STABILITY ANALYSIS OF SWITCHED LINEAR STOCHASTIC SYSTEMS WITH UNKNOWN SWITCHINGS

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Abstract: This paper is concerned with the stabilization problem of switched linear stochastic systems with unknown switchings. The system switching among a finite set of linear stochastic systems is considered. Since there are noise perturbations, the switching laws can not be exactly identified in a finite time horizon, we prove that if each individual subsystem is controllable and the switching duration uniformly has a strict positive lower bound, then the system can be stabilized by using on-line identification method and suitable designed controller.

Keywords: Switched systems, stability, stabilization, estimation.

1. INTRODUCTION

Consider the following switched linear stochastic system

$$dx(t) = A_{\theta_t}x(t)dt + B_{\theta_t}u(t)dt + F_{\theta_t}dw_t, \qquad (1)$$
$$E\|x(0)\|^2 < \infty$$

where $x(t) \in \mathbb{R}^n$ is the completely observable system state and $u(t) \in \mathbb{R}^r$ is the input. The $\{w_t\}$ is an l -dimensional standard Wiener process, which is independent of $\{x(s), s \leq t\}$. A_{θ_t}, B_{θ_t}, F_{θ_t} are coefficient matrices with suitable dimensions. The switching law $\theta_t : [0, \infty) \to \Theta$ is a piecewise constant function of time. In this paper, θ_t can not be directly observed and is independent of $\{x(s), s \leq t, w_t\}$. The set $\Theta \triangleq \{1, 2, \ldots, N\}$.

Switched systems are frequently encountered in practice, e.g., power systems (Sira-Ranirez, 1991), (Williams and Hoft, 1991), robot manipulators, traffic management (Varaiya, 1993), etc. Numerous results on switched systems have been achieved in recent years.

The stability analysis of switched systems is started with different premises. When the switching law is Markov process, this always been called as Markov jump systems. For this kind systems the stability analysis can be traced back to the work of Rosenbloom (Rosenbloom, 1954) in 1954. And then many researchers contributed to this thesis, e.g., (Kozin, 1969) and (Leizarowitz, 1990), etc. When θ_t is observable, the necessary and sufficient conditions for mean-square stabilization have been obtained by (Bouhtouti and Hadri, 2003) and (Fang and Loparo, 2002) for continuous-time deterministic and stochastic jump systems respectively. While θ_t is unobservable, some sufficient conditions for stabilization at the sense of average quadratic index for continuous-time jump stochastic systems have been given (cf. (Caines and Zhang, 1995)).

For the other switching laws, the study for stability (or stabilizability) can be classified into two

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types. One is to study the conditions under which the systems can be stabilized for any switch, for example, (Liberzon *et al.*, 1999) , (Molchanov and Pyatnitskiy, 1989), (Narendra and Balakrishman, 1994) and (Shim *et al.*, 1998), etc. The another is to find the restrictions of switching laws under which the given family of subsystems can maintain stability (or stabilizability). For deterministic systems, some researchers assume the systems have a finite switching frequency in a finite time horizon (e.g., (Peleties and Decarlo, 1991)). Some others suppose the switching duration is large enough (e.g. (Morse, 1996), (Zhai *et al.*, 2001)). For stochastic systems, using the concept of average dwell time put forward by (Hespanha and Morse, 1999), (Feng and Zhang, 2004) studies stability of disturbed switched systems with observable switching laws, they derive the conclusion that when the ratio of the total time the system dwelling on stable subsystems to that on unstable subsystems is not smaller than some given constant, the switched linear system can be stabilized though there exists unstable subsystems. When the switching laws are unobservable, (Guo *et al.*, 2004) shows that the switched deterministic system can be stabilized if each individual subsystem is controllable and the dwell time is uniformly positive.

This paper studies the stabilizability of (1) with unobservable switching laws. Since the switching law can no be directly unobserved, we ought to identify the parameters before designing the stabilizer. Comparing with the deterministic system, the influence of random noise retards us to identify the accurate state of the parameters in any finite duration. Therefore the following stabilizing control may proceed under wrong parameters. In this article we study the effects of misestimate, and conclude that the stabilization of the switched stochastic linear system with unknown system is possible.

The rest of the paper is organized as follows. In Section 2 the definitions of dwell time and mean-square stabilization, as well as the assumptions of this paper are introduced. The method of system identification and its probability of veracity are depicted in Section 3. And the main results are given in Section 4. The last section concludes the paper with some remarks and the prospects for the future work.

2. DEFINITIONS AND ASSUMPTIONS

First we will introduce the concept of dwell time.

Definition 1. The dwell time of a switching law, denoted by $\tau(\omega)$, is defined as $\tau(\omega) \triangleq \inf_k\{t_k(\omega) - t_{k-1}(\omega)\}$, where $t_k(\omega) \triangleq \inf\{t : t > t_{k-1}(\omega), \theta_t \neq \theta_{t_{k-1}}\}$, and ω represents sample trajectory.

The dwell time $\tau(\omega)$ restricts the class of admissible switching signals to signals with the property that the interval between any two consecutive switching times is no smaller than $\tau(\omega)$. There are many definitions for stabilizability(Feng *et al.*, 1992). This paper investigates the mean-square stabilizability of the systems given as follows.

Definition 2. Switched linear stochastic system (1) is mean-square stabilized, if for any $x_0 \in \mathbb{R}^n$ and any $\theta_0 \in \Theta$, there exists a feedback control $u(t)$ such that

$$\overline{\lim_{t\to\infty}} E\|x_t\|^2 < \infty. \tag{2}$$

Before stating our assumptions we display some properties of the system. For the finite set of gain matrices $\{F_1, F_2, \dots, F_N\}$, there always exists a constant σ such that $\|F(\cdot)\| \le \sigma$ uniformly with θ_t, where $\|\cdot\|$ denotes the operator norm induced by the Euclidean norm on $\mathbb{R}^n$. That is, for any $\alpha \in \mathbb{R}^l$, $\|F\|^2 \triangleq \max_{||\alpha||=1} \|F\alpha\|_2^2$.

Throughout the paper, we hold the following two assumptions:

H1) Each pare of $[A(i), B(i)]$ is completely controllable, $i \in \Theta$.

H2) Switching instants are available and $\tau \triangleq \inf_\omega \tau(\omega) > 0$.

Remark 1. The assumption **H2)** means for any θ_t, the time interval between any two consecutive switching times is no smaller than a strictly positive constant τ. This assumption can not be eliminated. Because for Markov jump processes, which don't satisfy this assumption, there are examples (see, e.g., Example 3.1 of (Fang and Loparo, 2002)) showing that even a deterministic linear switched system with observable parameters with H1) can not be mean-square stabilized.

3. IDENTIFICATION OF SWITCHING SIGNALS

To design a feedback stabilizer, we have to know which subsystem the switched system is dwelling on. So we need to estimate the unobservable parameter θ_t. Consider a time interval $[k\tau, (k+1)\tau)$, $k \in \{0, 1, 2, \dots\}$. The system will switch, if it does, only once by the definition of τ. θ_t will be estimated when the system reaches the moments $k\tau$, $(k+1)\tau$ and the switching time, denoted by t_{k_τ}. A identification process will expense a length of h time or be interrupted by a switching event, where $0 < h < \tau/2$. In the rest of $[k\tau, (k+1)\tau)$, $k \in \{0, 1, 2, \dots\}$, we will design a feedback control to stabilize the identified system .

Now we encounter a problem that the estimation usually does not equal to the true value of the parameter because there exists disturbance. In what follows, we will show that under certain class of excitation signals, the probability $P\{\hat{\theta}_{t_0} = \theta_{t_0}\}$ will tend to 1 as $\|x(t_0)\|$ going to infinity, where $\hat{\theta}_{t_0}$ is the estimate of θ_{t_0} given in (5), $t_0 = k\tau, t_{k_\tau},\ k = 0, 1, 2, \cdots$.

Let $C^{(n)}[0, h]$ be the space of $\mathbb{R}^r$-valued functions defined on [0,h], which have continuous derivatives up to order n. $\forall u \in C^{(n)}[0, h]$, let us denote $U(t)$ as

$$U(t) = (u(t), u^{(1)}(t), \cdots, u^{(n)}(t))^T. \quad (3)$$

Consider the following class of functions

$$U_0 = \{u \in C^{(n)}[0, h] | U(0) = 0, \\ \lambda_{min}(\int_0^h U(t)U(t)^T dt) > 0\} \quad (4)$$

where $\lambda_{min}(\cdot)$ denotes the minimum eigenvalue of a square matrix. U_0 is not empty (cf. The appendix of (Guo *et al.*, 2004)). The excitation signal can be taken as any fixed function in U_0, denoted as $u^0(t)$.

Let $\phi(t) = (x(t)^T, u(t)^T)^T$ and $H_i = (A_i, B_i)^T$. The estimate $\hat{\theta}_{t_0}$ for θ_{t_0} is given as

$$\arg \min_{1\le i\le N} \| \int_{t_0}^{t_0+h} [\phi(s)dx(s)^T - \phi(s)\phi(s)^T ds H_i^T]\|, \quad (5)$$

where $t_0 = k\tau, t_{k_\tau},\ k = 0, 1, 2, \cdots$,

$$u(s) = \beta\|x(t_0)\|u^0(s - t_0), \quad s \in [t_0, t_0 + h), \quad (6)$$

$\beta > 0$ is a constant such that $\beta \ge 2\eta_0\sqrt{\frac{h}{b_1}}$,

$$\eta_0 = \max_{1\le i\le N} \max_{0\le t\le h} \|e^{A_i t}\|, \quad (7)$$

and b_1 is constant depending on $(A_i, B_i), 1 \le i \le N, h$ and $u^0(t)$.

Let $c = max\{c', 1\}$, and

$$\overline{a} = \max_{i\ne j} \|H_i - H_j\|,\ \underline{a} = \min_{i\ne j} \|H_i - H_j\|, \quad (8)$$

where

$$c' = \max_{\substack{1\le i\le N\\ 0\le t\le h}} (\|e^{A_i t}\| + \beta\| \int_0^t e^{A_i(t-s)} B_i u^0(s)ds\|). \quad (9)$$

Let $\hat{x}(s)$ be the solution of the deterministic counterpart of (1), i.e.,

$$d\hat{x}(s) = A(\theta_s)\hat{x}(s)ds + B(\theta_s)u(s)ds,\ \hat{x}(t_0) = x(t_0). \quad (10)$$

Denote $\hat{\phi}(t) = (\hat{x}(t)^T, u(t)^T)^T$. Noticing that (10) is deterministic, the following lemma given in (Guo *et al.*, 2004) also holds here.

Lemma 1. Consider switched linear system (10). For a t_0, if the system does not switch on $[t_0, t_0 + h)$, then under the control defined in (6),

$$\lambda_{min}\{ \int_{t_0}^{t_0+h} \hat{\phi}(s)\hat{\phi}(s)^T ds\} \\ \ge \{\frac{1}{2} b_1\beta^2 - h\eta_0^2\}\|x(t_0)\|^2, \quad (11)$$

The estimate of the probability of $\hat{\theta}_t = \theta_t$ is given by the following lemma.

Lemma 2. Consider switched linear stochastic system (1). For a t_0, if the system does not switch on $[t_0, t_0 + h)$, then under the control defined in (6), we have

$$P\{\hat{\theta}_{t_0} \ne \theta_{t_0}\} \le \min\{1, a\|x(t_0)\|^{-1} + b\|x(t_0)\|^{-2}\}, \quad (12)$$

where $\hat{\theta}_t$ is given in (5) and a, b are two positive constants.

Proof. From (1) and (5) we have

$$\int_{t_0}^{t_0+h} \phi(s)dx(s)^T - \int_{t_0}^{t_0+h} \phi(s)\phi(s)^T ds H_i^T \\ = \int_{t_0}^{t_0+h} [\phi(s)\phi(s)^T ds(H - H_i) + \phi(s)dw_s^T F^T] \\ \triangleq \Phi(H_i), \quad (13)$$

where H_{θ_t}, F_{θ_t} denote as H, F respectively for short.

Since $P\{\hat{\theta}_{t_0} \ne \theta_{t_0}\} = \sum_{i\ne\theta_{t_0}} P\{\hat{\theta}_{t_0} = i\}$, we calculate $P\{\hat{\theta}_{t_0} = i\}$, $i \in \Theta \setminus \{\theta_{t_0}\}$ separately.

By (5) and (13), we have

$$P\{\hat{\theta}_{t_0} = i\} \le P\{\|\Phi(H_i)\| \le \min_{j\ne i}\{\|\Phi(H_j)\|\}. \quad (14)$$

Since $i \ne \theta_{t_0}$, this implies that

$$\min_{j\ne i} \|\Phi(H_j)\| \le \| \int_{t_0}^{t_0+h} \phi(s)dw_s^T F^T\|,$$

which yields

$$P\{\hat{\theta}_{t_0} = i\} \le P\{\|\Phi(H_i)\| \le \| \int_{t_0}^{t_0+h} \phi(s)dw_s^T F^T\|\}.$$

Then by

$$\|\Phi(H_i)\| \ge \| \int_{t_0}^{t_0+h} \phi(s)\phi(s)^T dt(H - H_i)\| \\ - \| \int_{t_0}^{t_0+h} \phi(s)dw_s^T F^T\|,$$

it follows that

$$P\{\hat{\theta}_t = i\} \le P\{2\| \int_{t_0}^{t_0+h} \phi(s)dw_s^T F^T\| \\ \ge \| \int_{t_0}^{t_0+h} \phi(s)\phi(s)^T ds(H - H_i)\|\}. \quad (15)$$

We decompose $x(t)$ on $[t_0, t_0+h)$ as $x(t) = \hat{x}(t) + \tilde{x}(t)$, in which $\hat{x}(t)$ is defined in (10), and set

$$\int_{t_0}^{t_0+h} \phi(s)\phi(s)^T ds(H - H_i) = \Psi_1 + \Psi_2 + \Psi_3, \tag{16}$$

where

$$\Psi_1 = \int_{t_0}^{t_0+h} \hat{\phi}(s)\hat{\phi}(s)^T ds(H - H_i),$$
$$\Psi_2 = \int_{t_0}^{t_0+h} \tilde{x}(s)\tilde{x}(s)^T ds(A^T - A_i^T),$$
$$\Psi_3 = \int_{t_0}^{t_0+h} [\hat{\phi}(s)\tilde{x}(s)^T + \tilde{x}(s)\hat{\phi}(s)^T] ds(H - H_i).$$

By Lemma 1 and (8) we have

$$\begin{aligned} ||\Psi_1|| &\geq \lambda_{min}\{\| \int_{t_0}^{t_0+h} \hat{\phi}(s)\hat{\phi}(s)^T ds\|\}\|(H - H_i)\| \\ &\geq \underline{a}(\frac{1}{2}b_1\beta^2 - h\eta_0^2)\|x(t_0)\|^2. \end{aligned} \tag{17}$$

By (10) and (1) we have

$$\tilde{x}(t) = \int_{t_0}^{t} e^{A(t-s)} F dw_s.$$

Then

$$\begin{aligned} E\|\tilde{x}(t)\|^2 &= E\| \int_{t_0}^{t} e^{A(t-s)} F dw_s\|^2 \\ &= \int_{t_0}^{t} tr(F^T e^{A^T(t-s)} e^{A(t-s)} F) ds \\ &\leq \int_{t_0}^{t} l\|e^{A(t-s)}F\|^2 ds \leq \sigma^2\eta_0^2 hl, \end{aligned} \tag{18}$$

where for the last inequality is from (7). Thus

$$\begin{aligned} E\{||\Psi_2||\} &\leq \int_{t_0}^{t_0+h} E\|\tilde{x}(s)\tilde{x}(s)^T\| ds(A^T - A_i^T) \\ &\leq \bar{a}\int_{t_0}^{t_0+h} E\|\tilde{x}(s)\|^2 ds \leq \bar{a}\sigma^2\eta_0^2 h^2 l. \end{aligned} \tag{19}$$

Since

$$\begin{aligned} E\|\hat{\phi}(t)\tilde{x}(t)^T\| &= E\|\tilde{x}(t)\begin{pmatrix}\hat{x}(t)^T & u(t)^T\end{pmatrix}\| \\ &\leq (\|\hat{x}(t)\| + \|u(t)\|)E\|\tilde{x}(t)\|. \end{aligned} \tag{20}$$

By (18) and Schwarz inequality, we have

$$E\|\tilde{x}(t)\| \leq (E\|\tilde{x}(t)\|^2)^{\frac{1}{2}} \leq \eta_0\sigma h^{\frac{1}{2}} l^{\frac{1}{2}}.$$

And by using c in (9) and u_t in (6),

$$\begin{aligned} \|\hat{x}(t)\| &= \|e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-s)} Bu_s ds\| \\ &\leq c\|x(t_0)\|. \end{aligned}$$

These imply that the right hand side of (20) is less than

$$\begin{aligned} &(\|\hat{x(t)}\| + \|u(t)\|)(E\|\tilde{x}(t)\|^2)^{\frac{1}{2}} \\ &\leq \Big(c\|x(t_0)\| + \beta\|x(t_0)\| \cdot \|u^0(t - t_0)\|\Big)\eta_0\sigma h^{\frac{1}{2}} l^{\frac{1}{2}} \\ &\leq \eta_0\eta_1\sigma h^{\frac{1}{2}} l^{\frac{1}{2}}\|x(t_0)\|, \end{aligned} \tag{21}$$

where $\eta_1 = c + \beta \max_{0\leq t\leq h} \|u^0(t)\|$. Therefore,

$$E\{||\Psi_3||\} \leq 2\bar{a}\eta_0\eta_1\sigma h^{\frac{3}{2}} l^{\frac{1}{2}}\|x(t_0)\|.$$

Analogously, noting that

$$\begin{aligned} &E\| \int_{t_0}^{t_0+h} \phi(s) dw_s^T F^T\| \\ &\leq \|F\|\Big(E\| \int_{t_0}^{t_0+h} \phi(s) dw_s^T\|^2\Big)^{\frac{1}{2}} \\ &\leq \sigma \int_{t_0}^{t_0+h} (\|\hat{x}(s)\| + \|u(s)\| + E\|\tilde{x}(s)\|)^2 ds \\ &\leq \sigma^2\eta_0 hl^{\frac{1}{2}} + \sigma\eta_1 h^{\frac{1}{2}}\|x(t_0)\|. \end{aligned} \tag{22}$$

and from (17), (19) and (22), and combining with Chebyshev inequality, we know that for $\|x(t_0)\| \geq 0$,

$$\begin{aligned} &P\{\hat{\theta}_{t_0} = i\} \leq P\{\|\Psi_1\| \\ &\leq P\{\| \int_{t_0}^{t_0+h} \phi(s) dw_s^T F^T\| \geq \frac{1}{6}\|\Psi_1\|\} \\ &+ P\{\|\Psi_2\| \geq \frac{1}{3}\|\Psi_1\|\} + P\{\|\Psi_3\| \geq \frac{1}{3}\|\Psi_1\|\} \\ &= a\|x(t_0)\|^{-1} + b\|x(t_0)\|^{-2}, \end{aligned} \tag{23}$$

where

$$a = \frac{6\sigma\eta_1 h^{\frac{1}{2}}(1 + \bar{a}\eta_0 hl^{\frac{1}{2}})}{\underline{a}(\frac{1}{2}b_1\beta^2 - h\eta_0^2)},$$
$$b = \frac{3\sigma^2 h\eta_0(2l^{\frac{1}{2}} + \bar{a}\eta_0 hl)}{\underline{a}(\frac{1}{2}b_1\beta^2 - h\eta_0^2)}.$$

It is obvious that $P\{\hat{\theta}_{t_0} = i\} \leq 1$, this implies the assertion of Lemma. ∎

4. MAIN RESULT

To obtain the main result of this paper, we will use the squashing lemma given in (Cheng *et al.*, 2004):

Lemma 3. Let $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times r}$ be matrices such that the pair (A, B) is completely controllable , then $\forall\lambda > 0$, there always exists a matrix $K \in \mathbb{R}^{r\times n}$ such that

$$\|e^{(A+BK)t}\| \leq M\lambda^L e^{-\lambda t}, \ \forall t \geq 0 \tag{24}$$

where $L = (n-1)(n+2)/2$, $M > 0$ are constants depending only on A, B and n .

Denote

$$\Lambda \triangleq \{\lambda : \log 32c + \max(\log M\lambda^L, 0) < \frac{1}{2}(\tau - 2h)\lambda\}$$

where ϵ is a given fixed small positive constant, and where η_2 is defined as

$$\eta_2 = \max_{1\leq i,j\leq N} \max_{0\leq t\leq \tau-h} \|e^{(A_i+B_iK_j)t}\|. \tag{25}$$

We have the following Theorem.

Theorem 1. If assumptions **H1)** and **H2)** hold, then the switched linear stochastic system (1) is mean-square stabilized.

Proof. Consider the system's behavior in the time interval $[k\tau, (k+1)\tau)$, $k \in \{0, 1, 2, \dots\}$. We will calculate the conditional square moment $E(\|x((k+1)\tau)\|^2 | x(k\tau))$.

First, for the identification processes during $[t_0, t_0 + \delta)$, $t_0 = k\tau$ or t_{k_τ}, $0 < \delta \le h$, by (1) and (6),

$$\begin{aligned} & E(\|x(t_0+\delta)\|^2 | x(t_0)) \\ & \le \Big(\|e^{A\delta}\| + \beta \| \int_0^\delta e^{A(\delta - s)} B u^0(s) ds \|\Big)^2 \|x(t_0)\|^2 \\ & \quad + \int_0^\delta l \|e^{A(\delta-s)} F\|^2 ds \\ & \le c^2 \|x(t_0)\|^2 + \eta_0^2 \sigma^2 h l \le c^2 \|x(t_0)\|^2 + c_1, \end{aligned} \quad (26)$$

where A, B and F denote the true coefficients of the system, c is defined in (9), η_0 is given by (7), and $c_1 = \max\{\eta_0^2 \sigma^2 \tau l, \eta_2^2 \sigma^2 \tau l\}$.

Second, for the stabilization process during $[t_*, t_* + \mu)$, $t_*, t_* + \mu \in [k\tau, (k+1)\tau)$, by Lemma 3 there are $\lambda \in \Lambda$, and corresponding K_i such that for each of $\{(A_i, B_i)\}$ (24) is satisfied. Then let $K = K(\hat{\theta}_{t_*})$, we have

$$\begin{aligned} & E\Big(\|x(t_* + \mu)\|^2 \Big| x(t_*)\Big) \\ & = \sum_{i=1}^N E\Big(\|x(t_*+\mu)\|^2 \Big| \hat{\theta}_{t_*} = i, x(t_*)\Big) \cdot P\{\hat{\theta}_{t_*} = i\} \\ & \le E\Big(\|x(t_*+\mu)\|^2 \Big| \hat{\theta}_{t_*} = \theta_{t_*}, x(t_*)\Big) \\ & + \sum_{i \ne \theta_{t_*}} E\Big(\|x(t_*+\mu)\|^2 \ \Big| \hat{\theta}_{t_*} = i, x(t_*)\Big) \\ & \cdot P\{\hat{\theta}_{t_*} = i\}. \end{aligned} \quad (27)$$

If $\hat{\theta}_{t_*} = \theta_{t_*}$, then

$$\begin{aligned} & E\Big(\|x(t_*+\mu)\|^2 \Big| \hat{\theta}_{t_*} = \theta_{t_*}, x(t_*)\Big) \\ & \le \|e^{(A+BK)\mu} x(t_*)\|^2 + E \int_0^\mu l \|e^{(A+BK)(\mu - s)} F\|^2 ds \\ & \le (M\lambda^L e^{-\lambda\mu})^2 \|x(t_*)\|^2 + \frac{l}{2\lambda} (\sigma M \lambda^L)^2 (1 - e^{-2\lambda\mu}). \end{aligned} \quad (28)$$

Otherwise,

$$\begin{aligned} & \sum_{i \ne \theta_{t_*}} E(\|x(t_*+\mu)\|^2 | \hat{\theta}_{t_*} = i, x(t_*)) \\ & = \sum_{i \ne \theta_{t_*}} E\{||e^{(A+BK_i)\mu} x(t_*) \\ & \qquad + \int_0^\mu e^{(A+BK_i)(\mu - s)} F dw_s||^2 \Big| x(t_*)\} \\ & \le (N-1)\eta_2^2 \{\|x(t_*)\|^2 + l\sigma^2 \mu\} \\ & \le (N-1)(\eta_2^2 \|x(t_*)\|^2 + c_1), \end{aligned} \quad (29)$$

where η_2 is given in (25).

From (26)-(29) and Lemma 2, after an identification process $[t_0, t_0 + h)$ followed with a stabilization process $[t_0 + h, t_1)$,

$$\begin{aligned} & E(\|x(t_1)\|^2 | x(t_0)) \\ & \quad \le [2cM\lambda^L e^{-\lambda(t_1 - t_0 - h)}]^2 \|x(t_0)\|^2 + C_1, \end{aligned} \quad (30)$$

where C_1 is a constant.

$E(\|x(k\tau+\tau)\|^2 | x(k\tau))$ is estimated in the following three situations.

1. For the situation that the system does not switch on the time interval $[k\tau, k\tau + \tau)$,

$$\begin{aligned} & E\big(\|x(k\tau+\tau)\|^2 \big| x(k\tau)\big) \\ & \quad \le 4c^2 (M\lambda^L)^2 e^{-2\lambda(\tau - h)} \|x(k\tau)\|^2 + C_1. \end{aligned} \quad (31)$$

2. For the situation that system switches one time on $[k\tau, k\tau + \tau)$ and $t_{k_\tau} - k\tau > h$, $(k+1)\tau - t_{k_\tau} > h$, by using (30) twice, we obtain

$$\begin{aligned} & E(\|x(k\tau+\tau)\|^2 | x(k\tau)) \\ & \quad \le 32 c^4 (M\lambda^L)^4 e^{-2\lambda(\tau - 2h)} \|x(k\tau)\|^2 + C_2 \end{aligned} \quad (32)$$

where C_2 is a positive constant.

3. For the situation that the system switches once on $[k\tau, (k+1)\tau)$ and $t_{k_\tau} - k\tau \le h$, or $k\tau + \tau - t_{k_\tau} \le h$, using (26) and (30), we have

$$\begin{aligned} & E(\|x(k\tau+\tau)\|^2 | x(k\tau)) \\ & \quad \le 8c^3 (M\lambda^L)^2 e^{-2\lambda(\tau - 2h)} \|x(k\tau)\|^2 + C_3, \end{aligned} \quad (33)$$

where C_3 is a positive constant.

From (31)-(33), it follows that

$$E\big(\|x(k\tau+\tau)\|^2 \big| x(k\tau)\big) \le \alpha_\lambda^2 \|x(k\tau)\|^2 + C, \quad (34)$$

where

$$\begin{aligned} & \alpha_\lambda = 32[c \max(M\lambda^L, 1)]^4 e^{-2\lambda(\tau - 2h)}, \\ & C = \max\{C_1, C_2, C_3\}. \end{aligned}$$

Since α_λ and C are independent of k, by iterating the inequality (34), we have

$$E\big(\|x(k\tau)\|^2 \big| x(0)\big) \le \alpha_\lambda^{2k} \|x(0)\|^2 + \frac{1 - \alpha_\lambda^{2k}}{1 - \alpha_\lambda^2} C.$$

Taking expectations on both sides, it follows that

$$E\|x(k\tau)\|^2 \le \alpha_\lambda^{2k} E\|x(0)\|^2 + \frac{1 - \alpha_\lambda^{2k}}{1 - \alpha_\lambda^2} C.$$

It can be inferred from the definition of Λ that $\alpha_\lambda < 1$. From the initial condition $E\|x(0)\|^2 < \infty$, and letting $k \to \infty$, it is easy to see that

$$\overline{\lim_{k \to \infty}} E\|x(k\tau)\|^2 \le \frac{C}{1 - \alpha_\lambda^2} < \infty. \quad (35)$$

As for any $t \in [0, \infty)$, there always exists $k \in \{1, 2, \dots\}$ such that $[k\tau, k\tau + \tau)$ covers t. By applying the inequalities (26) and (30) we obtain

$$E\big(\|x(t)\|^2 | x(k\tau)\big) \le c^4 \eta_2^4 \|x(k\tau)\|^2 + C_0,$$

where $C_0 = c^2 \eta_2^4 c_1 + c^2 \eta_2^2 c_1 + \eta_2^2 c_1 + c_1$.

Therefore

$$\overline{\lim_{t\to\infty}} E\|x(t)\|^2 \le \frac{c^4\eta_2^4 C}{1-\alpha_\lambda^2} + C_0 < \infty.$$

This completes the proof of Theorem. ■

5. CONCLUDING REMARKS

This paper has studied the stabilization problem of switched linear stochastic systems with unknown switchings but known switching instants. Our main assumptions are that each individual subsystem is controllable and the dwell time has a positive lower bound. Comparing with the corresponding results in deterministic systems, the difficulty is that the parameters can not be exactly identified within finite time. So the effect of misestimate would be considered during the analysis. We proved that the system is mean-square stabilized by using on-line estimation method and suitable designed linear feedback controllers. In this paper, the switching instants are available and the identification and control are two separated procedures based on this information, it is better to make these two purposes with a unified input for practicing application, this is a future research topic. Many researches such as (Fang and Loparo, 2002) and (Guo *et al.*, 2004) have shown that the conditions for almost surely stability are differ with that required for mean square stability. So this is also a further research topic to consider the condition for almost surely stability in stochastic systems.

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