback action. Sufficient conditions for the synthesis of the feedback action are provided in terms of an LMI feasibility problem. Constraints on the position of the observer poles are added to fasten the residual generation dynamics and to avoid low damped and/or high frequency modes. Numerical simulations on the model of the AP-68 Vulcanair small commercial aircraft are presented. *Copyright©2005 IFAC.*

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## 1. INTRODUCTION

In order to detect and isolate faults on aircrafts, system redundancy is necessary. Two main categories of redundancy are typically used: *direct* and *analytical redundancy*. Direct redundancy means that multiple independent hardware channels (e.g. duplex, triplex or quadruplex replication of sensors), with a procedure to *vote* the healthy system channels, are used. On the other side, analytical redundancy implies the use of mathematical relations to obtain the redundant measurements. Today in flight control system design, especially for the category of small commercial aircraft (SCA), the direct redundancy approach is typically used.

At present it seems unreasonable to completely replace hardware with virtual sensors due to flight certification problems. In this paper we propose to use a bank of nonlinear  $H_\infty$  observers to increase the reliability of the measurement systems and/or to reduce the hardware replications. In particular we cope with the redundancy management of the Air Data System (ADS) providing angle of attack and velocity measurements, and the Attitude and Heading System (AHS) providing yaw, pitch, roll angles and angular rates, and linear acceleration measurements.

Failure detection, identification and reconfiguration (FDIR) has been an active field of research over the past decades. Many techniques have been proposed especially for sensors (Sensor Fault Detection, Isolation and Accommodation, SFDIA), and/or actuators failures with application to the

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aeronautical field. Most of the current research activities are focused on the *analytical redundancy* techniques making use of a priori information on the system behaviour to detect and isolate faults. Different observer based techniques have been studied in the literature for SFDIA (Chen and Patton, 1999): multiple model Kalman filtering, *Thau* observers, *parity space* approaches, dedicated observers, and, more recently,  $H_2/H_\infty$  based techniques (Marcos *et al.*, 2004).

Sensor FDI problems are particularly challenging in complex multi-input multi-output systems with nonlinearities, uncertainties and external disturbances, and in the presence of different kind of faults. Satisfactory answers to the problem have been given in the case of linear time invariant plants. However, in the presence of nonlinearities with robustness, disturbance rejection and FDI requirements it is common practice to assume a linearized model of the plant calculated in the neighborhood of a certain working condition to design FDI observers. Indeed this approach can be successfully applied if nonlinearities are *moderate* and/or the excursion of the state, input, and output is small. In any case, stability and performance of the FDI scheme are guaranteed only in the neighborhood of the selected working conditions. Considerable research activity aimed at the design and analysis of observer based fault diagnosis schemes specific for nonlinear systems has been also produced (Chen and Patton, 1999), (Seliger and Frank, 1999), (Frank and Seliger, 2000), (Hammouri *et al.*, 1999), (Zhang *et al.*, 2002).

The LMI procedure described in this paper provides a practical approach to design a bank of nonlinear identity observers to be used for the FDI residual generation. Also healthy estimates of the faulted measurement signals can be produced with the proposed observers. These observers are composed of an open loop part replicating the system nonlinearities and a linear closed loop part designed with  $H_\infty$  criteria. The advantages of the proposed technique are that the system nonlinearities are directly accounted for in the observer structure; the design tools offered in the  $H_\infty$  setting can be used for the translation of the FDI systems requirements (Edelmayer *et al.*, 1997), (Chen *et al.*, 1998) while a *multi-model* approach for the synthesis of the feedback linear part of the observers allows a reduction of the design time.

## 2. THE PROBLEM STATEMENT AND THE OBSERVER STRUCTURE

Analytical sensor redundancy needs one or more signal reconstruction modules based on algorithms which are able to estimate the measured variables of interest from the knowledge of the

measured inputs and outputs of the system under consideration. In this paper we investigate the possibility to adopt a bank of extended  $H_\infty$  observers allowing the detection and isolation of faults on the AHS and ADS in the presence of external disturbances and uncertainties.

In particular we consider a small commercial aircraft (Amato *et al.*, 2003) where ADS and AHS are duplicated to achieve *direct redundancy*. This means that we can assume two vector of measurements  $y_{SM1} = (y_{AHS1}^T y_{ADS1}^T)^T$  and  $y_{SM2} = (y_{AHS2}^T y_{ADS2}^T)^T$  provided by the first and second pair of ADS and AHS respectively.

The basic idea to achieve FDI is shown in figure 1. A bank of two observers, one for each vector of measurements, provides estimates  $\hat{y}_{SM1}$  and  $\hat{y}_{SM2}$  of the monitored variables. The Decision Logic (DL), on the basis of the measured and estimated outputs, isolates the faulted AHS or ADS and votes the healthy measurement.

The nonlinear mathematical model of the aircraft can be written as follows (Stevens and Lewis, 1992) (see Table 1 for the symbol definitions):

$$\begin{aligned}
W\dot{V} &= T \cos(\alpha + \mu_T) \cos \beta - D + Wg_1 \\
VW\dot{\beta} &= -T \cos(\alpha + \mu_T) \sin \beta + Y + \\
&\quad - WVr + Wg_2 \\
WV \cos \beta \dot{\alpha} &= -T \sin(\alpha + \mu_T) - L + \\
&\quad + WVq + Wg_3 \\
J_x \dot{p} - J_{xz} \dot{r} &= R + qr(J_y - J_z) + pqJ_{xz} \quad (1) \\
J_y \dot{q} &= M + rp(J_z - J_x) + (r^2 - p^2)J_{xz} \\
&\quad - J_{xz} \dot{p} + J_z \dot{r} = N + pq(J_x - J_y) - qrJ_{xz} \\
\dot{\phi} &= p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \\
\dot{\theta} &= q \cos \phi - r \sin \phi \\
\dot{\psi} &= r \cos \phi \sec \theta + q \sin \phi \sec \theta
\end{aligned}$$

where

$$\begin{aligned}
g_1 &= g(-\cos \alpha \cos \beta \sin \theta + \sin \beta \sin \phi \cos \theta + \\
&\quad + \sin \alpha \cos \beta \cos \phi \cos \theta) \\
g_2 &= g(\cos \alpha \sin \beta \sin \theta + \cos \beta \sin \phi \cos \theta + \\
&\quad - \sin \alpha \sin \beta \cos \phi \cos \theta) \\
g_3 &= g(\sin \alpha \sin \theta + \cos \alpha \cos \phi \cos \theta),
\end{aligned}$$

and

$$\begin{aligned}
D &= \frac{1}{2} \rho V^2 S C_D & L &= \frac{1}{2} \rho V^2 S C_L \\
Y &= \frac{1}{2} \rho V^2 S C_Y & R &= \frac{1}{2} \rho V^2 S b C_R \\
M &= \frac{1}{2} \rho V^2 S \bar{c} C_M & N &= \frac{1}{2} \rho V^2 S b C_N
\end{aligned}$$

As for the main uncertainties affecting the aircraft we have aerodynamic and mass properties uncertainties. The aerodynamic coefficients can

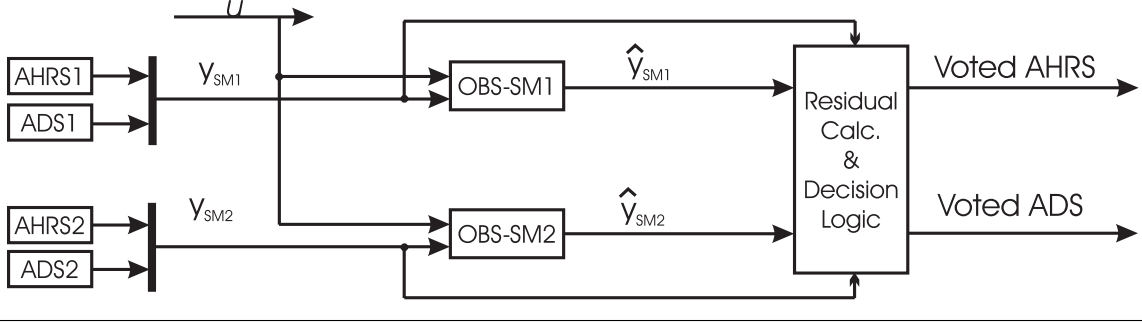


Fig. 1. Schematic of the FDI algorithm

be expressed as the sum of a number of terms depending on different variables. For example the lift coefficient, neglecting the contribute of the Reynolds number and of flaps, can be written as:

$$C_L = C_L(q, \alpha, \dot{\alpha}, Mach, \delta_e) = C_{Lbasic}(\alpha, Mach) + C_{L\alpha}(\alpha, Mach)(1 + \Delta C_{L\alpha})\alpha + C_{L\delta_e}(\alpha, Mach, \delta_e)(1 + \Delta C_{L\delta_e})\delta_e + \dots$$

The aerodynamic uncertainties are introduced as multiplicative perturbations affecting some of the addenda composing the total coefficients; these uncertainties introduce a percentage variation of the nominal value of the terms they affect.

The aircraft mass and the centre of gravity (CG) position are also uncertain in different ways. First, during a given flight, the mass varies due to fuel consumption; also, the mass distribution may vary with the consequence of changing the position of the aircraft CG.

We cope with possible faults both on the ADS and AHS. We consider the case that one or more measurements provided by these systems may be faulted, assuming as possible fault models *abrupt switches to zero* of the measured signal, *slow drifts* (a linearly increasing signal is added to the measurement), *abrupt constant bias* (additive step disturbance on the measured signal), *abrupt freezing* of the measured signal (the measured signal is frozen at its current value at a certain time instant), *increasing noise* (additive gaussian noise with increasing variance).

If we assume the presence of atmospheric disturbances  $w_p = (u_{wind} \ v_{wind} \ w_{wind})^T$ , and choose  $u = (\delta_e \ \delta_T \ \delta_a \ \delta_r \ \delta_f)^T$ ,  $y = x = (V \ \alpha \ q \ \theta \ p \ r \ \phi \ \beta)^T$ , the nonlinear model of the aircraft can be written in the form:

$$\begin{aligned} \dot{x} &= f(x, u) + B_{wp}(x, u)w_p \\ y &= g(x, u) + D_{wp}(x, u)w_p + w_f \end{aligned} \quad (2)$$

where  $x \in \mathbb{R}^n$  is the state vector,  $y \in \mathbb{R}^p$  is the output vector,  $u \in \mathbb{R}^{m_1}$  is the input vector,  $w_p \in \mathbb{R}^{m_2}$  is the process disturbance vector,  $w_f \in \mathbb{R}^p$  is the sensor fault vector. We also define an unfaulted output vector

$$y_{NF} = g(x, u) + D_{wp}(x, u)w_p.$$

The nonlinear observer structure proposed in this paper is the classical *nonlinear identity observer* structure firstly used by (Hengy and Frank, 1986) for FDI problems. We consider the following open loop nonlinear dynamic observer

$$\dot{\hat{x}} = f(\hat{x}, u) + \hat{u} \quad (3)$$

$$\hat{y} = \hat{x} \quad (4)$$

where  $\hat{x} \in \mathbb{R}^n$  is the estimated state vector,  $\hat{u} \in \mathbb{R}^n$  is an additional observer input that can be used to achieve a closed loop estimation in the form  $\hat{u} = L(y - \hat{y})$ .

In the literature Kalman filtering and  $H_\infty$  approaches have been proposed to obtain such feedback actions. In the next section we will provide a procedure to achieve  $H_\infty$  performance and pole clustering of the observer linearized dynamics. In particular, on the basis of the results given by (Chilali and Gahinet, 1996), sufficient conditions involving LMI feasibility problems (Boyd *et al.*, 1994) will be provided.

### 3. THE OBSERVER GAIN SYNTHESIS PROCEDURE

If observer (3) is used and a static linear feedback policy  $\hat{u} = L(y - \hat{y})$  is adopted, the state estimation error dynamics is the following:

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{\hat{x}} = f(x, u) - f(\hat{x}, u) + \\ &\quad - L(g(x, u) - g(\hat{x}, u)) + B_{wp}(x, u)w_p + \\ &\quad - L(D_{wp}(x, u)w_p + w_f) \end{aligned} \quad (5)$$

Now, if vector functions  $f(\cdot, \cdot), g(\cdot, \cdot)$  are sufficiently smooth, a first order approximation of (5) is

$$\begin{aligned} \dot{e} &\cong f(x, u) - \left( f(x, u) + \frac{\partial f(x, u)}{\partial x}(\hat{x} - x) \right) + \\ &\quad - L \left( g(x, u) - \left( g(x, u) + \frac{\partial g(x, u)}{\partial x}(\hat{x} - x) \right) \right) + \\ &\quad + B_{wp}(x, u)w_p - L(D_{wp}(x, u)w_p + w_f) \end{aligned} \quad (6)$$

where the *Jacobian* matrices are computed in the neighbourhood of the current real state. We approach to the following expression of the error dynamics:

$$\dot{e} = \left( A(x, u) - LC(x, u) \right) e + B_{wp}(x, u) w_p - L \left( D_{wp}(x, u) w_p + w_f \right) \quad (7)$$

with

$$A(x, u) = \frac{\partial f(x, u)}{\partial x}; \quad C(x, u) = \frac{\partial g(x, u)}{\partial x};$$

that can be more generally rewritten as:

$$\dot{e} = \left( A(\pi) - LC(\pi) \right) e + B_{wp}(\pi) w_p - L \left( D_{wp}(\pi) w_p + w_f \right) \quad (8)$$

$\pi$  being a parameter vector belonging to a certain set  $\mathcal{P}$  (see Remark 1). Now we recast the FDI design problem to obtain the observer matrix gain  $L$  in the classical  $H_\infty$  framework. We have to find an optimal closed loop control law for the following system:

$$\begin{aligned} \dot{e} &= A(\pi)e + \begin{bmatrix} B_{wp}(\pi) & 0 \end{bmatrix} w - I\hat{u} = \\ &= A(\pi)e + B_1(\pi)w - I\hat{u} \\ e_z &= (y_{NF} - \hat{y}) = \left( C(\pi)e + \begin{bmatrix} D_{wp}(\pi) & 0 \end{bmatrix} w \right) = \\ &= C_1(\pi)e + D_{11}(\pi)w \\ e_y &= (y - \hat{y}) = \left( C(\pi)e + \begin{bmatrix} D_{wp}(\pi) & I \end{bmatrix} w \right) = \\ &= C_2(\pi)e + D_{21}(\pi)w \end{aligned} \quad (9)$$

$w = (w_p^T \ w_f^T)^T$  being the vector of external disturbance,  $e_y$  the vector of measured outputs (output estimation errors),  $e_z$  the vector of controlled outputs. If a solution of the (LPV)  $H_\infty$  problem exists we have

$$\sup_{\substack{\pi \in \mathcal{P}, w \in L_2 \\ \|w\|_2 \leq I}} \frac{\|e_z\|_2}{\|w\|_2} < \gamma_{opt}$$

with  $\gamma_{opt}$  sufficiently small.

Since we are assuming as controlled output  $e_z = (y_{NF} - \hat{y})$ , assuming as FDI residual the linear combination  $r = (y - \hat{y})$  of the measured outputs,  $\|r\| = \|(y - \hat{y})\| \rightarrow \|w_f\|$  as  $\|e_z\|$  approaches to zero. On the other hand  $\hat{y} \rightarrow y_{NF}$  in the absence of faults, thus providing a nice output estimate that can be used as healthy measurement for accommodation (see experimental results in Section 4).

If the spectral components of the process disturbance and fault are well separated in the frequency domain, the application of standard  $H_\infty$  design methodologies allows us to give emphasis to this effect by the introduction of frequency weighting filters.

The following procedure can be used to design the observer gain matrix  $L$  guaranteeing an  $H_\infty$

performance level and that the linearised observer closed loop poles belong to a specified sub-region of the complex plane  $\mathcal{D}(\alpha_{min}, \zeta_{min}, \omega_{nmax})$  determined by a maximum natural frequency  $\omega_{nmax}$ , a minimum damping coefficient  $\zeta_{min}$  and a minimum real part  $\alpha_{min}$ .

### Procedure 1

- Determine a region of admissible poles  $\mathcal{D}(\alpha_{min}, \zeta_{min}, \omega_{nmax})$  and an  $H_\infty$  performance level  $\gamma$  on the  $w - z$  input-output channel on the basis of the FDI requirements.
- Find a positive definite matrix  $P$  and a gain matrix  $W$  such that the following set of LMIs is solved  $\forall \pi \in \mathcal{P}$ :

$$\begin{pmatrix} \mathcal{L}^+(A(\pi), P) + & & P B_1(\pi) + & & C_1^T(\pi) \\ +\mathcal{L}^+(C_2(\pi), W) + 2\alpha_{min}P & & +W D_{21}(\pi) & & \\ * & & -I & & D_{11}^T(\pi) \\ * & & * & & -\gamma^2 I \end{pmatrix} < 0 \quad (10)$$

$$\begin{pmatrix} -\omega_{nmax}P & P A(\pi) + W C_2(\pi) \\ * & -\omega_{nmax}P \end{pmatrix} < 0 \quad (11)$$

$$\begin{pmatrix} \sin \theta \left( \mathcal{L}^+(A(\pi), P) + \right) & \cos \theta \left( \mathcal{L}^-(A(\pi), P) + \right) \\ +\mathcal{L}^+(C_2(\pi), W) & +\mathcal{L}^-(C_2(\pi), W) \\ * & \sin \theta \left( \mathcal{L}^+(A(\pi), P) + \right) \\ & +\mathcal{L}^+(C_2(\pi), W) \end{pmatrix} < 0 \quad (12)$$

with  $\mathcal{L}^+(X, Y) = X^T Y^T + Y X$ ,  $\mathcal{L}^-(X, Y) = X^T Y^T - Y X$ ,  $\theta = \cos^{-1}(\zeta_{min})$ ;

- compute matrix  $L = P^{-1}W$ .

*Remark 1.* LMIs (10)-(12) are only *sufficient* conditions for the solution of the observer gain synthesis problem. The conservatism of the proposed approach strongly relies on the choice of the system parameterization. Furthermore, due to the parameter dependence of the system matrices, the proposed LMI problem turns out to be infinite dimensional. If the dependence of the system matrices on the parameter vector is not multi-affine it is in general not possible to reduce the problem to a finite dimensional one. A practical approach may be that of solving LMIs for a family of significant plant models obtained for a finite number of parameter values within  $\mathcal{P}$  (*multi-model* approach).

## 4. SIMULATION RESULTS

A wide campaign of simulations on the non-linear complete model of the AP-68 Vulcanair small commercial aircraft has shown the effectiveness of the proposed FDI scheme in a large region of the operating envelope guaranteeing a very low number of false alarms due to external disturbances/uncertainties and detecting most of

the sensor faults falling in the categories presented in Section 2. All the simulation were performed in the Matlab-Simulink environment using a complete nonlinear model of the aircraft with nonlinear actuators and sensors dynamics. Also flight configurations with flaps, slats, airbrakes and landing gears out have been considered and different kind of maneuvers has been assumed to excite the nonlinear behavior of the plant.

To allow fault isolation a bank of two observers was built:

- a so-called *SM1* observer based on the AHS1 and ADS1 measurements,
- a so-called *SM2* observer observer based on the AHS2 and ADS2 measurements.

As detection residual we adopted:

$$e_{\text{AHS}} = \frac{y_{\text{AHS1}} - y_{\text{AHS2}}}{\mu_1}; \quad e_{\text{ADS}} = \frac{y_{\text{ADS1}} - y_{\text{ADS2}}}{\mu_2}.$$

If one of the AHSs (ADSs) is faulted, the norm of  $e_{\text{AHS}}$  ( $e_{\text{ADS}}$ ) exceeds a certain threshold.

For isolation we assume

$$e_{\text{SM1}} = \frac{y_{\text{SM1}} - \hat{y}_{\text{SM1}}}{\lambda_1}; \quad e_{\text{SM2}} = \frac{y_{\text{SM2}} - \hat{y}_{\text{SM2}}}{\lambda_2}.$$

If the norm of  $e_{\text{SM1}}$ , ( $e_{\text{SM2}}$ ) exceeds a certain threshold a fault is declared on AHS1 (AHS2) or ADS1 (ADS2) depending on the detection result. In facts the output of the SM1 observer is not affected by faults on AHS2 and ADS2, while the output of the SM2 observer is not affected by faults on AHS1 and ADS1.

The estimations provided by the bank of observers can also be used by the DL as healthy measurements (Amato *et al.*, 2003).

Thanks to the normalization factors  $\lambda$  and  $\mu$ , thresholds were fixed to 1 and the usual infinity norm of vectors was used to detect and isolate faults. In particular if  $\|e_{\text{SM1}}\|_\infty (\|e_{\text{SM2}}\|_\infty) > 1$  for a time interval of 0.1 seconds, a fault is declared on SM1(SM2) measurement set. The normalization factors  $\lambda_1 = \lambda_2$  were chosen on the basis of the physical consideration that errors of about 0.5 *deg* on  $[\phi, \theta, \psi, \alpha]$ , 1 *deg/s* on  $[p, q, r]$  and 1 *m/s* on  $V$  measurements can be tolerated, while the values of  $\mu_1 = \mu_2$  where chosen assuming that half of these errors are suited for isolation.

The design of the nonlinear observers was carried out assuming an admissibility region  $\mathcal{D}(10, 0.707, 30)$  for both the observers. As significant parameters  $\pi$  the variables  $\alpha$  [-5 14] *deg*, *Mach* [0.15 0.6],  $h$  [100 7000] *m*,  $W$  [1600 2700] *kg*,  $XCG$  [24 32]% of the mean aerodynamic chord were assumed to cover a wide range of flight conditions. Also uncertainties on the aerodynamic coefficient up to 30% of the nominal conditions were considered. The design was performed on a family of 30 plants representative of the flight envelope. Figures 2 to 5 show some numerical results.

## 5. CONCLUSIONS

The problem of detecting and isolating sensor faults, both incipient and abrupt kind, on the AHS and ADS of a AP-68 Vulcanair commercial aircraft in the presence of external disturbances and uncertainties has been considered. A procedure to design a bank of extended  $H_\infty$  observers for sensor FDI has been carried out. The application of  $H_\infty$  techniques allows a decoupling of the disturbance effect from the effect of the fault on the FDI residual. A multi-model approach on a family of linearized plants has been adopted to extend the applicability of the linear design to a wide aircraft operating envelope. Sufficient conditions have been provided in terms of the solvability of an LMI feasibility problem. Constraints on the position of the observer poles have also been added to avoid low damped or high frequency modes. A wide campaign of simulations on a nonlinear model of the aircraft have shown the effectiveness of the proposed technique.

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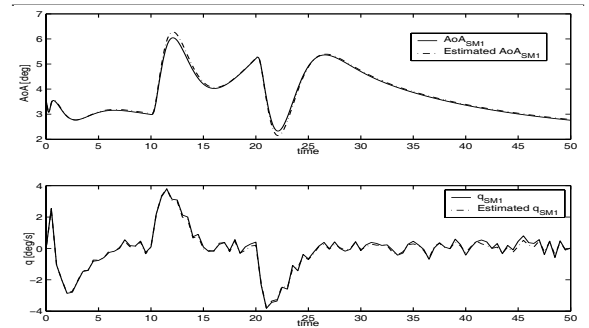


Fig. 2. Estimates behavior of AoA in the presence of mass uncertainties and of  $q$  measures in the presence of a light atmospheric turbulence.

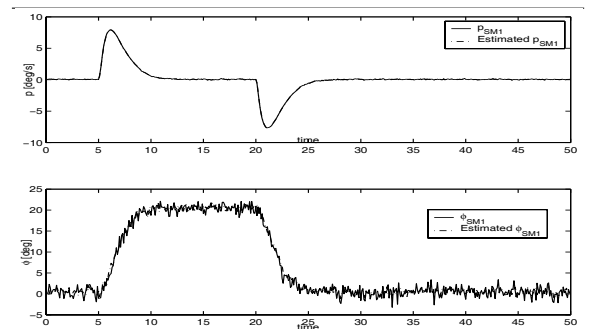


Fig. 3. Estimates of  $p$  in the presence of aerodynamic uncertainties and of  $\phi$  in the presence of severe atmospheric turbulence.

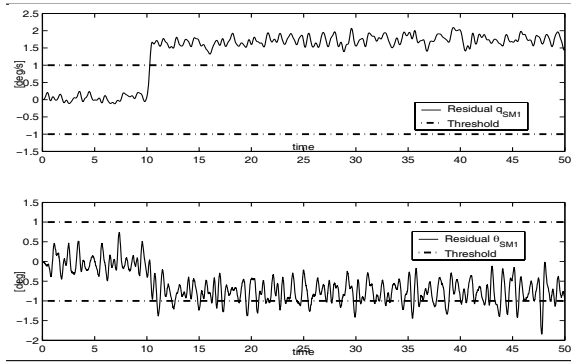


Fig. 4. Abrupt fault on  $q$  measurement:  $q$  and  $\theta$  residual. The first residual allows a sudden isolation

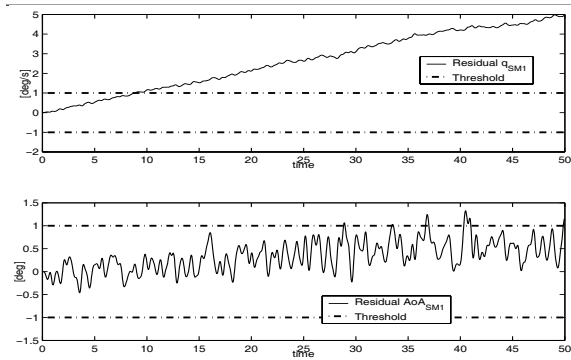


Fig. 5. Slow drift on AoA measurement:  $q$  and AoA residual. The first residual allows an isolation in about 10 s

Table 1. Nomenclature

Variable	
$[p \ q \ r]^T$	Roll, Pitch Yaw rates
$[\phi \ \theta \ \psi]^T$	Roll, Pitch Yaw angles
$V$	True air speed
$\alpha$	Angle of attack
$\beta$	Sideslip angle
$h$	Altitude
$T$	Thrust
$\mu_T$	Angle between the thrust direction and the $X$ -body axis
$W$	Aircraft weight
$g$	Gravity acceleration constant
$J_x, J_y, J_z$	Momentum of inertia about $(X, Y, Z)$ -body axes
$J_{xz}$	Cross product of inertia
$C_D, C_L, C_Y$	Drag, Lift, Side force coefficients
$C_R, C_M, C_N$	Roll, Pitch, Yaw moment coefficients
$\rho = (\rho(h))$	Air density
$\bar{c}$	Mean aerodynamic chord
$b$	Wing span
$S$	Wing reference area
$\delta_e, \delta_a, \delta_r$	Elevator, Ailerons, Rudder deflection
$\delta_T$	Throttle command
$\delta_f$	Flap deflection

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