ADAPTIVE TRACKING CONTROL OF FUZZY TIME-DELAY SYSTEMS USING VARIABLE STRUCTURE CONTROL APPROACH

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Abstract: Based on fuzzy T-S model and variable structure control technique, adaptive output tracking control problem for fuzzy time-delay systems in the existence of parameter perturbations is developed. Firstly, the sliding mode was chosen by variable structure control theory. By applying the Lypunov stability theory, the design method of adaptive variable structure control is proposed. The use of adaptive technique is to overcome the unknown upper bound of perturbation so that the reaching condition can be satisfied. The method guaranteed the trajectory of system to arrive the slide surface in finite time interval and be kept here thereafter. Secondly, the sufficient condition for globally bound of the state is presented by using ISS theory and LMI method. Finally, simulation example is employed to illustrate the validity and effectiveness of the proposed method.

Keywords*:* Output tracking control; T-S fuzzy model; Adaptive control; Variable structure control; Robust control

INTRODUCTION

The robust stabilization and tracking control of nonlinear systems are important topics in the field of control theory and many approaches have been introduced to treat control problems in the past few decades. In general, tracking problems are more difficult than stabilization problems especially for nonlinear systems. For nonlinear system design, various control schemes are introduced including exact feedback linearization, adaptive control, siding model control and fuzzy control. The technique of exact feedback linearization needs perfect knowledge of the nonlinear system and uses that knowledge to cancel the nonlinearities of the system. However, as pointed out in [Ying H (1999)], the controller derived by feedback linearization may not be bounded, i.e. the controller is not guaranteed to be stable for no minimum phase systems. Since perfect knowledge of the system is impossible, exact feedback linearization seems impractical for nonlinear systems with uncertainty [Khalil H K, (1996, Slotine J J E (1992)]. Recently, based on feedback linearization technique, $H\infty$ adaptive fuzzy control schemes have been introduced to deal with nonlinear systems [Chen B S (1996),Chen B S (1998).]. However, the complicated parameter update law and control algorithm make this control scheme impractical. Especially in the case of considering the projection algorithm for the parameter update law can avoid the singularity of feedback linearization control. Variable structure system (VSS) control scheme has been developed as a popular robust strategy to treat uncertain control systems (see, for example, [Feng G (1995), Utkin V I (1978), Yoo D S, (1992)]. The main feature of such VSS control is its ability to deal with external disturbances, quickly varying parameters and unmodeled dynamics. However, it can’t avoid displaying chattering phenomena and exciting high-frequency unmodeled dynamics. So a conservative design may be obtained since the VSS control algorithm should be designed to treat the worst situation of uncertainties. On the other hand, Fuzzy control has recently found extensive applications for a wide variety of industrial systems and consumer products. It has attracted the attention of many control researchers due to its model free approach

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[Jadbabaie A (1998). Takagi T (1985)]. According to the universal approximation theorem, the T–S fuzzy model has been proved to be a very good representation for a certain class of nonlinear dynamic systems [Wang W J (1999)]. In their studies, a nonlinear plant was represented by a set of linear fuzzy models interpolated by membership . functions and then a model-based fuzzy controller was developed to stabilize the T-S fuzzy model. Currently, the stabilization problem for the system represented in T-S fuzzy model have been well addressed [Takagi T, (1985), Wang H O, (1996), Wang W J (1999)], but the studies concerning with tracking controller design based on T-S fuzzy model are relatively few. [Tseng C S, (2001)] proposed fuzzy tracking control design for nonlinear dynamic systems via T-S fuzzy model. However their work did only concern on the tracking control of nominal T-S fuzzy model without time delay and uncertainty. So the robustness of the whole tracking control system can't be guaranteed. In addition, dynamic systems with time delay are common in chemical processes and long transmission lines in pneumatic, hydraulic, or rolling mill systems. Nonlinear systems with time delay constitute basic mathematical models of real phenomena in biology, mechanics, economics, etc. Generally speaking, the dynamic behaviors of systems with delays are more complicated than that of systems without any delay. Fuzzy time-delay systems of T–S model provide a method of using local linear delayed systems combined with fuzzy linguistic descriptions to achieve nonlinearity.

Motivated by the aforementioned concerns, adaptive output tracking control problems for fuzzy time-delay systems in the existence of parameter perturbations was developed based on the combination of sliding mode control and fuzzy control techniques. The method conquers the deficiency of LMI theory and H_∞ theory on this problem. Furthermore, fuzzy control techniques can weaken immanent chattering phenomenon in the sliding mode control.

2. FUZZY TIME-DELAY SYSTEM FORMATION

A fuzzy dynamic model has been proposed by Takagi and Sugeno [Takagi T (1985)] to represent a nonlinear system. The T-S fuzzy model is a piecewise interpolation of several linear models through membership functions. The fuzzy model is described by fuzzy If-Then rules and will be employed here to deal with the control design problem for nonlinear time-delay systems. The i th rule of the fuzzy model for nonlinear systems is of the following form:

Plant Rule i:

If $\theta_1(t)$ is μ_{i1} and...and $\theta_p(t)$ is μ_{ip} , Then

$$\dot{x}(t) = (A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t - \tau(t)) + Bu \tag{1}$$

$$y = Cx(t), \quad (i = 1,2...r) \tag{2}$$

where μ_{ij} denotes fuzzy set; $x(t) \in R^n$ denotes state vector; $u(t) \in R^m$ denotes the control input; A_{1i}, A_{2i} denotes some constant matrices of compatible dimensions; $\tau(t)$ is time-delay in the state. It is assumed that $0 \le \tau \le d$, $\dot{\tau}(t) \le l < 1$ where d , l denote a constant; ΔA_{1i} , ΔA_{2i} denote the perturbations of system. It is assumed that the premise variables are independent of the input variables $u(t)$.

The overall fuzzy system is achieved by fuzzy blending of each individual rule as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))[(A_{1i} + \Delta A_{1i})x(t) + (A_{2i} + \Delta A_{2i})x(t - \tau(t)) + Bu(t)] \tag{3}$$

$$y = Cx(t) \tag{4}$$

where $\theta_i(t)$ $(i = 1,2.....p)$ are the premise variables; $\theta(t) = [\theta_1(t), \theta_2(t).......\theta_p(t)]$, $w_i(\theta(t)) = \prod_{j=1}^{r} \nu_{ij}(\theta_j(t))$, $h_i(\theta(t)) = \dfrac{w_i(\theta(t))}{\sum_{j=1}^{r} w_j(\theta(t))}$, $\nu_{ij}(\theta_j(t))$ denotes the grade of membership of $\theta_j(t)$. The following assumptions are made.

Assumption 1: The matrix CB is nonsingular.

Assumption 2: All the perturbations $\Delta A_{1i}, \Delta A_{2i}$ satisfy the following matching condition, there exist $\|\Delta \bar{A}_{1i}\| \le M_{\Delta A_1}$, $\|\Delta \bar{A}_{2i}\| \le M_{\Delta A_2}$, such that

$$\Delta A_{1i}(x) = B\Delta \bar{A}_{1i}, \Delta A_{2i}(x) = B\Delta \bar{A}_{2i},$$

where $M_{\Delta A_1}$, $M_{\Delta A_2}$ is known number.

Assumption 3: [Chou C H (2001)]**:** There exists unknown positive constant q_0, q_1 , such that

$$\|x(t - \tau(t))\| \le q_0 + q_1 \|x(t)\|$$

for $\tau(t) \in [0, d]$, d is a constant.

The object in this paper is to find a variable structure controller such that the output $y(t)$ of (3) will track a desired reference trajectory $y_r(t)$.

3. OUTPUT TRACKING CONTROL

In the following, we introduce the definition of ISS and a necessary and sufficient condition for ISS. Consider the general nonlinear system

$$\dot{x} = f(x,u) \tag{5}$$

where $x(t) \in R^n, u(t) \in R^m$, $f:\ R^n \times R^m \to R^n$ is continuous and satisfies $f(0,0) = 0$. According to [9], we have

Lemma1: System (5) is ISS if and only if there is a smooth function $V:\ r^n \to R_+$ such that there exist K_∞ function ν_1, ν_2 and K function ν_3, ν_4 , such that

$$\nu_1(\|\xi\|) \le V(\xi) \le \nu_2(\|\xi\|) \qquad \forall \xi \in R^n$$

$\dot{V}(\xi) \le -\nu_3(\|\xi\|)$, so that $V(\xi) \ge \nu_4(\|\xi\|)$

A function γ: $\mathrm{R}_+ \to \mathrm{R}_+$ is called K function if it is continuous, strictly increasing and $\gamma(0)=0$; if γ further satisfies $\lim_{t\to\infty}\gamma(t)=\infty$, it is called K_∞ function.

The sliding mode was selected for system (3) by variable structure control theory:

$$s(t)=(CB)^{-1}(y(t)-y_r(t))$$

Defining tracking error

$$e(t)=y(t)-y_r(t)$$

Then, the following variable structure controller is:

$$\begin{aligned}u(t)=&-\sum_{i=1}^r h_i(\theta(t))(CB)^{-1}CA_{1i}x(t)-[\sum_{i=1}^r h_i\left\|(CB)^{-1}CA_{2i}\right\|\\&(\hat{q}_0+\hat{q}_1\|x(t)\|)+(\hat{q}_1(t)M_{\Delta A_2}+M_{\Delta A_1})\|x(t)\|+\\&\hat{q}_0(t)M_{\Delta A_2}]\operatorname{sgn}s-\alpha_1 s-\alpha_2\operatorname{sgn}s+(CB)^{-1}\dot{y}_r(t)\end{aligned} \tag{6}$$

where α_1, α_2 are two positive numbers, the parameter adaptive laws are designed as

$$\dot{\hat{q}}_0(t)=\sum_{i=1}^r h_i\left\|s^T(CB)^{-1}CA_{2i}\right\|,$$

$$\dot{\hat{q}}_1(t)=\sum_{i=1}^r h_i\left\|s^T(CB)^{-1}CA_{2i}x(t)\right\| \tag{7}$$

Substituting the controller (6) into (3), yields

$$\begin{aligned}\dot{x}(t)=&\sum_{i=1}^r h_i(\theta(t))[(A_{1i}-B(CB)^{-1}CA_{1i})x(t)+A_{2i}x(t-\tau)+\\&B(\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau))]-B[(\sum_{i=1}^r h_i\left\|(CB)^{-1}CA_{2i}\right\|\\&(\hat{q}_0(t)+\hat{q}_1(t)\|x(t)\|)+(\hat{q}_1(t)M_{\Delta A_2}+M_{\Delta A_1})\|x(t)\|+\\&\hat{q}_0(t)M_{\Delta A_2})\operatorname{sgn}s+\alpha_1 s+\alpha_2\operatorname{sgn}s-(CB)^{-1}\dot{y}_r(t)]\end{aligned}$$

$$y=Cx(t) \tag{8}$$

Since matrix C is of full row rank, a nonsingular matrix T_1 can be found such that

$$CT_1=[0\ \ \bar{C}_2]=\bar{C},$$

where $\bar{C}_2$ is nonsingular. Let $T_1^{-1}B=\begin{bmatrix}\bar{B}_1 & \bar{B}_2\end{bmatrix}^T=\bar{B}$, where $\bar{B}_1\in R^{(n-m)\times m}$, $\bar{B}_2\in R^{m\times m}$, since $CB=\bar{C}\bar{B}=\bar{C}_2\bar{B}_2$ it follows that matrix $\bar{B}_2$ is nonsingular. Define

$$T_2=\begin{bmatrix}I & -\bar{B}_1\bar{B}_2^{-1}\\ 0 & \bar{C}_2\end{bmatrix},\ T_0=T_2T_1^{-1}=\begin{bmatrix}T_{01}\\ T_{02}\end{bmatrix}$$

where $T_{01}=\begin{bmatrix}I & -\bar{B}_1\bar{B}_2^{-1}\end{bmatrix}T_1^{-1}$, $T_{02}=\begin{bmatrix}0 & \bar{C}_2\end{bmatrix}T_1^{-1}$. It shows that $T_{02}=C$, $T_{01}B=0$. Let $T_0^{-1}=\begin{bmatrix}T_{0inv1} & T_{0inv2}\end{bmatrix}$, where $T_{0inv1}\in R^{(n-m)\times m}$ and $T_{0inv2}\in R^{n\times m}$. For closed-loop system (8), we can obtain that

Theorem 1: Consider (3) satisfy assumptions 1-3, suppose that there are positive-define matrix P and R, such that the following:

$$\begin{pmatrix}PN_{1i}+N_{1i}^{\mathrm{T}}P+\frac{1}{(1-l)}R & PT_{01}A_{2i}T_{0inv1}\\ T_{0inv1}^{\mathrm{T}}A_{2i}^{\mathrm{T}}T_{01}^{\mathrm{T}}P & -R\end{pmatrix}<0$$

holds, where $N_{1i}=T_{01}A_{1i}T_{0inv1}$, the variable structure controller(6) with the adaptive laws(7) will make output of the closed-loop system (8) track reference signal $y_r(t)$.

Proof: the proof is divided into two parts. In part a), we show that output of (3) can follow exactly the desired signal $y_r(t)$ after a finite time interval and obtain adaptive laws. In part b), we show that the state of (3) is bounded globally.

a) The derivative of $s(t)$ along the trajectory of closed-loop system (7) is

$$\begin{aligned}\dot{s}(t)=&(CB)^{-1}C\dot{x}(t)-(CB)^{-1}\dot{y}_r(t)\\=&\sum_{i=1}^r h_i(\theta(t))(CB)^{-1}C[A_{1i}x(t)+A_{2i}x(t-\tau)]+u(t)+\\&\sum_{i=1}^r h_i(\theta(t))[\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau)]-(CB)^{-1}\dot{y}_r(t)\\=&\sum_{i=1}^r h_i(\theta(t))(CB)^{-1}CA_{2i}x(t-\tau)-\\&[\sum_{i=1}^r h_i\left\|(CB)^{-1}CA_{2i}\right\|(\hat{q}_0(t)+\hat{q}_1(t)\|x(t)\|)+\\&(\hat{q}_1(t)M_{\Delta A_2}+M_{\Delta A_1})\|x(t)\|+\ \hat{q}_0(t)M_{\Delta A_2}]\operatorname{sgn}s+\\&\sum_{i=1}^r h_i(\theta(t))[\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau)]-\\&\alpha_1 s-\alpha_2\operatorname{sgn}s\end{aligned}$$

If $s_j>0$, we hold

$$\begin{aligned}\dot{s}_j&=-\alpha_1 s_j-\alpha_2-M_{\Delta A_1}\|x(t)\|-M_{\Delta A_2}\|x(t-\tau)\|\\&+(\sum_{i=1}^r h_i(\theta(t))[\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau)])_j\\&\le-\alpha_1 s_j-\alpha_2-M_{\Delta A_1}\|x(t)\|-M_{\Delta A_2}\|x(t-\tau)\|\\&+\left|(\sum_{i=1}^r h_i(\theta(t))[\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau)])_j\right|\\&\le-\alpha_1 s_j-\alpha_2-M_{\Delta A_1}\|x(t)\|-M_{\Delta A_2}\|x(t-\tau)\|\\&+\sum_{i=1}^r h_i(\theta(t))\left\|\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau)\right\|\\&\le-\alpha_1 s_j-\alpha_2-M_{\Delta A_1}\|x(t)\|-M_{\Delta A_2}\|x(t-\tau)\|\\&+\sum_{i=1}^r h_i(\theta(t))(M_{\Delta A_1}\|x(t)\|+M_{\Delta A_2}\|x(t-\tau)\|)\\&=-\alpha_1 s_j-\alpha_2\end{aligned}$$

if $s_j<0$, we hold $\dot{s}_j\ge-\alpha_1 s_j+\alpha_2$ From above, it can be seen that all s_j will arrive at zero in finite time interval and kept here thereafter.

The adaptation error of each gain is defined as

$$\tilde{q}_0(t)=\hat{q}_0(t)-q_0,\ \ \tilde{q}_1(t)=\hat{q}_1(t)-q_1$$

Consider a Lypunov candidate function given by

$$V(t)=\tfrac{1}{2}(s^Ts+\tilde{q}_0^2(t)+\tilde{q}_1^2(t))$$

The derivative of $V(t)$ along the trajectory of system (8) is

$$\dot{V}(t)=s^T\dot{s}+\tilde{q}_0(t)\dot{\tilde{q}}_0(t)+\tilde{q}_1(t)\dot{\tilde{q}}_1(t)$$
$$\le\|s^T\|\sum_{i=1}^{r}h_i(\theta(t))\|(CB)^{-1}CA_{2i}\|(q_0(t)+q_1(t)\|x(t)\|)-$$
$$[\sum_{i=1}^{r}h_i\|(CB)^{-1}CA_{2i}\|(\hat{q}_0(t)+\hat{q}_0(t)M_{\Delta A_2})+$$
$$\sum_{i=1}^{r}h_i\|(CB)^{-1}CA_{2i}\|(\hat{q}_1(t)+\hat{q}_1(t)M_{\Delta A_2}+$$
$$M_{\Delta A_1})\|x(t)\|]\sum_{i=1}^{r}|s_i|-\alpha_1 s^T s-\alpha_2\sum_{i=1}^{m}|s_i|+$$
$$(\hat{q}_0-q_0)\dot{\hat{q}}_0(t)+(\hat{q}_1-q_1)\dot{\hat{q}}_1(t)$$

from (7), we obtain

$$\dot{V}(t)\le-\sum_{i=1}^{r}h_i(\theta(t))\|(CB)^{-1}CA_{2i}\|\hat{q}_0(t)(\sum_{i=1}^{m}(|s_i|-\|s^T\|))-$$
$$[(M_{\Delta A_2}\hat{q}_0(t)+\hat{q}_1(t)M_{\Delta A_2}+M_{\Delta A_1})\|x(t)\|]\sum_{i=1}^{r}|s_i|-\alpha_1 s^T s-$$
$$\alpha_2\sum_{i=1}^{m}|s_i|<0$$

from above, we show that output of (3) can follow exactly the desired signal $y_r(t)$ after a finite time interval and obtain adaptive laws simultaneously.

b) The state of system (3) is bounded globally.

Under the following state transformation:

$$z=\begin{bmatrix}z_1\\z_2\end{bmatrix}=T_0x(t)=\begin{bmatrix}T_{01}x\\T_{02}x\end{bmatrix}$$

we obtain

$$\dot{z}_1(t)=\sum_{i=1}^{r}h_i(\theta(t))[T_{01}A_{1i}T_{0inv1}z_1(t)+T_{01}A_{1i}T_{0inv2}z_2(t)+$$
$$T_{01}A_{2i}T_{0inv1}z_1(t-\tau)+T_{01}A_{2i}T_{0inv2}z_2(t-\tau)] \qquad (9)$$
$$\dot{z}_2(t)=\sum_{i=1}^{r}h_i(\theta(t))[CB(\Delta\bar{A}_{1i}x(t)+\Delta\bar{A}_{2i}x(t-\tau))]-CB[\alpha_1 s+$$
$$\alpha_2\operatorname{sgn}s+(M_{\Delta A_1}\|x(t)\|+M_{\Delta A_2}\|x(t-\tau)\|)\operatorname{sgn}s-(CB)^{-1}\dot{y}_r(t)] \qquad (10)$$

Since $z_2=T_{02}x=Cx=y$, notice that in the previous argument, when $t\ge t_r$, $y(t)=y_r(t)$. So our attention can be concentrated in (9).

Now view $z_2(t)$ as the input of (9) and choose an ISS-Lyapunov function candidate as

$$V(z_1(t))=z_1^{\mathrm{T}}Pz_1+\tfrac{1}{(1-l)}\int_{t-\tau}^{t}z_1^{\mathrm{T}}(s)Rz_1(s)ds$$

The derivative of $V(z_1(t))$ along the trajectory of system (9) is

$$\dot{V}(z_1)=\sum_{i=1}^{r}h_i(\theta(t))z_1^T(t)(PN_{1i}+N_{1i}^TP)z_1(t)+2\sum_{i=1}^{r}h_i(\theta(t))z_1^T(t)\times$$
$$T_{01}A_{1i}T_{0inv2}z_2(t)+2\sum_{i=1}^{r}h_i(\theta(t))z_1^T(t)PT_{01}A_{2i}T_{0inv1}z_1(t-\tau)$$
$$+2\sum_{i=1}^{r}h_i(\theta(t))z_1^T(t)PT_{01}A_{2i}T_{0inv2}z_2(t-\tau)+\tfrac{1}{(1-l)}z_1^T(t)Rz_1(t)$$
$$-z_1^T(t-\tau)Rz_1(t-\tau)$$
$$=\sum_{i=1}^{r}h_i(\theta(t))\begin{bmatrix}z_1^T(t) & z_1^T(t-\tau)\end{bmatrix}\begin{pmatrix}PN_{1i}+N_{1i}^{\mathrm{T}}+\tfrac{1}{(1-l)}R & PT_{01}A_{2i}T_{0inv1}\\ T_{0inv1}^{\mathrm{T}}A_{2i}^{\mathrm{T}}T_{01}^{\mathrm{T}}P & -R\end{pmatrix}\times$$
$$\begin{bmatrix}z_1(t)\\z_1(t-\tau)\end{bmatrix}+2\sum_{i=1}^{r}h_i(\theta(t))z_1^T(t)PT_{01}A_{1i}T_{0inv2}z_2(t)+$$
$$2\sum_{i=1}^{r}h_i(\theta(t))z_1^T(t)PT_{01}A_{2i}T_{0inv2}z_2(t-\tau)$$

Defining

$$W_i=\begin{pmatrix}PN_{1i}+N_{1i}^{\mathrm{T}}P+\tfrac{1}{(1-l)}R & PT_{01}A_{2i}T_{0inv1}\\ T_{0inv1}^{\mathrm{T}}A_{2i}^{\mathrm{T}}T_{01}^{\mathrm{T}}P & -R\end{pmatrix}$$

let $\lambda=\min\{\lambda_{\min}(-W_i),1,2,\cdots r\}$, $\lambda_{\min}(\cdot)$ denote the minimum eigenvalue of concerned matrix, we have

$$\dot{V}(z_1)\le-\lambda(\|z_1\|^2+\|z_1(t-\tau)\|^2)+\gamma_1\|z_1\|\|z_2\|+$$
$$\gamma_2\|z_1\|\|z_2(t-\tau)\|$$

where γ_1, γ_2 are two positive numbers. From assumption 3, There exists known number q_0, q_1, such that

$\|z(t-\tau(t))\|\le q_0+q_1\|z(t)\|$, so we obtain

$$\dot{V}(z_1)\le-(\lambda+\lambda q_1^2)\|z_1\|^2+(\gamma_1+\gamma_2q_1)\|z_1\|\|z_2\|-\lambda q_0^2+$$
$$(-2\lambda q_0q_1+\gamma_2q_0)\|z_1\|$$
$$<-(\lambda+\lambda q_1^2)\|z_1\|^2+(\gamma_1+\gamma_2q_1)\|z_1\|\|z_2\|+$$
$$(-2\lambda q_0q_1+\gamma_2q_0)\|z_1\|$$
$$=-\tfrac{1}{2}\beta_1\|z_1\|^2+(-\tfrac{1}{2}\beta_1\|z_1\|^2+\beta_2\|z_1\|\|z_2\|+\beta_3\|z_1\|)$$

where

$\beta_1=\lambda+\lambda q_1^2$, $\beta_2=\gamma_1+\gamma_2q_1$, $\beta_3=-2\lambda q_0q_1+\gamma_2q_0$, when $\|z_1\|\ge\dfrac{2\beta_2\|z_2\|+2\beta_3}{\beta_1}$, such that $\dot{V}(z_1)\le-\tfrac{1}{2}\beta_1\|z_1\|^2$

According to lemma1, when $t\ge t_r$, the system(9) is ISS. This completes the proof.

4. SIMULATION EXAMPLE

Consider a continuous stirred tank reactor (CSTR), according to [Cao Y Y (2000)], the material and energy balance equation are

$$V\frac{dA}{dt}=\lambda qA_0+q(1-\lambda)A(t-\alpha)-qA(t)-VK_0\exp(\frac{-E}{RT(t)})A(t)$$
$$VC_\rho\frac{dA}{dt}=qC_\rho[\lambda T_0+(1-\lambda)T(t-\alpha)-T(t)]-$$
$$V(-\Delta H)K_0\exp(\frac{-E}{RT(t)})A(t)-U(T(t)-T_\omega)$$

By transformation, we have

$$\dot{x}_1(t)=f_1(x)+(\tfrac{1}{\lambda}-1)x_1(t-\tau(t))$$
$$\dot{x}_2(t)=f_2(x)+(\tfrac{1}{\lambda}-1)x_2(t-\tau(t))+\beta u(t)$$

where

$x(t)=\begin{bmatrix}x_1(t) & x_2(t)\end{bmatrix}^T$, $x_1(t)=\dfrac{A_0-A(t)}{A_0}$,

$x_2(t)=\dfrac{T(t)-T_0}{T_0}(\dfrac{-E}{RT(t)})$, $\theta_1(t)=\dfrac{A_0-\varphi_1(t)}{A_0}$,

$\theta_2(t)=\dfrac{\varphi_2(t)-T_0}{T_0}(\dfrac{-E}{RT(t)})$, $\upsilon=\dfrac{V}{q\lambda}$, $\tau=\dfrac{\alpha}{\upsilon}$,

$\gamma_0=\dfrac{E}{RT(t)}$,

$$f_1(x) = -{}^{1}\!/_{\lambda}\, x_1(t) + D_\alpha (1 - x_1(t)) \exp\left(\frac{x_2(t)}{1 + x_2(t)/\gamma_0}\right),$$

$$f_2(x) = -({}^{1}\!/_{\lambda} + \beta) x_2(t) + HD_\alpha (1 - x_1(t)) \exp\left(\frac{x_2(t)}{1 + x_2(t)/\gamma_0}\right)$$

.

The parameters are given as

$$\gamma_0 = 20;\ H = 8;\ \beta = 0.3;\ D_\alpha = 0.072;\ \lambda = 0.8;\ \tau = 2.$$

Choose a steady-state as $x_e(t) = [0.1440;\ 0.8862]$, defining a new state variable

$$\delta x_1 = x_1 - x_e(1),\ \delta x_2 = x_2 - x_e(2)$$

As in [Cao Y Y (2000)], we present the following fuzzy control law

Rule 1: If δx_2 is small (i.e. δx_2 is -1) Then

$$\delta\dot{x}(t) = (A_{11} + \Delta A_{11})\delta x(t) + (A_{21} + \Delta A_{21})\delta x(t - \tau(t)) + B\delta u$$

Rule 2: If δx_2 is middle (i.e. δx_2 is 0) Then

$$\delta\dot{x}(t) = (A_{12} + \Delta A_{12})\delta x(t) + (A_{22} + \Delta A_{22})\delta x(t - \tau(t)) + B\delta u$$

Rule 3: If δx_2 is large (i.e. δx_2 is 1)Then

$$\delta\dot{x}(t) = (A_{13} + \Delta A_{13})\delta x(t) + (A_{23} + \Delta A_{23})\delta x(t - \tau(t)) + B\delta u$$

where

$$B = \begin{bmatrix} 0 & \beta \end{bmatrix}^T,\ A_{1j} = \begin{bmatrix} a_{11j} & a_{12j} \end{bmatrix}^T,$$

$$a_{1ij} = \frac{\partial f_i}{\partial x}(\delta x_j) + \frac{\partial f_i(\delta x_j) - (\delta x_j)^T \frac{\partial f_i}{\partial x}(\delta x_j)}{\left\|\delta x_j\right\|^2}\delta x_j$$

$$(i = 1, 2),\ (j = 1, 2, 3)$$

From above, we obtain

$$A_{11} = \begin{bmatrix} -1.4274 & 0.0757 \\ -1.4189 & -0.9442 \end{bmatrix},\ A_{12} = \begin{bmatrix} -2.0508 & 0.3958 \\ -6.4066 & 1.6168 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} -4.5279 & 0.3167 \\ -26.2228 & 0.9837 \end{bmatrix},\qquad B = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix},$$

$$A_{21} = A_{22} = A_{23} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\Delta A_{11} = \begin{bmatrix} 0 & 0 \\ -1.4189 & -0.9442 \end{bmatrix},$$

$$\Delta A_{12} = \begin{bmatrix} 0 & 0 \\ -1.4189 & -0.9442 \end{bmatrix}$$

$$\Delta A_{13} = \begin{bmatrix} 0 & 0 \\ -1.4189 & -0.9442 \end{bmatrix},$$

$$\Delta A_{21} = \Delta A_{22} = \Delta A_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0.25 \end{bmatrix}.$$

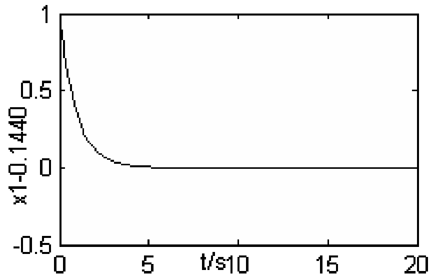
The membership function are selected as

$$\alpha_1 = \frac{\exp(\frac{-(\theta+1)^2}{\sigma^2})}{\exp(\frac{-(\theta+1)^2}{\sigma^2}) + \exp(\frac{-\theta^2}{\sigma^2}) + \exp(\frac{-(\theta-1)^2}{\sigma^2})},$$

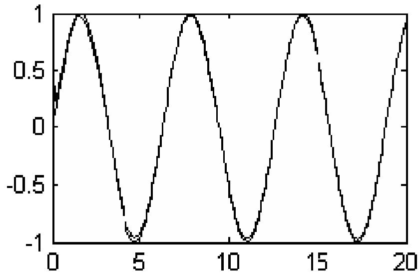
$$\alpha_2 = \frac{\exp(\frac{-\theta^2}{\sigma^2})}{\exp(\frac{-(\theta+1)^2}{\sigma^2}) + \exp(\frac{-\theta^2}{\sigma^2}) + \exp(\frac{-(\theta-1)^2}{\sigma^2})}\partial,$$

$$\alpha_3 = \frac{\exp(\frac{-(\theta-1)^2}{\sigma^2})}{\exp(\frac{-(\theta+1)^2}{\sigma^2}) + \exp(\frac{-\theta^2}{\sigma^2}) + \exp(\frac{-(\theta-1)^2}{\sigma^2})}.$$

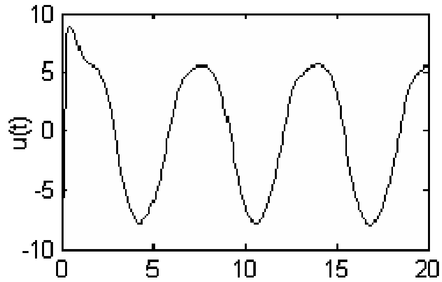
Let $y(t) = \sin(t)$, $\delta x(0) = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$, the trajectory of system are shown in Fig1.



(a) The trajectory of δx_1

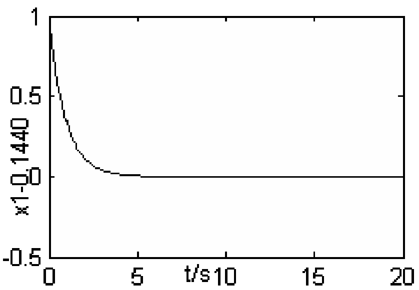


(b) The trajectory of output tracking



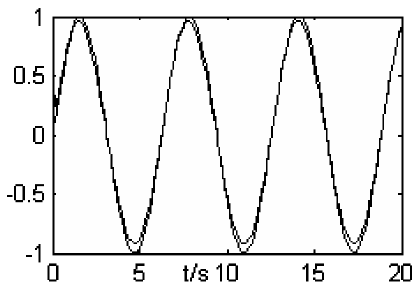
(c) Response of system control

Fig1 The response of system with adaptive variable structure controller (6)

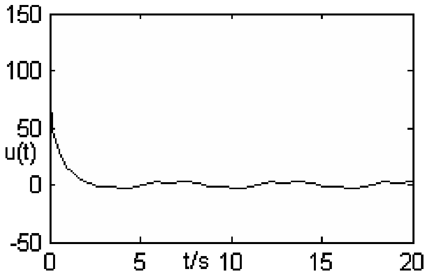


,

(a) The trajectory of δx_1



(b) The trajectory of output tracking



(c) The response of system control

Fig2 The response of system with conventional variable structure controller

From Fig 1, it can be seen that the control techniques proposed in this paper guarantee globally bound of the state and the output of this system possesses well tracking performance. Comparing Fig 1 with Fig 2, our control techniques can track the reference signal more accurately and weaken chattering phenomenon more effectively than conventional control technique.

5. CONCLUSION

Based on fuzzy T-S model and variable structure control technique, adaptive output tracking control problem for fuzzy time-delay systems in the existence of parameter perturbations was developed. The method conquers the deficiency of LMI theory and H_∞ theory in this field. Furthermore, fuzzy control techniques can weaken immanent chattering phenomenon in the sliding mode control.

REFERENCES

Cao, Y. Y. and P. M. Frank (2000). Analysis and Synthesis of Nonlinear Time-Delay Systems via Fuzzy Control Approach. *IEEE Trans on Fuzzy Systems*. Vol. 8 (2) 200–211.

Chen, B. S., C. H. Lee and Y. C. Chang (1996). $H\infty$ tracking design of uncertain nonlinear SISO systems: Adaptive fuzzy approach, *IEEE Trans on Fuzzy Systems*. Vol. 4 (1) 32–43.

Chen, B. S., H. J. Uang and C. S. Tseng (1998). Robust tracking enhancement of robot systems including motor dynamics: A fuzzy-based dynamic game approach. *IEEE Trans on Fuzzy Systems*. Vol.6(2) 538–552.

Chou, C. H. and C. C. Cheng (2001). Design of adaptive variable structure controllers for perturbed time-varying state delay systems, Journal of the Franklin Institute. Vol. 338, 35-46.

Feng, G. and Y. A. Jiang (1995). Variable structure based decentralized adaptive control, *IEEE Proc. Control Theory*. Vol. 142 (5) 439-443.

Jadbabaie, A., M. Jamshidi and A. Titli (1998). Guaranteed-cost design of continuous-time Takagi-Sugeno fuzzy controllers via linear matrix inequalities, In IEEE World Congress Computational Intelligence, Anchorage, AK. 5(2) 268–273.

Khalil, H. K. Nonlinear Systems, Englewood Cliffs, NJ: Prentice-Hall, 1996.

Slotine, J. J. E and W. Li (1992). Applied Nonlinear Control, Englewood Cliffs, NJ: Prentice-Hall, 51(3) 249–258.

Sontag, E. D. and Y. Wang (1995). On characterization of input-to-state stability property, System and control letter. Vol.34, 351-359.

Takagi, T. and M. Sugeno (1985). Fuzzy identification of systems and its application to modeling and control, IEEE Trans on Systems, Man and Cybernetics. Vol.15 (1), 116 - 132.

Tseng, C. S., B. S. Chen and H. J. Uang (2001). Fuzzy tracking control design for nonlinear dynamic systems via T–S fuzzy model, *IEEE Trans on Fuzzy Systems*. Vol. 9 (3), 381–392.

Utkin, V. I. Sliding Modes and their Application in Variable Structure Systems, MIR, Moscow, 1978.

Wang, H. O., K. Tanaka and M. F. Griffin (1996). An approach to fuzzy control of nonlinear systems: Stability and design issues, *IEEE Trans on Fuzzy Systems*. Vol.4 (1) 14–23.

Wang, W. J. and H. R. Lin (1999). Fuzzy control design for the trajectory tracking on uncertain nonlinear systems. *IEEE Trans on Fuzzy Systems*. Vol.7(1), 53–62.

Ying, H (1999). Analytical analysis and feedback linearization tracking control of the general Takagi-Sugeno fuzzy dynamical systems, *IEEE Trans on Systems. Man and Cybernetics.* Vol. 29(2) 290–298.

Yoo, D. S. and M. J. Chung (1992). Variable structure control with simple adaptation laws for upper bounds on the norm of the uncertainties, *IEEE Trans on Automatic Control*. Vol. 37 (6) 860-86.