DESIGN AND EXPERIMENTAL TESTING OF A MULTIVARIABLE SHAPE CONTROLLER FOR THE JET TOKAMAK

Giuseppe Ambrosino [*] **Marco Ariola** [*,1]
Alfredo Pironti [*] **Filippo Sartori** [**]

[*] *Dipartimento di Informatica e Sistemistica, Università degli Studi di Napoli Federico II, Via Claudio 21, 80125, Napoli, Italy*
[**] *Euratom/UKAEA Fusion Association, Culham Science Centre, Abingdon, Oxon OX14 3DB, UK*

Abstract: This paper describes the design and the experimental validation of a multivariable shape controller for the JET tokamak. This new controller allows the control of extremely shaped plasmas with high values of elongation and triangularity. The problem has been formulated as an output regulation problem for an LTI plant whose controllable outputs are more than the control inputs. For the case of constant references, we propose a control scheme which minimizes a quadratic cost function. This cost function weights the tracking error at steady-state. Our methodology is based on the singular value decomposition of the static gain matrix of the plant. In the controller design we also take into account the steady-state control effort. Some experimental results are presented.

Keywords: Output regulation, power plants, experimental results

1. INTRODUCTION

The increasing energy consumption pushes toward the search of new resources. Nuclear fusion seems to offer great possibilities since the fuel sources are essentially inexhaustible, the fusion process is inherently safe, and no harmful greenhouse gases are produced. One possible approach for nuclear fusion is the magnetic confinement of a fully ionized gas called plasma in suitable devices. Among the various possible configurations, the most promising approach has proved to be the tokamak.

Tokamaks were first developed in the ex Soviet Union in the late Sixties and are characterized by toroidal symmetry. The confinement of the plasma is obtained via the interaction of the plasma with an external electromagnetic field, produced by toroidal coils. Among the many different physical and engineering problems connected with nuclear fusion in tokamaks, plasma control has gained more and more importance because of the need of achieving always better performance.

High performance in the next generation tokamaks shall be achieved by elongated, vertically unstable plasmas, placed as close as possible to the plasma metallic facing components. Although the plasma facing components are designed to withstand high heat fluxes, contact with the plasma is always a major concern in tokamak operations

[1] This author is responsible for correspondence. Ph.: +39 081 76 83862; fax: +39 081 76 83816; e-mail: ariola@unina.it

and, therefore, adequate plasma-wall clearance must be guaranteed. This is obtained by means of additional magnetic fields produced by suitable currents flowing in a number of poloidal field coils surrounding the plasma ring. These currents are generated by a power supply system driven in feedback by a plasma shape control system. Figure 1 shows the poloidal field coils of the Joint European Torus (JET) tokamak.

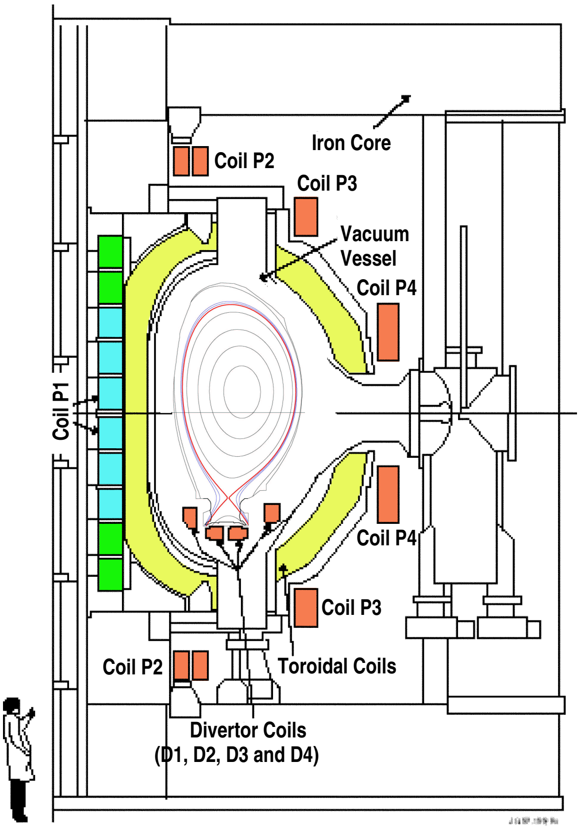

Fig. 1. The JET cross section. The plasma boundary is shown in red. The poloidal coils (P1-P4 and D1-D4) and the toroidal coils, which surround the plasma ring, produce the necessary confinement magnetic field (courtesy of EFDA-JET).

The work described in this paper has been carried out in the framework of a project aimed at investigating the possibility of obtaining extremely shaped plasmas with the existing active circuits and control hardware (Crisanti *et al.*, 2003). One of the steps needed to achieve this objective is the redesign of the JET shape controller, since the present controller does not consider the possibility of operating with highly shaped plasmas.

In the experiments with highly shaped plasmas, usually several geometric parameters are controlled, such as the plasma center, elongation and triangularity. Alternatively the distance between the plasma boundary and the vessel at some specific points can be controlled. Whatever choice is made, the strong output coupling calls for a model-based MIMO approach to obtain improvements in closed-loop performance. The controller is also restricted to demand as little power as possible, to limit surges in the total power required for the poloidal field system.

Multivariable control design approaches have been used recently to control the plasma vertical position in (Vyas *et al.*, 1998), where the authors use the H_∞ technique; in (Gossner *et al.*, 1999), where predictive control is adopted; in (Scibile and Kouvaritakis, 2001), where a nonlinear, adaptive controller is designed. In (Ariola *et al.*, 2001) the authors propose a controller designed using the H_∞ technique, which has been used during normal tokamak operation to control at the same time the plasma current, vertical position and some geometrical parameters.

This paper describes the features of the new JET controller, which has been called eXtreme Shape Controller (XSC). This new controller is the first example of multivariable tokamak controller that allows to control with a high accuracy the overall plasma boundary, specified in terms of a certain number of gaps. The problem is formulated as an output regulation problem for a non right-invertible plant, i.e. a plant that where the number of *independent* control variables is less than the number of *independent* outputs to regulate. In this case it is not possible to guarantee that the difference between the reference and the controlled plant output (*tracking error*) is zero at steady-state. To tackle this problem, we essentially adopt the singular value decomposition in order to isolate the part of the plant output which can be better regulated at steady-state. Moreover the singular value decomposition gives us an insight into the steady-state control effort: since some of the singular values of the plant static gain are *small*, we truncate these singular values introducing a trade-off between the tracking error and the control effort.

The paper is divided as follows: in Section 2 we discuss the control requirements and we describe the plant model; in Section 3 we present the technique we have adopted to design the controller; Section 4 includes some experimental results; the conclusions are drawn in Section 5.

2. CONTROL REQUIREMENTS AND SIMPLIFIED PLASMA MODELLING

In the JET tokamak (see Figure 1) there are eight poloidal field coils available to the plasma shape control system. These coils are denoted by P1, ...,P4, and D1, ...,D4. The P-coils are connected to form five circuits. The currents flowing in these circuits are indicated by I_{P1E}, I_{PFX}, I_{SHA}, I_{P4T},

I_{P4i}, whereas the currents flowing in the D-coils are indicated by I_{Di}, with $i = 1, \ldots, 4$. Therefore we have nine circuits available to the plasma control system. One of these circuits, P1E, is used to control the plasma current, whereas the other eight circuits can be used to control the plasma shape.

The controller we want to design should be able to control the plasma shape. One problem regarding the plasma shape control is the choice of the controlled variables. In this case the plasma shape has been characterized by a finite number of parameters which are identified on the basis of the available magnetic measurements. More specifically, the geometrical parameters controlled by the XSC are a set of 28 gaps, the radial and vertical position of the X-point, and finally 2 parameters describing the strike point positions. The system we want to control plant is characterized by the fact that the number of controlled outputs, equal to 32, is much larger than the number of control inputs, which is equal to 8.

A tokamak device is a rather complex system: it includes the plasma, the active coils, and the metallic structures (hereafter named passive conductors). It is a distributed parameter system whose dynamic behavior is described by a set of nonlinear PDEs, whereas most controller design techniques consider ODE models, usually linear and time invariant. The main problem is then that of introducing some simplifying physical assumptions and of using approximate numerical methods to obtain a model detailed enough to catch the principal phenomena, but reasonably simple to make the controller design straightforward and fast. In this paper we use the model derived by Albanese and Villone (1998).

As shown by Ariola *et al.* (2003), making use of some preliminary compensation loops, for the design of the plasma shape controller we are reduced to the following linearized simplified model

$$Y(s) = P(s)U(s) \tag{1}$$

where $Y(s)$ are the controlled parameters, $U(s)$ are the current references for the $m = 8$ circuits which are available to the shape controller, and

$$P(s) = \frac{C_l}{1 + s\tau}$$

with $\tau = 0.1$s, and $C \in \mathbb{R}^{p\times m}$ with $p = 32$.

3. THE CONTROLLER DESIGN

The shape control problem basically consists in determining the circuit currents that can reduce the errors on the geometrical descriptors to zero at steady-state. Since we are using m currents, only m linear combinations of the geometrical descriptors can reduced to zero at steady-state. Our problem then becomes that of determining the m linear combinations of the errors on the geometrical parameters that minimize the overall error in a quadratic sense. On the other hand, once these m linear combinations have been selected, the m values of the control circuit currents at steady-state are univocally determined. Hence this approach could lead to high values of the currents; these values could possibly exceed the saturation limits. To overcome this problem, the number of linear combinations of geometrical descriptor errors to reduce to zero could be chosen to be less than m. This extra degree-of-freedom can be used to reduce the amplitude of the requested currents. A straightforward solution to both these optimization problems is given by the following singular value decomposition approach, which can be applied to more general cases as shown by Ambrosino *et al.* (2003).

We want a constant reference $\bar{r}$ to be tracked by the controlled output variable $y(t)$. We will denote the tracking error by

$$e(t) := \bar{r} - y(t)\,.$$

Let us consider a controller $K(s)$ with input $e(t)$ and output $u(t)$. Therefore the closed-loop system is defined by the equations

$$y = Pu\,, \quad u = Ke\,, \quad e = \bar{r} - y \tag{2}$$

Our aim is to find a controller $K(s)$ which internally stabilizes the closed-loop system (2) and makes the error $e(t)$ *small* in some sense at steady-state. Since the number p of output variables to regulate is greater than number of control inputs m, we consider the problem of minimizing a steady-state performance index in the form

$$J = \lim_{t\to+\infty} e^T(t)Qe(t)\,, \tag{3}$$

where $Q \in \mathbb{R}^{p\times p}$ is a positive definite weighting matrix.

Let us consider the singular value decomposition of the following matrix

$$\tilde{C} = Q^{1/2}CR^{-1/2} = U\Sigma V^T\,, \tag{4}$$

where $\Sigma = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_m) \in \mathbb{R}^{m\times m}$, $U \in \mathbb{R}^{p\times m}$, $V \in \mathbb{R}^{m\times m}$, and $R \in \mathbb{R}^{m\times m}$ is a positive definite weighting matrix. An analysis of the singular values showed that $\sigma_1 > \ldots \sigma_k \gg \sigma_{k+1} > \ldots \sigma_m$ for $k = 5$. We recall the following properties of the SVD (4)

$$V^TV = VV^T = I\,, \tag{5a}$$

$$U^TU = I\,. \tag{5b}$$

The properties of the SVD imply that the columns of the matrix $Q^{-1/2}U\Sigma$ form a basis for the subspace of the obtainable steady-state output values. The reference signal can be splitted into two components: one which lies in this subspace, and the other which is orthogonal to it; therefore we can write

$$\bar{r} = Q^{-1/2} U \Sigma \bar{w} + \bar{b}\,, \tag{6}$$

where $\bar{w} \in \mathbb{R}^m$ is defined as

$$\bar{w} = \Sigma^{-1} U^T Q^{1/2} \bar{r}\,, \tag{7}$$

and $\bar{b}$ satisfies $\bar{b}^T Q^{1/2} U = 0$.

Now let us decompose the plant output accordingly to what has been done for the reference (6). Therefore let us define

$$z(t) = \Sigma^{-1} U^T Q^{1/2} y(t)\,. \tag{8}$$

The signal $z(t)$ represents the component of the output signal $y(t)$ that can be actually regulated; it has the same dimension of $u(t)$.

Denoting by $\bar{z}$ the steady-state value of $z(t)$, we have

$$\begin{aligned} \bar{z} &= \Sigma^{-1} U^T Q^{1/2} \bar{y} = \Sigma^{-1} U^T Q^{1/2} C \bar{u} \\ &= \Sigma^{-1} U^T U \Sigma V^T R^{1/2} \bar{u} = V^T R^{1/2} \bar{u}\,, \end{aligned} \tag{9}$$

where we have used (1), (4) and (5b). From (9), using (5a), we obtain

$$\bar{u} = R^{-1/2} V \bar{z}\,.$$

Finally using (1) we have

$$\bar{y} = C\bar{u} = Q^{-1/2} U \Sigma V^T R^{1/2} \bar{u} = Q^{-1/2} U \Sigma \bar{z}. \tag{10}$$

The decomposition (6) has a direct consequence on the cost function (3); indeed using (6) and (10) it is possible to write

$$\bar{e} = \bar{r} - \bar{y} = Q^{-1/2} U \Sigma \bar{w} + \bar{b} - Q^{-1/2} U \Sigma \bar{z}\,.$$

In this way we obtain

$$\begin{aligned} J &= (\bar{w} - \bar{z})^T \Sigma^2 (\bar{w} - \bar{z}) + \bar{b}^T Q \bar{b} \\ &= \sum_{i=1}^{m} \sigma_i^2 (\bar{w}^i - \bar{z}^i)^2 + \bar{b}^T Q \bar{b}\,, \end{aligned} \tag{11}$$

where $\bar{w}^i$ (resp. $\bar{z}^i$) indicate the components of $\bar{w}$ (resp. $\bar{z}$). The quadratic term involving the vector $\bar{b}$ in (11) does not depend on the choice of the controller, but only on the reference signal $\bar{r}$ to be tracked. Therefore minimizing J is equivalent to minimize the cost function

$$\tilde{J} = (\bar{w} - \bar{z})^T \Sigma^2 (\bar{w} - \bar{z}) = \sum_{i=1}^{m} \sigma_i^2 (\bar{w}^i - \bar{z}^i)^2\,.$$

In our case for $k = 5$, $\sigma_k \gg \sigma_{k+1}$. This suggests that we modify the cost function (11) neglecting the terms corresponding to the singular values σ_i with $i > k$ (the smallest ones). In this way we are using just k linear combinations of the inputs and therefore we can minimize a weighted norm of the steady-state control vector $\bar{u}$. To this aim we consider the new cost function (with $k < m$ terms)

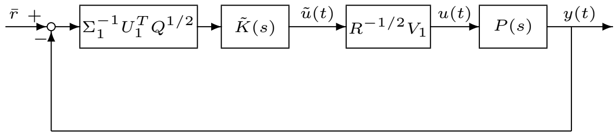

Fig. 2. The feedback scheme with the controller (18)

$$\tilde{J}_1 = \sum_{i=1}^{k} \sigma_i^2 (\bar{w}^i - \bar{z}^i)^2\,. \tag{12}$$

Hence our aim becomes to find a controller structure which solves the following *optimization problem*

$$\boxed{\min_{\bar{u}} \bar{u}^T R \bar{u} \quad \text{such that } \tilde{J}_1 = 0\,.} \tag{13}$$

Let us introduce the following partitions

$$U = \begin{pmatrix} U_1 & U_2 \end{pmatrix}, \quad V = \begin{pmatrix} V_1 & V_2 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix},$$

$$z(t) = \begin{pmatrix} z_a(t) \\ z_b(t) \end{pmatrix}, \quad \bar{z} = \begin{pmatrix} \bar{z}_a \\ \bar{z}_b \end{pmatrix}, \quad \bar{w} = \begin{pmatrix} \bar{w}_a \\ \bar{w}_b \end{pmatrix},$$

where $U_1 \in \mathbb{R}^{p\times k}$, $V_1 \in \mathbb{R}^{m\times k}$, $\Sigma_1 \in \mathbb{R}^{k\times k}$, $z_a(t) \in \mathbb{R}^k$, $\bar{z}_a \in \mathbb{R}^k$ and $\bar{w}_a \in \mathbb{R}^k$. Using these partitions, (5) become

$$\begin{pmatrix} V_1^T V_1 & V_1^T V_2 \\ V_2^T V_1 & V_2^T V_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \tag{14a}$$

$$V_1 V_1^T + V_2 V_2^T = I\,, \tag{14b}$$

$$\begin{pmatrix} U_1^T U_1 & U_1^T U_2 \\ U_2^T U_1 & U_2^T U_2 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}. \tag{14c}$$

Our performance index (12) can be rewritten as

$$\tilde{J}_1 = (\bar{w}_a - \bar{z}_a)^T \Sigma_1^2 (\bar{w}_a - \bar{z}_a)\,. \tag{15}$$

From (7) we have that

$$\bar{w}_a = \Sigma_1^{-1} U_1^T Q^{1/2} \bar{r}\,,$$

whereas from (8) we have

$$z_a(t) = \Sigma_1^{-1} U_1^T Q^{1/2} y(t)\,.$$

Finally making use of (1), (4) and of (14c) we have

$$\begin{aligned} \bar{z}_a &= \Sigma_1^{-1} U_1^T Q^{1/2} \bar{y} \\ &= \Sigma_1^{-1} U_1^T Q^{1/2} Q^{-1/2} U \Sigma V^T R^{1/2} \bar{u} = V_1^T R^{1/2} \bar{u}\,. \end{aligned} \tag{16}$$

Now let

$$u(t) = R^{-1/2} V_1 \tilde{u}(t)\,, \tag{17}$$

and

$$K(s) = R^{-1/2} V_1 \tilde{K}(s) \Sigma_1^{-1} U_1^T Q^{1/2}\,. \tag{18}$$

In this way we arrive to the feedback scheme of Figure 2.

Let us choose $\tilde{K}(s)$ in the form

$$\tilde{K}(s) = \tilde{K}_a(s) + \frac{\tilde{K}_b(s)}{s}\,, \qquad (19)$$

so that (18) becomes

$$K(s) = R^{-1/2} V_1 \left(\tilde{K}_a(s) + \frac{\tilde{K}_b(s)}{s} \right) \Sigma_1^{-1} U_1^T Q^{1/2}\,. \qquad (20)$$

In this way, provided that $K(s)$ defined in (20) internally stabilizes the closed-loop (2) system, the performance index (15) is equal to zero.

Now we can prove the following result.

Theorem 1. Any controller with the structure (20), provided that it internally stabilizes the closed-loop system (2), solves the optimization problem (13).

PROOF. The optimization problem (13) consists in finding the minimum of $\bar{u}^T R u$ with the constraint that $\tilde{J}_1 = 0$. It is easy to show by standard static optimization techniques that this minimum value, that must satisfy (16), is attained when

$$\bar{u} = R^{-1/2} V_1 \bar{w}_a\,.$$

On the other hand, using the controller structure (20) $\tilde{J}_1 = 0$ and from (16) and (17), using the fact that $V_1^T V_1 = I$ (see (14a)) we have

$$\bar{u} = R^{-1/2} V_1 \bar{z}_a\,.$$

The fact that $\bar{z}_a = \bar{w}_a$ (see (15)) completes the proof.

Now we need to design a stabilizing controller with the structure (18). Let us choose $\tilde{K}(s)$ (see (19)) in the simplified form

$$\tilde{K}(s) = K_P + \frac{K_I}{s}\,,$$

with $K_P\,, K_I \in \mathbb{R}^{k\times k}$. In order to find a convenient choice for K_P and K_I, let us evaluate the loop gain transfer matrix $F(s)$; using (14a) and (14c) we have

$$\begin{aligned} F(s) &= \Sigma_1^{-1} U_1^T Q^{1/2} P(s) R^{-1/2} V_1 \tilde{K}(s) \\ &= \Sigma_1^{-1} U_1^T Q^{1/2} \frac{Q^{-1/2} U \Sigma V^T R^{1/2}}{1+s\tau} R^{-1/2} V_1 \tilde{K}(s) \\ &= \Sigma_1^{-1} \begin{pmatrix} I & 0 \end{pmatrix} \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix} \frac{\tilde{K}(s)}{1+s\tau} \\ &= \frac{\tilde{K}(s)}{1+s\tau} \end{aligned}$$

Therefore exploiting the properties of the singular value decomposition, if we choose K_P and K_I as diagonal matrices

$$K_P = \mathrm{diag}(K_{P_1}, \dots K_{P_k})\,,\ K_I = \mathrm{diag}(K_{I_1}, \dots K_{I_k})\,,$$

we reduce our problem to k decoupled SISO problems (see Figure 3). Since we are controlling

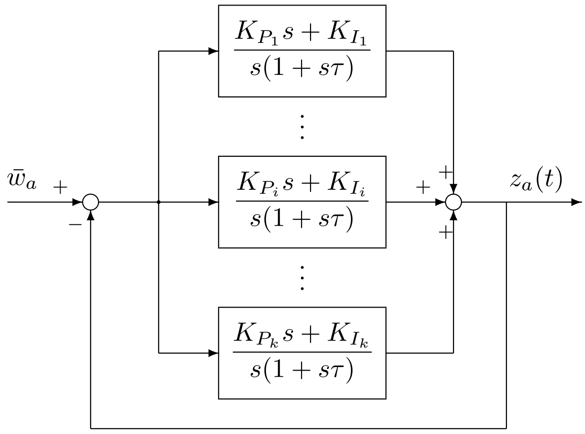

Fig. 3. The decoupled scheme for the PI design

k linear combinations of the output $y(t)$, the most reasonable choice is to let $K_P = k_p I$, $K_I = k_i I$. The values of the two scalars k_p and k_i have been chosen so as to assign to each SISO loop the behavior of a second-order system with a natural frequency of 25 rad/s and a damping factor of 0.7.

4. EXPERIMENTAL RESULTS

The XSC has been implemented at JET on a 400 MHz G4 PowerPC. The controller software architecture has been designed so that it allows to test all the software off-line, since at JET long commissioning periods are not available.

Tokamak reactors are pulsed machines; in each pulse the plasma is created, ramped up to the reference flat-top current, heated, maintained in a constant state and finally cooled down and terminated. The XSC has been designed to control the plasma shape during the flat-top phase, when the plasma current has a constant magnitude; as a matter of fact it has been used at JET also during large excursions of the plasma current: maintaining the plasma shape constant during such large excursions is very demanding, since all the plasma parameters are changing and the assumption that a single linearized model can describe the plasma behavior is no longer valid.

Hereafter we show the results obtained during the JET shot number 61995, where the XSC took control during the plasma flat-top at $t = 68$ s. Figure 4 shows the reference shape and the shape that has been obtained with the XSC: the shape is reached with a small error. Figure 5a shows the average value of the error on the 28 controlled gaps. As it can be seen, initially, when the XSC is switched on, the mean error is of about 4 cm, then the XSC reduces this error to about 1 cm; eventually the error slightly increases since the plasma current is changing significantly (Figure 5b). The XSC has been successfully used also in the presence of injection of heating power with the neutral beams; in these experiments, the maintenance of a constant shape is very important for the physicist to carry out their analyses.

5. CONCLUSIONS

In this paper we have described the eXtreme Shape Controller which has been recently designed and implemented on the JET tokamak. This new controller gives the possibility of controlling the plasma shape, specified in terms of some plasma-wall distances, and of maintaining it even in the presence of significant variations of critical plasma parameters. The design procedure is essentially based on the singular value decomposition of the plant output matrix. This procedure allows us to take into account all the different requirements, specified in terms of accuracy on the controlled variables and of maximum allowable control effort. The new controller has been fully commissioned on the JET tokamak; now it is in operation and it delivers the performance that were expected.

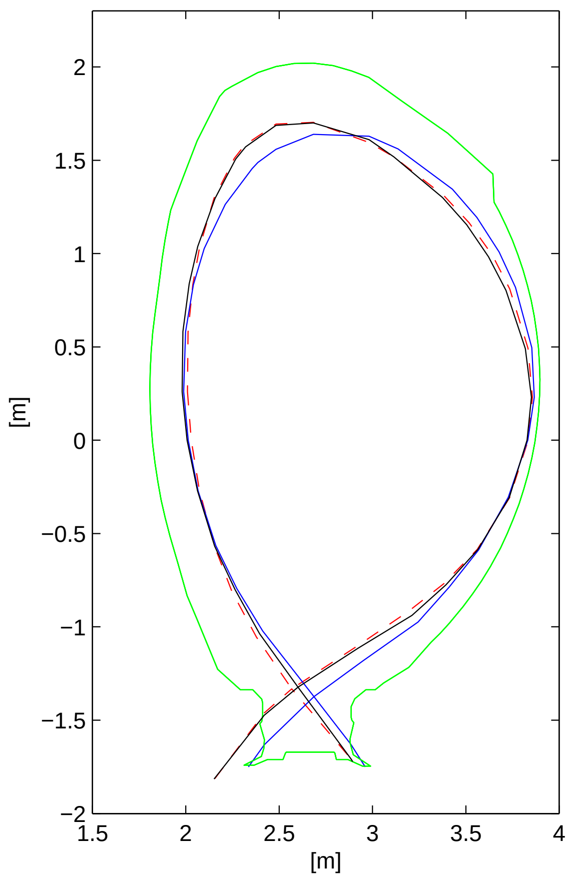

Fig. 4. Experimental test of the XSC during the shot #61995: *i*) the reference shape to be tracked (dashed in red); *ii*) the plasma boundary before the XSC is active (solid in blue); *iii*) the plasma boundary two seconds after the XSC takes control (solid in black). The vessel is shown in green.

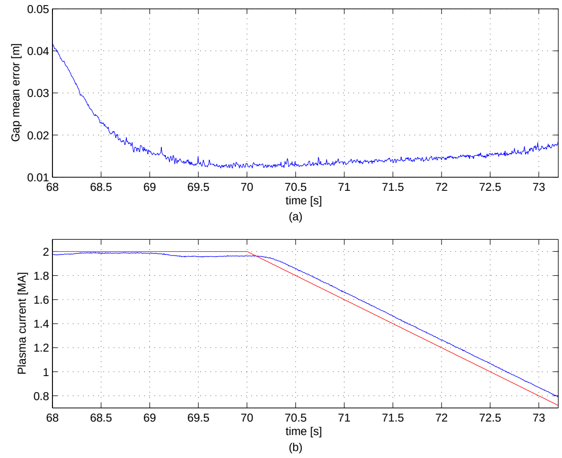

Fig. 5. Experimental test of the XSC during the shot #61995: (a) shows the average error on the controlled gaps when the XSC is active; (b) shows the plasma current I_p (blue) and its reference.

REFERENCES

Albanese, R. and F. Villone (1998). The Linearized CREATE-L Plasma Response Model for the Control of Current, Position and Shape in Tokamaks. *Nuclear Fusion* **38**(5), 723–738.

Ambrosino, G., M. Ariola and A. Pironti (2003). Optimal regulation for linear non right-invertible plants. In: *Proceedings of the 42nd IEEE Conference on Decision and Control*. Maui, Hawaii USA. pp. 281–286.

Ariola, M., G. Ambrosino, A. Pironti, J. B. Lister and P. Vyas (2001). Design and Experimental Testing of a Robust Multivariable Controller on a Tokamak. *IEEE Transactions on Control Systems Technology* **9**(6), 831–838.

Ariola, M., G. De Tommasi, A. Pironti and F. Sartori (2003). Controlling extremely shaped plasmas in the JET tokamak. In: *Proceedings of the 42nd IEEE Conference on Decision and Control*. Maui, Hawaii USA. pp. 4611–4615.

Crisanti, F. et al. (2003). Upgrade of the present JET shape and vertical stability controller. *Fusion Engineering and Design* **66–68**, 803–807.

Gossner, J. R., P. Vyas, B. Kouvaritakis and A. W. Morris (1999). Application of Cautious Stable Predictive Control to Vertical Positioning in COMPASS-D Tokamak. *IEEE Transactions on Control Systems Technology* **7**(5), 580–587.

Scibile, L. and B. Kouvaritakis (2001). A discrete adaptive near-time optimum control for the plasma vertical position in a Tokamak. *IEEE Transactions on Control Systems Technology* **9**(1), 148–162.

Vyas, P., A. W. Morris and D. Mustafa (1998). Vertical Position Control on COMPASS-D. *Fusion Technology* **33**(2), 97–105.